Mean Value Theorem for a System of Differential Equations for the Stress Tensor and Pore Pressure

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A system of second-order differential equations for the stress tensor and pore pressure for the poroelasticity statics in the absence of mass forces and energy dissipation is obtained. The stress tensor is shown to be a biharmonic function. Integral mean value relations in explicit form for the obtained systems of differential equations are found.

Keywords: differential equation, poroelasticity statics, stress tensor.

It is well known that static simulation methods are used to solve multidimensional boundary value problems at a small number of points, especially if the domain boundary shape is rather complex [1–3].

If a boundary value problem has stochastic parameters (for instance, the equation coefficients or right-hand side are random), Monte Carlo methods are an especially convenient tool to calculate both average characteristics of the solution and other static characteristics [4].

Such theorems were proved for many basic equations and systems of equations (see [1–15]). In papers [16–18], systems of differential equations in terms of displacements of particles of an elastic porous body and pore pressure for stationary processes in the porous medium were obtained [19, 20]. Mean value relations for such systems of differential equations were also established [16–18].

In the present paper, a system of differential equations in terms of the stress tensor and pore pressure for stationary processes in the porous medium is obtained. Mean value relations for the obtained system of differential equations are found.

Problem statement

Assume that a bounded domain $\Omega \subset R^3$ is a porous medium filled with a homogeneous isotropic saturated fluid. In the reversible case the elastic-porous static state of the medium Ω is described by the following system of differential equations [16–18]:

$$\mu \, \Delta \mathbf{U} + (\tilde{\lambda} + \mu) \, \nabla \mathrm{div} \, \mathbf{U} = 0, \tag{1}$$

$$\Delta p = 0 \,, \tag{2}$$

where $\mathbf{U} = (U_1, U_2, U_3)$ is the displacement vector of an elastic porous body with partial density ρ_s, p is the pore pressure, $\tilde{\lambda} = \lambda - (\rho^2 \, \alpha)^{-1} \, K^2$, $K = \lambda + 2 \, \mu/3$, λ, μ, $\alpha = \rho \, \alpha_3 + K/\rho^2$ are the

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constants of the equation of state [20–23], $\rho = \rho_s + \rho_l$, and ρ_l is the partial density of the fluid. In paper [16], a formula was obtained relating the stress tensor with the deformation tensor of an elastic-porous body and pore pressure:

$$\sigma_{ik} = 2\mu\varepsilon_{ik} + \tilde{\lambda}\delta_{ik}\varepsilon_{mm} - \hat{\alpha}\delta_{ik}\,p, \quad i,k = 1,2,3, \tag{3}$$

$$\varepsilon_{ik} = \frac{1}{2}(U_{i,k} + U_{k,i}), \quad i,k = 1,2,3, \quad \varepsilon_{mm} = \sum_{n=1}^{3}\varepsilon_{nn},$$

where δ_{ik} is the Kronecker symbol, $v_{,k} = \dfrac{\partial v}{\partial x_k}$, $\hat{\alpha} = 1 - \dfrac{K}{\alpha\rho^2}$.

Solving system (3) with respect to the deformation tensor, we obtain

$$\varepsilon_{ik} = \frac{1}{2\mu}\sigma_{ik} - \frac{\delta_{ik}}{3\tilde{\lambda} + 2\mu}\left(\frac{\tilde{\lambda}}{2\mu}\sigma_{mm} - \hat{\alpha}\,p\right), \quad i,k = 1,2,3. \tag{4}$$

System of differential equations for the stress tensor and pore pressure

Substituting (3) into the consistency condition of the deformation tensor $\varepsilon_{ij,kk} + \varepsilon_{kk,ij} = \varepsilon_{ik,kj} + \varepsilon_{kj,ik}$, $i,j,k = 1,2,3$ we obtain the following system of second-order differential equations for the stress tensor and pore pressure:

$$\sigma_{ij,kk} + \sigma_{kk,ij} - \frac{1}{3\tilde{\lambda} + 2\mu}\left[\tilde{\lambda}\delta_{ij}\sigma_{mm,kk} + \tilde{\lambda}\delta_{kk}\sigma_{mm,ij} - 2\mu\hat{\alpha}\delta_{ij}\,p_{,kk} - 2\mu\hat{\alpha}\delta_{kk}\,p_{,ij}\right] =$$

$$= \sigma_{ik,kj} + \sigma_{kj,ik} - \frac{1}{3\tilde{\lambda} + 2\mu}\left[\tilde{\lambda}\delta_{ik}\sigma_{mm,kj} + \tilde{\lambda}\delta_{kj}\sigma_{mm,ik} - 2\mu\hat{\alpha}\delta_{ik}\,p_{,kj} - 2\mu\hat{\alpha}\delta_{kj}\,p_{,ik}\right].$$

Let us perform summation over k:

$$\Delta\sigma_{ij} + \sigma_{mm,ij} - \frac{1}{3\tilde{\lambda} + 2\mu}\left[\tilde{\lambda}\delta_{ij}\Delta\sigma_{mm} + 3\tilde{\lambda}\sigma_{mm,ij} - 2\mu\hat{\alpha}\delta_{ij}\,\Delta\,p - 6\mu\hat{\alpha}\,p_{,ij}\right] =$$

$$= \sum_{k=1}^{3}(\sigma_{ik,kj} + \sigma_{kj,ik}) - \frac{2}{3\tilde{\lambda} + 2\mu}\left[\tilde{\lambda}\sigma_{mm,ij} - 2\mu\hat{\alpha}\,p_{,ij}\right], \quad i,j = 1,2,3. \tag{5}$$

From equation (1), with allowance for (3), for σ_{ik} we obtain the first-order equation

$$\sum_{k=1}^{3}\sigma_{ik,k} + \hat{\alpha}\,p_{,i} = 0, \quad i = 1,2,3. \tag{6}$$

From (5), with harmonicity of the pore pressure p and the equilibrium equation (6), we obtain

$$\Delta\sigma_{ij} + \frac{2(\tilde{\lambda} + \mu)}{3\tilde{\lambda} + 2\mu}\sigma_{mm,ij} - \frac{\tilde{\lambda}\delta_{ij}}{3\tilde{\lambda} + 2\mu}\Delta\sigma_{mm} = 2\hat{\alpha}\,\frac{3\tilde{\lambda} + \mu}{3\tilde{\lambda} + 2\mu}\,p_{,ij}, \quad i,j = 1,2,3. \tag{7}$$

Hence, at $i = j$ and summing over i from 1 to 3, we obtain harmonicity of the stress tensor trace σ_{ik}, that is,

$$\Delta\sigma_{mm} = 0. \tag{8}$$

With allowance for this equality, relation (7) takes the following form:

$$\Delta\sigma_{ij} + \beta\sigma_{mm,ij} = \gamma\,p_{,ij}, \quad i,j = 1,2,3, \tag{9}$$

$$\beta = \frac{2(\tilde{\lambda} + \mu)}{3\tilde{\lambda} + 2\mu}, \quad \gamma = 2\hat{\alpha}\,\frac{3\tilde{\lambda} + \mu}{3\tilde{\lambda} + 2\mu}.$$

Thus, the pore pressure and stress tensor satisfy the system of second-order differential equations (2) and (9). It follows from system (9) that the stress tensor components are biharmonic functions. In fact, let the Laplace operator Δ act on the both sides of equality (9), and, taking into account properties (2) and (8), we obtain $\Delta^2 \sigma_{ij} = 0$.

Mean value relation for system (2), (9)

Now we introduce, in accordance with [9], $N(\mathbf{u})$, the averaging operator of the vector function $\mathbf{u} = (u_1, u_2, \ldots, u_n)^T$ over the surface of a sphere $S(\mathbf{x}, R)$ with respect to the uniform measure $d\Omega$, that is, $N(\mathbf{u}) = \frac{1}{\omega_n\, r^{n-1}} \int \mathbf{u}(\mathbf{x} + r\,\mathbf{y})\,d\Omega(\mathbf{y})$, where ω_n is the unit sphere area, and $\{s_i\}_{i=1}^n$ are the direction cosines.

For the harmonic function $p(\mathbf{x})$, $\mathbf{x} \in \Omega$, the mean value relations [10]

$$p(0) = \frac{\int_{S(0,R)} p\,d\Omega}{\int_{S(0,R)} 1\,d\Omega} = \frac{1}{\omega_3} \int_{S(0,1)} p\,d\Omega(s), \tag{10}$$

$$p(0) = \frac{3}{4\,\pi\,R^3}\, N^{(W)} p(\mathbf{x}), \tag{11}$$

are valid. Here $N^{(W)} p(\mathbf{x})$ is the integral of P over the ball $W(\mathbf{x}, R)(= \{|\mathbf{x} - \mathbf{y}| < R\}$. For the harmonic function $\dfrac{\partial^2 p}{\partial x_k \partial x_i}$, we use relation (11)

$$\frac{\partial^2 p(0)}{\partial x_k \partial x_i} = \frac{3}{4\,\pi\,R^3} \int_{W(0,R)} p_{,ik}\,dW = \frac{3}{4\,\pi\,R^3} \int_{S(0,1)} p\,\frac{x_k x_i}{R^2}\,d\Omega, \quad i, k = 1, 2, 3,$$

or

$$\frac{\partial^2 p(0)}{\partial x_k \partial x_i} = \frac{3}{4\,\pi\,R^3} \int_{S(0,1)} p\,s_{,i}\,s_{,k}\,d\Omega(s), \quad i, k = 1, 2, 3. \tag{12}$$

As shown in [10], the mean value relation is valid for the biharmonic function. Applying formula (2.5) from [10] to equation (6), we obtain

$$\sigma_{ij}(0) = \frac{3}{2\,\omega_3} \left[\frac{5}{R^3} \int_{W(0,R)} \sigma_{ij}\,dW - \int_{S(0,1)} \sigma_{ij}\,d\Omega \right]. \tag{13}$$

From the equilibrium equation (6), we have

$$\int_{W(0,\eta)} \sigma_{ij}\,dW = \int_{W(0,\eta)} (\sigma_{ik} x_j)_{,k}\,dW - \int_{W(0,\eta)} \sigma_{ik,k} x_j\,dW =$$

$$= \eta^3 \int_{S(0,1)} \sigma_{ik}\frac{x_j x_k}{\eta^2}\,d\Omega + \hat{\alpha} \int_{W(0,\eta)} (p\,x_j)_{,i}\,dW - \hat{\alpha}\delta_{ij} \int_{W(0,\eta)} p\,dW = \tag{14}$$

$$= \eta^3 \int_{S(0,1)} \sigma_{ik}\frac{x_j x_k}{\eta^2}\,d\Omega + \hat{\alpha}\eta^3 \int_{S(0,1)} p\frac{x_i x_j}{\eta^2}\,d\Omega - \hat{\alpha}\delta_{ij} \int_{W(0,\eta)} p\,dW.$$

From (13) and (14), we obtain

$$\sigma_{ij}(0) = \frac{3}{2\,\omega_3} \left[5\int_{S(0,1)} \sigma_{ik}\frac{x_j x_k}{\eta^2}\,d\Omega - \int_{S(0,1)} \sigma_{ij}\,d\Omega + 5\hat{\alpha}\int_{S(0,1)} p\frac{x_i x_j}{\eta^2}\,d\Omega - \frac{5\hat{\alpha}}{\eta^3}\delta_{ij}\int_{W(0,\eta)} p\,dW \right].$$

Now we multiply the both sides of this equality by η^4, integrate from 0 to R and, with allowance for (11), obtain

$$\frac{R^5}{5}\sigma_{ij}(0) = \frac{3}{2\,\omega_3}\left[5\int_{S(0,\,1)}\sigma_{ik}\frac{x_jx_k}{\eta^2}d\Omega - \int_{S(0,\,1)}\sigma_{ij}\,d\Omega + 5\hat{\alpha}\int_{S(0,\,1)}p\frac{x_ix_j}{\eta^2}d\Omega\right] - \frac{R^5\hat{\alpha}}{2}\delta_{ij}p(0). \tag{15}$$

From the equilibrium equation (6), as in (14), we obtain

$$\int_{W(0,R)}\eta^2\sigma_{ij}d\,W = R^5\int_{S(0,\,1)}\sigma_{ik}\frac{x_jx_k}{R^2}d\Omega - 2\int_{W(0,R)}\sigma_{ik}x_kx_jd\,W +$$
$$+\hat{\alpha}R^5\int_{S(0,\,1)}p\frac{x_ix_j}{R^2}d\Omega - \hat{\alpha}\delta_{ij}\int_{W(0,R)}\eta^2pd\,W - 2\hat{\alpha}\int_{W(0,R)}px_ix_jd\,W. \tag{16}$$

From (15) and (16) we have

$$\sigma_{ij}(0) = \frac{15}{2\,\omega_3R^5}\left[7\int_{W(0,R)}\sigma_{ik}x_kx_jd\,W - R^5\int_{S(0,\,1)}\sigma_{ik}\frac{x_jx_k}{R^2}d\Omega -\right.$$
$$\left. -R^5\hat{\alpha}\int_{S(0,\,1)}p\frac{x_ix_j}{R^2}d\Omega + \hat{\alpha}\delta_{ij}\int_{W(0,R)}\eta^2pd\,W + 7\hat{\alpha}\int_{W(0,R)}px_ix_jd\,W\right] - \frac{5\hat{\alpha}}{2}\delta_{ij}p(0). \tag{17}$$

Let us multiply (9) by η^2 and integrate with respect to the ball,

$$0 = \int_{W(0,\varsigma)}\eta^2\left[\Delta\sigma_{ij} + \beta\sigma_{mm,ij} - \gamma\,p_{,ij}\right]d\,W = \varsigma^2\int_{S(0,\varsigma)}\left[\sigma_{ij,k}\frac{x_k}{\varsigma} + \beta\sigma_{mm,i}\frac{x_j}{\varsigma} - \gamma\,p_{,i}\frac{x_j}{\varsigma}\right]dS -$$
$$-2\int_{W(0,\varsigma)}\left[\sigma_{ij,k}\frac{x_k}{\varsigma} + \beta\sigma_{mm,i}\frac{x_j}{\varsigma} - \gamma\,p_{,i}\frac{x_j}{\varsigma}\right]d\,W =$$
$$= -2\varsigma^3\int_{S(0,\,1)}\left[\sigma_{ij} + \beta\sigma_{mm}\frac{x_ix_j}{\varsigma^2} - \gamma\,p\frac{x_ix_j}{\varsigma^2}\right]d\Omega - \int_{W(0,\varsigma)}\left[3\sigma_{ij} + \beta\delta_{ij}\sigma_{mm} - \gamma\,\delta_{ij}p\right]d\,W. \tag{18}$$

Here we use the fact that the surface integral is zero and the Gaussian formula.

Assuming in (14) that $i = j = k$, we obtain

$$\int_{W(0,\eta)}\sigma_{kk}d\,W = \eta^3\int_{S(0,\,1)}\sigma_{kl}\frac{x_kx_l}{\eta^2}d\Omega + \hat{\alpha}\eta^3\int_{S(0,\,1)}pd\Omega - 3\hat{\alpha}\int_{W(0,\eta)}pd\,W. \tag{19}$$

From (18) and (19), using simple transformations, we have

$$3\int_{W(0,R)}\sigma_{ik}x_jx_kd\,W + 3\hat{\alpha}\int_{W(0,R)}px_ix_jd\,W -$$
$$-(\hat{\alpha} + \frac{\gamma}{3})\delta_{ij}\int_{W(0,R)}\eta^2pd\,W + \beta\delta_{ij}\int_{W(0,R)}\sigma_{kl}x_kx_ld\,W = \tag{20}$$
$$= \int_{W(0,R)}\eta^2\sigma_{ij}d\,W + \beta\int_{W(0,R)}\sigma_{mm}x_ix_jd\,W - \gamma\int_{W(0,R)}px_ix_jd\,W.$$

It can be shown by direct calculations that $\sigma_{kl}x_kx_l$ is a biharmonic function. Using for it formula (13), we obtain

$$0 = \frac{5}{R^3}\int_{W(0,R)}\sigma_{kl}x_kx_ld\,W - \int_{S(0,\,1)}\sigma_{kl}x_kx_l\,d\Omega. \tag{21}$$

From (20), with allowance for (16) and (21), we have

$$5\int_{W(0,R)}\sigma_{ik}x_jx_kdW = \beta\int_{W(0,R)}\sigma_{mm}x_ix_jdW + R^5[\int_{S(0,1)}\sigma_{ik}\frac{x_jx_k}{R^2}d\Omega-$$

$$-\frac{\beta}{5}\delta_{ij}\int_{S(0,1)}\sigma_{kl}\frac{x_kx_l}{R^2}d\Omega] - (5\hat{\alpha}+\gamma)\int_{W(0,R)}px_ix_jdW- \tag{22}$$

$$-(\hat{\alpha}+\frac{\gamma}{3})\delta_{ij}\int_{W(0,R)}\eta^2pdW + \frac{\gamma}{3})\delta_{ij}\int_{W(0,R)}\eta^2pdW + \hat{\alpha}R^5\int_{S(0,1)}p\frac{x_ix_j}{R^2}d\Omega.$$

The volume integral in (22) can be transformed as follows:

$$\int_{W(0,R)}\sigma_{kk}x_ix_jdW = \int_{W(0,R)}(\sigma_{kl}x_mx_ix_j)_{,l}dW - \int_{W(0,R)}\sigma_{kl,l}x_kx_ix_jdW-$$

$$-\int_{W(0,R)}\sigma_{ki}x_kx_jdW - \int_{W(0,R)}\sigma_{kj}x_kx_idW = R^5\int_{S(0,1)}\sigma_{kl}\frac{x_ix_jx_kx_l}{R^4}d\Omega+$$

$$+\hat{\alpha}R^5\int_{W(0,1)}p\frac{x_ix_j}{r^2}d\Omega - 3\hat{\alpha}\int_{W(0,R)}px_ix_jdW - \int_{W(0,R)}\sigma_{ki}x_kx_jdW - \int_{W(0,R)}\sigma_{kj}x_kx_idW. \tag{23}$$

In the derivation of (23), we used the equilibrium equation (6).

From (22) and (24) we obtain, after simple transformations,

$$\int_{W(0,R)}\sigma_{ik}x_jx_kdW = \frac{R^5}{5+2\beta}[\beta\int_{S(0,1)}\sigma_{kl}\frac{x_ix_jx_kx_l}{R^4}d\Omega + \int_{S(0,1)}\sigma_{ik}\frac{x_jx_k}{R^2}d\Omega-$$

$$-\frac{\beta}{5}\delta_{ij}\int_{S(0,1)}\sigma_{kl}\frac{x_kx_l}{R^2}d\Omega] + \frac{\hat{\alpha}(\beta+1)}{5+2\beta}R^5\int_{S(0,1)}p\frac{x_ix_j}{R^2}d\Omega+ \tag{24}$$

$$+\frac{\gamma}{3(5+2\beta)}\delta_{ij}\int_{S(0,1)}\eta^2p\frac{x_ix_j}{R^2}dW - \frac{(5+3\beta)\hat{\alpha}+\gamma}{5+2\beta}\int_{W(0,R)}px_ix_jdW.$$

From (18) and (24), after simple transformations, we obtain integral mean value relations for the stress tensor

$$\sigma_{ij}(0) = \frac{3}{2\omega_3(5+2\beta)}[10(1-\beta)\int_{S(0,1)}\sigma_{ik}\frac{x_kx_j}{R^2}d\Omega - 7\beta\delta_{ij}\int_{S(0,1)}\sigma_{kl}\frac{x_kx_l}{R^2}d\Omega+$$

$$+35\beta\int_{S(0,1)}\sigma_{kl}\frac{x_ix_jx_kx_l}{R^4}d\Omega] + \frac{15\hat{\alpha}(2+5\beta)}{2\omega_3(5+2\beta)}\int_{S(0,1)}p\frac{x_ix_j}{R^2}d\Omega+$$

$$+\frac{15}{2\omega_3R^5}\left(\hat{\alpha}+\frac{7\gamma}{3(5+2\beta)}\right)\delta_{ij}\int_{W(0,R)}\eta^2pdW + \frac{105(\hat{\alpha}\beta+\gamma)}{2\omega_3(5+2\beta)R^5}\int_{W(0,R)}px_ix_jdW - \frac{5\hat{\alpha}}{2}\delta_{ij}p(0). \tag{25}$$

Let $B(R) = \{x \in R^3 : |x| < R\}$ be a ball of radius R. S is its boundary, ω_3 is the unit sphere area, and $d\Omega$ is the measure on S.

Theorem 1. *For the system of differential equations (9), the following mean value relations are valid:*

$$\sigma_{ij}(0) = \frac{3}{2\omega_3(5+2\beta)}\left[10(1-\beta)\int_{S(0,1)}\sigma_{ik}\frac{x_kx_l}{R^2}d\Omega - 7\beta\delta_{ij}\int_{S(0,1)}\sigma_{ki}\frac{x_kx_l}{R^2}d\Omega+\right.$$

$$+35\beta \int_{S(0,1)} \sigma_{ki} \frac{x_i x_j x_k x_l}{R^4} \, d\Omega] + \frac{15\widetilde{\alpha}\,(2+5\beta)}{2\omega_3\,(2+5\beta)} \int_{S(0,1)} p\frac{x_i x_j}{R^2} d\Omega + + \frac{15}{2\omega_3 R^5}\,(\widetilde{\alpha} +$$

$$+\frac{7\gamma}{3\,(5+2\beta)}\Bigg)\delta_{ij}\int_{W(0,R)} \eta^2 p dW + \frac{105\,(\widetilde{\alpha}\beta+\gamma)}{3\,(5+2\beta)\,R^5}\int_{W(0,R)} p x_i x_j dW - \frac{5\widetilde{\alpha}}{2}\delta_{ij} p\,(0)\,. \tag{26}$$

Thus, we obtained integral mean value relations (7), (8), and (26) for a system of poroelasticity equations. To determine the dilatancy zone, it is necessary to have integral characteristics of the medium being considered. In mathematical simulation, the averaging method - the mean value theorem is used for this. The relations obtained for the stress tensor of a porous body and pore pressure allow using dilatancy zones in problems of monitoring Earth's crust technogenic processes and earthquake prediction [24].

Since in (26) $\hat{\alpha}$ tends to zero, we obtain mean value relations for the stress tensor components for the static equations of classical elasticity [10].

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Теоремы о среднем значении для системы дифференциальных уравнений для тензора напряжения и порового давления

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Рассмотрена система уравнений второго порядка для тензора напряжения и порового давления для пороэластичной статики в отсутствие сил масс и энергии диссипации. Тензор напряжения является бигармонической функцией. Найдено соотношение для интегрального среднего значения в точной форме при рассмотренных полученных систем дифференциальных уравнений.

Ключевые слова: дифференциальное уравнение, пороэластичная статика, тензор напряжения.