Nonparametric dual control algorithm for discrete linear dynamic systems

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Abstract

The article devotes to the problem of controlling discrete linear dynamic systems under non-parametric uncertainty. Control action are calculated in which the difference equation degree of a dynamic process model is refined based on the rule of selection of significant variables. The computational experiments confirmed the efficiency of using non-parametric algorithms to control dynamic systems in comparison with the PID algorithm and the quasi-optimal control system.

Keywords: non-parametric algorithm, discrete dynamic system, priory information.

Introduction

Designing intelligent systems for controlling dynamic objects is one of the important task of system analysis. Previously, algorithms of control dynamic objects were developed, in particular, the most widely used standard control algorithms. In some cases, their use is not effective enough. Into contemporary scientific approaches, optimal control algorithms are used. However, for their application, as a rule, a priori knowledge of the structure and parameters of the controlled object is necessary.

In the conditions when there is no prior information, the development of new control algorithms is a significant scientific problem. One of the ways to solve this task to use non-parametric methods [1]. For the application of non-parametric methods, it is necessary to know only about the quality characteristics of the object under study.

1 Dual control

Dual control was suggested by A.A. Feldbaum [2] and developed on the basis of the theory of statistical solutions. The theory of dual control was further developed in the studies of various authors [3], in particular B. Wittenmark [4]. It should be noted that a system in which dual control algorithms are used is an adaptive system, because as current information is received from an object, the quality of functioning increases.

2 Problem set-up and algorithm

The paper considers classes of control objects that can be described by linear difference equations of the form (1).

$$x_t = F(x_{t-1}, ..., x_{t-k}, u_t, \xi_t). \tag{1}$$

where F is an unknown linear functional, k is the degree of a difference equation, which is limited $k \leq k_{max}$. The input u_t and output x_t of a dynamic object are represented by measurements that form a sample of the form $x_i, u_i, i = \overline{1, s}$, where s is the sample size, u_i, x_i are the measurements of the input and output of the object at a time instant t_i.

For a dynamic object that can be described by difference equation (1), the control problem is to find the control functions u_t. The control function translates the output of an object x_t to a specified value x_t^* in some finite time t_p. In this case, the functional F is assumed to be unknown from a priori information, but there is a sample of observations $x_i, u_i, i = \overline{1, s}$. Non-parametric dual control algorithm has the following form (2) [5]:

$$u_{s+1} = u_s^* + \Delta u_{s+1}, \tag{2}$$

where u_s^* is the component accumulating information about the object under study, $\Delta u_{s+1} = \varepsilon(x_{s+1}^* - x_s)$ is the "studying" search step. The dual control scheme is shown in Figure 1.

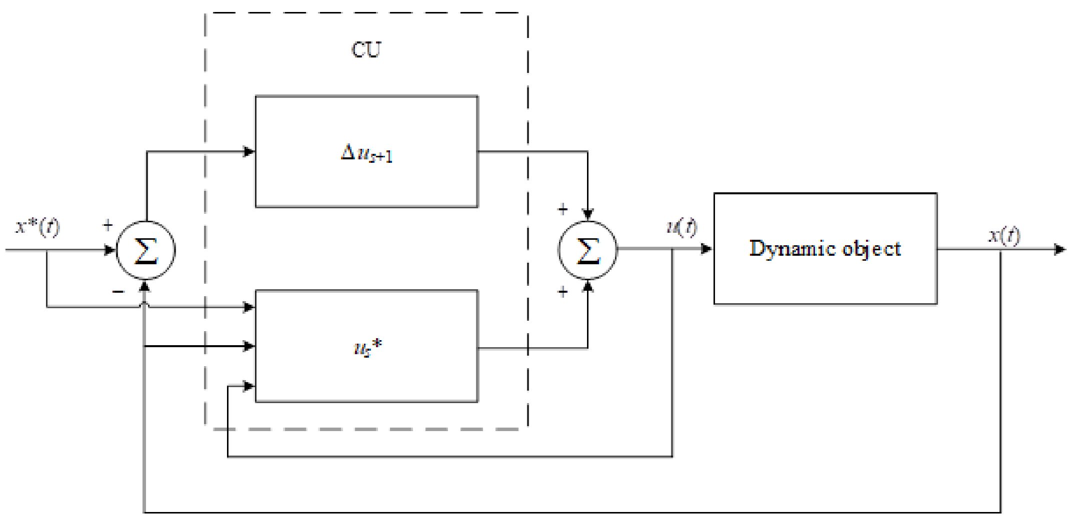

Figure 1: Dual control scheme of a dynamic object

In this case, we use the following estimation to get the value u_s^* from equation (3):

$$u_s^* = \frac{\sum_{i=1}^{s} u_i \cdot \Phi\left(\frac{x_{s+1}^* - x_i}{c_s}\right) \cdot \prod_{j=1}^{k}\left(\frac{x_{s-j} - x_{i-j}}{c_s}\right)}{\sum_{i=1}^{s} \Phi\left(\frac{x_{s+1}^* - x_i}{c_s}\right) \cdot \prod_{j=1}^{k}\left(\frac{x_{s-j} - x_{i-j}}{c_s}\right)}, \tag{3}$$

where u_s^* is a kernel function, c_s^u, $c_s^{x[j]}$ are bandwidths. The optimal bandwidths are found by minimizing a quadratic error function by using the sliding exam method.

3 Essential variables

The control algorithm for dynamic systems is constructed as follows. The differential equation degree of the dynamic process model k is determined on the bases of the rule of selection of essential variables. The vale k is further used in the calculation of control actions in (3), where only selected variables are present.

Formulation of the rule: in formula (3), each variable $x_{s-1}, ..., x_{s-k}$ is assigned its own bandwidths $c_s^{x[1]}, ..., c_s^{x[j]}$, the greater bandwidths, the less influence this variable has on the output of the object.

The algorithm for calculating significant variables x_{i-j} is based on the following scheme. First, the initial value of k is given. The model is constructed by equation.

$$x_s^t = \frac{\sum_{i=1}^{s} x_i \cdot \Phi\left(\frac{u_s - u_i}{c_s^u}\right) \cdot \prod_{j=1}^{k}\left(\frac{x_{s-j} - x_{i-j}}{c_s}\right)}{\sum_{i=1}^{s} \Phi\left(\frac{u_s - u_i}{c_s}\right) \cdot \prod_{j=1}^{k}\left(\frac{x_{s-j} - x_{i-j}}{c_s}\right)}, \tag{4}$$

and the relative error W_0 is calculated:

$$W = \sqrt{\frac{1}{s}\sum_{i=1}^{s}\left(x_i - x_i^s\right)^2 / \sum_{i=1}^{s}\frac{1}{s-1}\left(m_x - x_i\right)^2} \tag{5}$$

where m_x is an expected value.

For each i - th iteration, the following set of actions is performed:

1. For each coefficient $c_s^{x^1}, ..., c_s^{x^k}$ the optimal value is found: $c_s^{x^1} = c_s^{*x^1}$, $c_s^{x^2} = c_s^{*x^2}$,...,$c_s^{x^k} = c_s^{*x^k}$;
2. The maximum of all the values obtained is found: $c_{max_s}^{x^j}$;
3. The model is constructed by the equation (4). The multiplier $\Phi\left(\frac{u_s - u_i}{c_s^u}\right)$ is excluded, taking into account that j is a number for $c_{max_s}^{x^j}$.
4. A relative error W_i is calculated.

These actions will be repeated until $W_i \geq W_{i-1}$.

We choose a non-gradient multidimensional optimization the Nelder-Mead method as an optimization algorithm, since this method is effective at a low speed of calculation of the minimized function. To select the initial vertices of a deformable polyhedron, a region of possible values of bandwidths of the kernel functions was set, from which $n+k+1$ points were chosen arbitrarily, where n is the number of input variables, k is the degree of the difference equation, which form the simplex $n+k$.

In the case of the relation of an object to a class of linear, the algorithm allows one to determine the structure of the model with an accuracy of parameters.

4 Computational experiment

To illustrate the performance of the proposed algorithm, an example will be considered.The results of controlling a dynamic object using a non-parametric dual control algorithm (3) were compared with the control results of a typical control algorithm (PID), and with the results of using an quasi-optimal control system (criterion is control time).

The control quality was estimated by the control time (t_p) —the time from the beginning of the control to the moment when the output quantity differs from of no more than some given value α. ($\alpha = 5\%$). As an example, we give the work of three control algorithms. The control object is a series connection of three aperiodic links. Detailed control results are shown in table.

Table 1: Comparison results of the non-parametric dual control algorithm (NDCA) with the PID controller and the quasi-optimal control system (QOCS)

	Type of control systems	t_p
1	PID	7.9
2	NDCA	8.1 (at the stage of information accumulation) 1.2 (after passing the stage of accumulation of information)
3	QOCS	3.5

The dualism of the algorithm (3) is as follows. At the first control cycles, the main role in the formation of control actions is played by the term Δu_{s+1} from formula (3). But with the accumulation of information about the object, the role of the term u_s^* increases. Thus, the use of the non-parametric algorithm after passing through the stage of accumulation of information alows it possible to reduce the control time under equal conditions of noise and sample size compared to a typical PID controller and a quasi-optimal control system.

Conclusions

Nonparametric dual control algorithms for dynamic objects have been developed. A distinctive feature of the algorithms is the use of information about the order of a difference equation of a dynamic object when calculating control actions. The task of controlling dynamic objects is most effectively solved by the proposed algorithm, as compared with typical control algorithms, in particular, the PID algorithm and the quasi-optimal control system.

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