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Influence of the Thermophysical Properties of a Liquid Coolant on Characteristics of the 3D Flows with Phase Transition

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Regimes of the joint flows of the evaporating liquid and gas – vapor mixture induced by the action of a longitudinal temperature gradient in a three-dimensional channel of a rectangular cross-section in the terrestrial gravity field are studied in the present paper. The theoretical investigations are carried out on the basis of the partially invariant solution of rank 2 and defect 3 of the Boussinesq approximation of the Navier – Stokes equations. This solution allows one to correctly describe the two-layer flows with evaporation/condensation at the thermocapillary interface and to take into account the effects of thermodiffusion and diffusive thermal conductivity in the gas–vapor phase. The exact solution of governing equations are characterized by dependence of the velocity components on the transverse coordinates only. The functions of pressure, temperature and concentration of vapor linearly depend on the longitudinal coordinate and have the summands which are functions on transverse coordinates. The required functions satisfy the set of differential equations, boundary and interface conditions followed from the original three-dimensional problem statement and are found as a result of numerical technique. The presented solution of the evaporative convection problem is very contensive. It permits to specify the 3D flow regimes with different topology, thermal and concentration characteristics observed in physical experiments. Differences of flows in the ethanol–nitrogen, HFE-7100 – nitrogen and FC-72 – nitrogen systems are studied. Impact of the thermophysical properties of the working liquids on the basic characteristics of the fluid motions (hydrodynamical structure, temperature distribution, vapor content in the nitrogen, evaporative mass flow rate) is analyzed.

Keywords: evaporative convection, thermocapillary interface, three-dimensional flow, mathematical model, exact solution.
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Introduction

Processes of convection in the fluids accompanied by evaporation or condensation at the interface are actively studied experimentally and theoretically in the presence. In experiments

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the dynamics of an evaporating horizontal fluid layers is investigated, the evaporative mass flow rate of the liquid layers blown by the flux of dry or wet gas, average velocities of vortex structures and surface temperature gradients are measured [1–3]. The necessity for a theoretical study of convective flows of liquids and co-current gas fluxes in conditions of evaporation or condensation at the interface is explained by the need to develop the concepts of the planned experiments and to forecast their outcomes. From mathematical point of view, the problems of evaporative convection are very complex; they are still insufficiently explored and substantiated in the theory of convection [4].

In the present paper the joint flows of viscous fluids (of the liquid and gas – vapor medium) filling a three-dimensional rectangular channel caused by the combined action of thermal load and gravity field are studied in the frame of the Boussinesq approximation of the Navier – Stokes equations. The effects of thermodiffusion and diffusive thermal conductivity are taken into account additionally in modelling dynamics in the gas – vapor phase, and in boundary conditions imposed on the internal interface and external solid walls. Characteristics of the arising flow regimes in the two-layer system with evaporation/condensation at the thermocapillary interface are analyzed on the basis an exact solution of the governing equations. Employed solution is the partially invariant solution of rank 2 and defect 3 of the original equations and generalizes the well-known Ostroumov – Birikh solution [5, 6] for three-dimensional non-axis-symmetrical case [7] of the thermoconcentration convection problem [8]. It allows one to evaluate the extent of influence of various factors on flow characteristics and to obtain the qualitative and quantitative flow features. Impact of thermophysical properties of a liquid-phase coolant on the flow pattern and specifics of mass transfer due to the liquid evaporation or vapor condensation are discussed for systems with different types of working liquid. The results help us to develop a classification of the 3D flow regimes similarly to the two-dimensional flow classification [9, 10].

1. Problem statement

The Cartesian coordinate system will be chosen so that the gravity acceleration vector $\mathbf{g}$ is directed opposite to the Ox axis ($\mathbf{g} = -g\mathbf{i}$, $\mathbf{i}$ is the unit vector of Ox). Domain of two viscous incompressible fluid flow is an infinite horizontal channel with solid boundaries. Liquid and gas-vapor mixture fill layers Ω_1 and Ω_2, respectively: $\Omega_1 = \{(x, y, z) : -x_0 < x < 0,\ 0 < y < 1,\ -\infty < z < \infty\}$; $\Omega_2 = \{(x, y, z) : 0 < x < x^0,\ 0 < y < 1,\ -\infty < z < \infty\}$. The layers are immiscible and have a common thermocapillary interface Γ that is defined by equation $x = 0$. The outer boundaries of the channel ($x = -x_0$, $x = x^0$, $y = 0$ $y = 1$) are the fixed impermeable walls.

The stationary three-dimensional convective flows are described by the Oberbeck – Boussinesq approximation of the Navier – Stokes equations. The vapor is assumed a passive component, the vapor diffusion in the gas phase is described by diffusion equation. The governing system in the non-dimensional form is written as follows:

$$(\mathbf{v}_j \cdot \nabla)\mathbf{v}_j = -\eta_j^p \nabla p_j + \frac{\eta_j^{\mathbf{v}}}{\mathrm{Re}} \Delta \mathbf{v}_j + \mathbf{G}_j, \quad \operatorname{div} \mathbf{v}_j = 0, \quad \mathbf{v}_j \cdot \nabla T_j = \frac{\eta_j^T}{\mathrm{RePr}} (\Delta T_j + \underline{\bar{\delta} \Delta C}), \quad (1.1)$$

$$\mathbf{v}_2 \cdot \nabla C = \frac{1}{\mathrm{Pe}} (\Delta C + \bar{\alpha} \Delta T_2). \quad (1.2)$$

Here and subsequently index $j = 1, 2$ denotes the characteristics of the liquid (in Ω_1) and gas – vapor mixture (in Ω_2), respectively; $\mathbf{v}_j$ is the velocity vector, T_j is the temperature, p_j is the pressure (deviation of pressure from the hydrostatic one), C is the vapor concentration function,

$\mathbf{G}_1 = \mathbf{i}(\mathrm{Gr}/\mathrm{Re}^2)\,T_1$, $\mathbf{G}_2 = \mathbf{i}\big(\bar{\beta}(\mathrm{Gr}/\mathrm{Re}^2)\,T_2 + \gamma(\mathrm{Ga}/\mathrm{Re}^2)\,C\big)$. Equation (1.2) and marked term in (1.1) are taken into account to model the gas-vapor flows in Ω_2. The governing dimensionless parameters are introduced by standard way: the Reynolds number $\mathrm{Re} = u_* h/\nu_1$, the Prandtl number $\mathrm{Pr} = \nu_1/\chi_1$, the Grashof number $\mathrm{Gr} = \beta_1 T_* g h^3/\nu_1^2$, the Galilei number $\mathrm{Ga} = gh^3/\nu_1^2$, the diffusive Peclet number $\mathrm{Pe} = u_* h/D$. The following notation are used: $\eta_1^p = 1$, $\eta_2^p = 1/\bar{\rho}$, $\eta_1^{\mathbf{v}} = 1$, $\eta_2^{\mathbf{v}} = \bar{\nu}$, $\eta_1^T = 1$, $\eta_2^T = \bar{\chi}$, where $\bar{\rho} = \rho_2/\rho_1$, $\bar{\nu} = \nu_2/\nu_1$, $\bar{\chi} = \chi_2/\chi_1$, $\bar{\beta} = \beta_2/\beta_1$ are ratios of the densities ρ_j, coefficients of kinematic viscosity ν_j, thermal diffusivity χ_j and thermal expansion β_j respectively; γ is the concentration coefficient of the gas density; $\bar{\alpha} = \alpha T_*$, $\bar{\delta} = \delta/T_*$, the coefficients α and δ characterize the Soret and Dufour effects in the gas-vapor layer, D is the coefficient of vapor diffusion in the gas.

The problem statement includes the interface and boundary conditions formulated on the basis of the conservation laws and some additional assumptions [8, 11]. On the thermocapillary interface the continuity conditions of tangential velocities and temperature and the kinematic and dynamic conditions are assumed to be fulfilled. The heat transfer condition with respect to the diffusive mass flux due to evaporation and the mass balance equation are formulated. The linearized form of an equation for saturated vapor concentration at the interface is used to define the vapor concentration function [4]. The no-slip conditions for velocity vectors and the conditions of thermal insulating of the channel walls are set on the outer boundaries; the condition of absence of vapor flux on the upper and lateral rigid boundaries is used in the present statement. Conditions for the temperature and vapor concentration function on the fixed walls provide a fulfillment of the full heat flux condition with respect to the Dufour effect and of full mass flux condition with respect to the Soret effect [12]. The full problem statement for the evaporative convection problem both in the dimensional and non-dimensional form as well as detailed substantiation of the formulation are given in [8].

System (1.1), (1.2) admits the following exact solution [8]:

$$u_j = u_j(x,y), \quad v_j = v_j(x,y), \quad w_j = w_j(x,y), \quad T_j = -Az + \Theta_j(x,y), \quad C = Bz + \Phi(x,y),$$

$$p_1 = -A\frac{Gr}{Re^2}xz + q_1(x,y), \quad p_2 = -A\bar{\rho}\bar{\beta}\frac{Gr}{Re^2}xz + B\bar{\rho}\gamma\frac{Ga}{Re^2}xz + q_2(x,y). \tag{1.3}$$

The coefficients $A = A_* h/T_*$ and $B = B_* h$ determine the constant longitudinal temperature and concentration gradients along the interface; A_* and B_* are the corresponding dimensional gradients. This solution of special form are not the exact solution in usual sense. We use this term not only in the so-called "group sense" but also because the solution exactly satisfy all the governing equations and interface and boundary conditions. It should be emphasize, that solution (1.3) does not presuppose the axial symmetry. Furthermore, even without using any assumption about non-deformability of the interface solution (1.3) dictates relation $x = 0$ as an equation of the interface. Note, that there are no any other restricted at all y, z solutions of the equation of minimal surfaces. We interpret (1.3) as a solution describing the three-dimensional flow with the phase transition in the working area in a sufficiently long cuvette.

2. Numerical results of investigations of fluid flow regimes

The structure of exact solution (1.3) enables to reduce the original 3D problem to the set of the 2D problems to find the required functions u_j, v_j, w_j, Θ_j, Φ. Instead of the transverse components of the velocity vectors the stream functions $\psi_j(x,y)$ and vorticity $\omega_j(x,y)$ are introduced according to usual rules. The boundary conditions in terms of these new unknown functions are reformulated.

Numerical algorithm based on the longitudinal transverse finite difference scheme known as the method of alternating directions was constructed and code with visualization procedure was developed by the authors to perform the fluid flow simulations. A general scheme of solution of the coupled problem and the method parameters are outlined in [8]. Numerical investigations are carried out to describe the flow topology, thermal pattern and characteristics of vapor content as well as to analyze the possible differences associated with the physico-chemical properties of liquid coolant for the liquid – gas systems with ethanol/HFE-7100/FC-72 fluids as the liquid and nitrogen as the gas. Physical parameters of the working media are given here in the order (1) ethanol, (2) HFE-7100, (3) FC-72 and (4) nitrogen or only (1) ethanol, (2) HFE-7100, (3) FC-72, respectively: $\rho = \{0.79 \cdot 10^3,\ 1.5 \cdot 10^3,\ 1.72 \cdot 10^3,\ 1.2\}$ (kg/m^3); $\nu = \{0.15 \cdot 10^{-5},\ 0.38 \cdot 10^{-6},\ 0.47 \cdot 10^{-6},\ 0.15 \cdot 10^{-4}\}$ (m^2/s); $\beta = \{0.108 \cdot 10^{-2},\ 0.18 \cdot 10^{-2},\ 0.152 \cdot 10^{-2},\ 0.367 \cdot 10^{-2}\}$ (K^{-1}); $\chi = \{0.89 \cdot 10^{-7},\ 0.4 \cdot 10^{-7},\ 0.33 \cdot 10^{-7},\ 0.3 \cdot 10^{-4}\}$ (m^2/s); $\gamma = \{-0.62,\ -0.5,\ -0.5\}$. Values of other physical parameters are also presented: thermal conductivity $k = \{0.1672,\ 0.7 \cdot 10^{-1},\ 0.58 \cdot 10^{-1},\ 0.27 \cdot 10^{-1}\}$ (W/(m·K)); temperature coefficient of surface tension $\sigma_T = \{0.8 \cdot 10^{-4},\ 0.114 \cdot 10^{-3},\ 0.5 \cdot 10^{-4}\}$ (N/(m·K)); diffusion coefficient $D = \{0.135 \cdot 10^{-4},\ 0.7 \cdot 10^{-5},\ 0.7 \cdot 10^{-5}\}$ (m^2/s); latent heat of evaporation $L = \{8.55 \cdot 10^5,\ 1.42 \cdot 10^5,\ 9.7 \cdot 10^4\}$ (W·s/kg); coefficients in the relation for interface vapor concentration [4] (at $T_0 = 10^o C$) $C_* = \{0.1,\ 0.6,\ 0.6\}$ and $\bar{\varepsilon} = \{0.1, 0.04, 0.04\}$. The Soret and Dufour coefficients are taken to be equal to $\alpha = 10^{-4}$ K^{-1} and $\delta = 10^{-4}$ K, respectively, for all the systems of working media. The values of the characteristic temperature drop T_* and characteristic length h are equal to $T_* = 10$ K and $h = 10^{-2}$ m. The Reynolds number was chosen equal to 1. Computations are performed for the case of normal gravity with $g = 9.81$ m/s^2.

Examples of the flow pictures (trajectories of fluid particles and projections of stream tubes on transverse sections) and corresponding temperature distributions are shown in Figs. 1–3. Presented characteristics are observed in the systems with the liquid and gas-vapor layer thicknesses $h_l = 0.25$, $h_g = 0.5$, respectively, and the longitudinal temperature gradient $A = 0.3$.

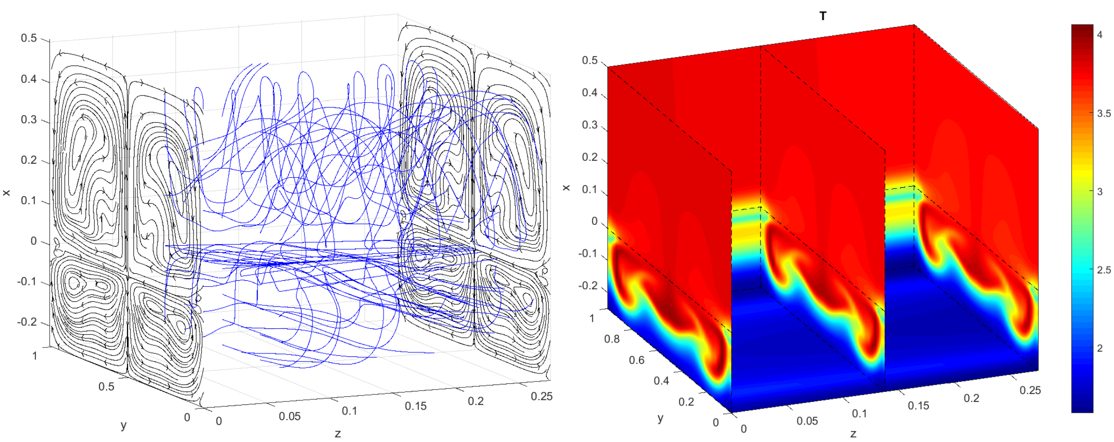

Fig. 1. Stream lines and trajectories (left) and distribution of temperature (right) in ethanol – nitrogen system

In all the cases the flow topology is symmetric with respect to the plane $y = 0.5$ in both phases. But essential differences in the patterns of the hydrodynamical and thermal fields can appear depending on the type of a liquid coolant. The different types of the planforms (configurations of stream lines in the (x, y)-plane) are observed in each system for considered configuration, especially with regard to lower layer. In the system ethanol – nitrogen we observe the roll-type convection characterized by the formation double-vortex structure both in the gas-vapor phase and in the lower liquid layer (Fig. 1, left). Vortices in the ethanol have two non-symmetric cores in the upper part of the liquid layer, and whirl cores in the nitrogen are arranged in central part of the gas layer and have deformed shape. Similar elements of the space-periodic structure of the flow in the liquid can be designated as thermocapillary rolls generated by the action of the Marangoni forces. The temperature field in the ethanol – nitrogen system is characterized by formation of the heat structures near the lateral walls which can be referred to as the thermal horns (Fig. 1, right). Upon that, the temperature maximum is observed within the "horns", it leads to the appearance of hot thermocline in the liquid and co-existence of domains with the stable and unstable temperature stratification in ethanol. So, both thermocapillary and convective mechanisms act in the system.

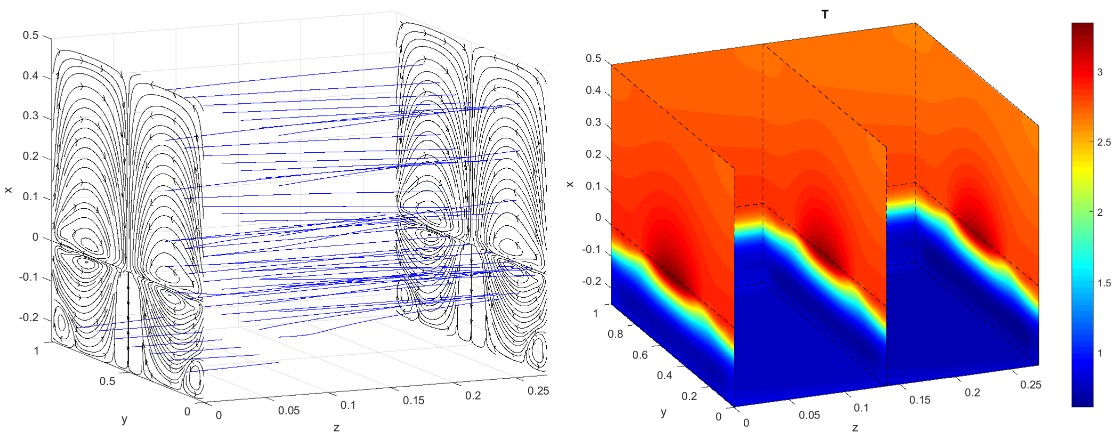

Fig. 2. Stream lines and trajectories (left) and distribution of temperature (right) in HFE-7100 – nitrogen system

With change of coolant the topological picture is varied. In the HFE-7100 layer the six-vortex flow structure is formed (Fig. 2, left). Two additional corner vortices appear in the liquid layer. Furthermore, main swirls are splitted into central compressed whirls and external ones occupying a great part of the liquid layer in its width. Formation of extra vortexes is explained by weaker viscous effects in HFE-7100 in comparison with ethanol. Furthermore, the near-bottom-wall reverse flows in HFE-7100 appear (see trajectories in right picture of Fig. 2). Double-vortex pattern of flow is kept in the nitrogen, upon that the whirl cores is displaced to the interface. Significant alteration of thermal field takes place also. The thermal roll occurs in the central part of the channel in the HFE-7100 – nitrogen system, and stable temperature stratification is formed (Fig.2, right). Thus, the determining mechanism for the system is the thermocapillary one.

Other type of the planforms is observed in the FC-72 layer. Six vortices with other forms and location of cores in comparison with pattern in HFE-7100 appear (Fig. 2, left). In fact, it can be said that a liquid flow stratification in the vertical direction occurs. Thermal shaft arising in the FC-72 – nitrogen system has bigger transverse size than similar roll in the HFE-7100 – nitrogen system, so that near-surface hot "film" appears in FC-72, and temperature stratification of the

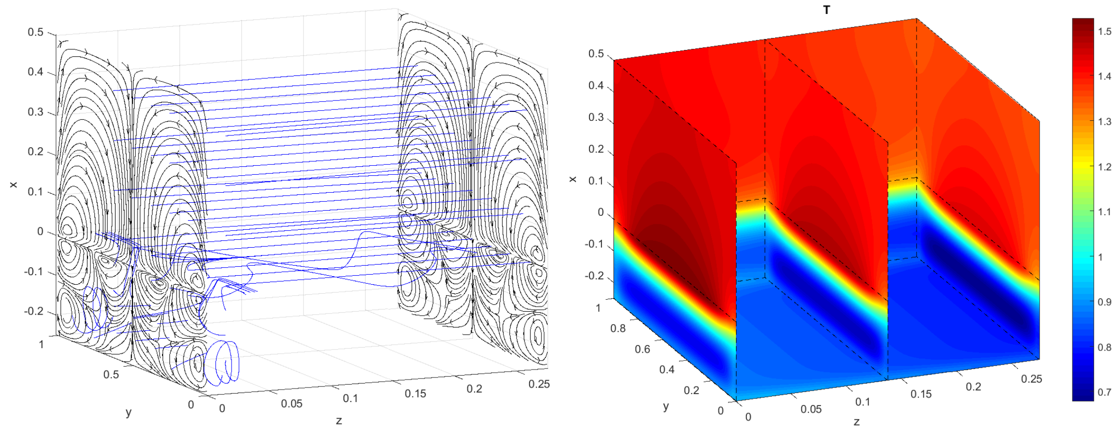

Fig. 3. Stream lines and trajectories (left) and distribution of temperature (right) in FC-72 – nitrogen system

liquid layer is the gravity-stable.

Despite similar thermophysical properties of the hydrofluoroether HFE-7100 and fluorinert FC-72 visual differences in the vapor concentration distribution are observed (Fig. 4). Concentration rolls whose spatial size depends on the pattern of the thermal field appear in both systems. Characteristics of vapor content for these liquids are essentially different. One can conclude that quantity of vapor in the gas depends on the volatility of the liquid (it is characterized by the parameter L which is the latent heat of evaporation). Furthermore, the higher viscosity of FC-72 can impedes to evaporation.

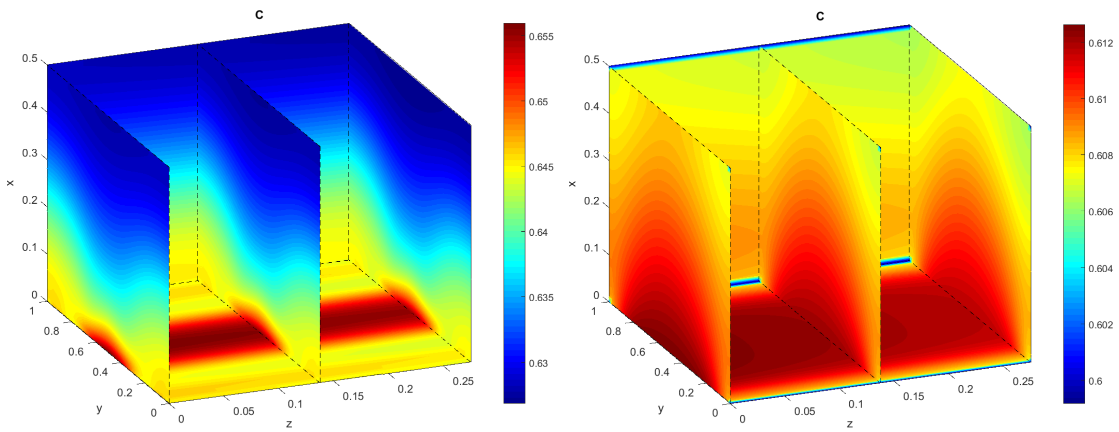

Fig. 4. Distribution of vapor concentration in HFE-7100 – nitrogen (left) and FC-72 – nitrogen (right) systems at $A = 0.3$

Thus, we have different flow topology and essentially varied temperature patterns for the various gas – liquid systems being in the same conditions. The major features are related with different viscous and transport properties such as heat conductivity/diffusivity and with significantly different values of the latent heat of evaporation, which are essentially lower in HFE-7100/FC-72 than in ethanol.

Conclusions

The fundamental meaning of the exact solution under consideration is that it allows one to represent the real flows with evaporation in long cavities. The group nature of the solution provides automatically the symmetry maintenance. The translational-rotational flow character in systems with ethanol/HFE-7100/FC-72 liquids is realized in various forms (double, quadruple or sextuple vortex patterns) depending on intensity of thermal load defined by value of the longitudinal temperature gradient, gravity, system geometry and type of heat transfer fluids. The exact solution (1.3) allows one to elucidate the substantial differences in structures of the hydrodynamic, thermal and vapor concentration fields and behavior of evaporative mass flow rate. Furthermore, classification of the flow types with respect both to the planform types and thermal pattern forms can be suggested. In these cases topologically and thermally different classes of flows are specified, respectively.

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Влияние теплофизических свойств жидкого теплоносителя на характеристики трехмерных течений с фазовым переходом

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Изучаются режимы совместных течений испаряющейся жидкости и газопаровой смеси в трехмерном канале прямоугольного поперечного сечения, находящемся в условиях нормальной гравитации под действием продольного градиента температуры. Теоретические исследования проводятся на основе частично инвариантного решения ранга 2 и дефекта 3 уравнений Навье – Стокса в аппроксимации Обербека – Буссинеска. Данное решение позволяет корректно описать двухслойные течения с испарением и/или конденсацией на термокапиллярной границе раздела и учесть при моделировании эффекты термодиффузии и диффузионной теплопроводности в газопаровой среде. Точное решение исходных уравнений характеризуется зависимостью компонент скорости только от поперечных координат. Функции давления, температуры и концентрации пара линейно зависят от продольной координаты и также имеют слагаемые, являющиеся функциями поперечных координат. Все подлежащие определению функции находятся в результате численного решения систем дифференциальных уравнений и граничных условий, выступающих следствием трехмерной постановки. Изучаемое точное решение задачи испарительной конвекции очень содержательно и позволяет получать трехмерные режимы течений различной топологии с такими тепловыми и концентрационными характеристиками, которые наблюдаются в физических экспериментах. Изучаются различия в структуре течений систем этанол – азот, HFE-7100 – азот и FC-72 – азот. В работе представлен анализ влияния термофизических свойств различных рабочих жидкостей на основные характеристики движений (на гидродинамическую структуру, распределение температуры и содержание пара в азоте, а также на массовую скорость испарения).

Ключевые слова: испарительная конвекция, термокапиллярная граница, трехмерное течение, математическая модель, точное решение.