Journal of Siberian Federal University. Mathematics & Physics 2018, 11(5), 569–580

УДК 519.6

Numerical Modelling of the Hydrothermal Regime of the Krasnoyarsk Reservoir

Victor M. Belolipetskii*
Pavel V. Belolipetskii†
Svetlana N. Genova‡
Institute of Computational Modelling SB RAS
Akademgorodok 50/44, Krasnoyarsk, 660036
Russia

Received 18.05.2017, received in revised form 25.09.2017, accepted 10.07.2018

Mathematical models of hydrothermal processes for stratified flows in flow-through reservoirs are considered in the paper. Models are based on the equations of fluid mechanics and heat transfer. They are applied to describe the hydrothermal regime of the Krasnoyarsk Reservoir. It is assumed that the reservoir depth is varied continuously, and the reservoir width is varied in a stepwise way. A numerical algorithm to study two-dimensional vertically stratified flows in terms of the streamfunction and the vorticity is considered. Parametrization of coefficients of the vertical turbulent exchange is proposed. Numerical results for currents at the dam area of the Krasnoyarsk reservoir are presented. The numerical model allows one to determine the temperature of water entering the intake openings in relation to the stratification, the position of water intake and the flow rate of water.

Keywords: numerical modelling, hydro-physics, Krasnoyarsk reservoir, stratified flows.
DOI: 10.17516/1997-1397-2018-11-5-569-580.

Introduction

One of the most important problems is the problem of "clean water". Water has a unique feature among the natural resources of the Earth – it is indispensable. The depletion of water resources is caused not only by the growth of water consumption but also by the water pollution. There are chemical, physical, biological, thermal and radioactive types of pollution. If the flow of contaminants exceeds the self-clean ability of the reservoir ecosystem then contaminants are accumulated in the bottom sediments, and negative processes are exacerbated by so-called secondary pollution. The ecological state of water bodies depends on hydro-physical, hydro-biological, hydro-chemical, meteorological and anthropogenic processes. Hydro-physical processes form the habitat of hydrobionts, and to a large extent determine the transfer and sedimentation of substances, the intensity of the processes of pollution and self-purification of water bodies.

The construction of dams introduces significant changes in the natural conditions of the adjacent areas. The temperature and flow regimes of the river vary both above and below the dam. The change in the temperature regime influences the development of river flora and fauna. A decrease in temperature in summer reduces the self-cleaning ability of the river. After the construction of the Krasnoyarsk hydroelectric power station hydro-ice-thermal regime of the Yenisei river has considerably changed both above and below the dam. The water temperature

*belolip@icm.krasn.ru
†pbel@icm.krasn.ru
‡sv@icm.krasn.ru

below the dam fell by 10–12 degrees in summer and increased by 1.5–3.0 degrees in winter. A sharp change in the temperature regime of the Yenisei river in the lower bay of the Krasnoyarsk HPP deprived the residents of Krasnoyarsk of their usual river recreation in summer.

For the numerical analysis of processes in the upper and lower bay mathematical models of various levels of complexity are used. Mathematical models of hydrothermal processes in water reservoirs are based on the equations of fluid mechanics and heat transfer. Mathematical models of stratified flows have been extensively studied [1–5]. Stratified flows in elongated water bodies were considered [6, 7]. The mathematical model is based on the width-averaged basic equations in Boussinesq and hydrostatic approximations. Various numerical methods for simulation of stratified flow dynamics were developed [8, 9]. The turbulent stratified flows in flowing water bodies are investigated with single and two parameter models of turbulence. The results of simulations are compared with measured data.

In many practical cases, it is necessary to take water masses from certain layers in steady stratified reservoirs. The study of such problems is possible in the framework of the theory of perfect fluid [6, 10, 11]. The densimetric Froude number is the criterion for selective water intake from a stratified reservoir: $Fr = \dfrac{u_0}{\sqrt{gH\Delta\rho/\rho_0}}$, where u_0 is the characteristic flow velocity magnitude, H is the depth, $\Delta\rho = \rho_{max} - \rho_{min}$, ρ_0 is the characteristic density. It was shown that for linear stratification selective extraction is possible when $Fr \leqslant Fr_{cr} = 1/\pi$ [10, 11]. Fig. 1 shows the flow patterns (streamlines $\psi = const$) for various values of the densimetric Froude number. For values $Fr > Fr_{cr}$ the flow zone extends to the entire depth (Fig. 1.a). In the case of the surface location of the water intake for $Fr \leqslant Fr_{cr}$ flow is stratified into two regions. In the upper region water flows into the drain hole but in the lower region water circulates with low velocities (Fig. 1b).

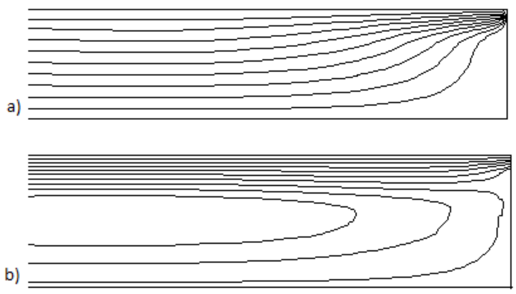

Fig. 1. Flow patterns in the near-dam section of the reservoir in the case of the surface water intake: a) $Fr > Fr_{cr}$, b) $Fr \leqslant Fr_{cr}$

The water temperature coming from the reservoir to the lower bay of the HPP depends on the flow pattern at the dam site. Krasnoyarsk reservoir is extended in the meridional direction and it is in the middle part of the Yenisei river. The reservoir is conventionally divided into 8 hydrological regions as shown in Tab. 1 [12].

The scheme of the reservoir is presented in Tab. 2. The reservoir depth is varied continuously, and the reservoir width is varied in a stepwise way.

1. Two-dimensional non-hydrostatic model of stratified flows

The mathematical model of stratified flows in the reservoir near a dam is based on the equations of inhomogeneous fluid motion in Boussinesq approximation., In the streamfunction-vorticity formulation they are [3]:

Table 1. Characteristics of the Krasnoyarsk reservoir regions [12]

Number of section	Length L, km	Average depth H, m	Average width without bays B, km
1	32	5.6	6.9
2	12	14.2	3.3
3	66	19.0	5.2
4	28	26.0	3.0
5	93	31.5	3.7
6	42	43.5	6.2
7	45	57.2	5.5
8	68	96.0	1.3

Table 2. Scheme of the reservoir *

Distance to the cross-section, km	0	25.9	32	36.3	44	83.2	110	122.3	138
Depth, m	4.6	5.6	9.9	14.2	16.6	19	22.5	26	28.75
Width without bays, km	6.9	6.9	6.9 (3.3)	3.3	3.3 (5.2)	5.2	5.2 (3.0)	3.0	3.0 (3.7)

Distance to the cross-section, km	201.8	231	253.4	273	306.3	318	339.6	386
Depth, m	31.5	37.5	43.5	50.35	57.2	76.6	96	105
Width without bays, km	3.7	3.7 (6.2)	6.2 (5.5)	5.5	5.5	5.5 (1.3)	1.3	1.3

* In the cross-sections where width is abruptly changed two values are shown.

$$\frac{\partial \omega}{\partial t} + U\frac{\partial \omega}{\partial x} + W\frac{\partial \omega}{\partial z} = K_x\frac{\partial^2 \omega}{\partial x^2} + K_z\frac{\partial^2 \omega}{\partial z^2} + g\frac{\partial \hat{\rho}}{\partial x}, \tag{1}$$

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial z^2} = -\omega, \tag{2}$$

$$U = \frac{\partial \psi}{\partial z}, \qquad W = -\frac{\partial \psi}{\partial x}, \tag{3}$$

$$\frac{\partial T}{\partial t} + U\frac{\partial T}{\partial x} + W\frac{\partial T}{\partial z} = K_{xT}\frac{\partial^2 T}{\partial x^2} + K_{zT}\frac{\partial^2 T}{\partial z^2} + \alpha\beta\frac{F_I\, e^{-\beta z}}{c_p\rho_0}, \tag{4}$$

$$\rho = \rho_0[1 + \hat{\rho}(T)]. \tag{5}$$

Here ψ is the streamfunction, ω is the vorticity U, W are components of the water flow velocity in the directions x, z, respectively, t is time, ρ is the water density, ρ_0 is the characteristic water density, g is gravity acceleration, T is the water temperature, K_x, K_z, K_{xT}, K_{zT} are constant coefficients of turbulent exchange, F_I is incoming short-wave solar radiation, β is the radiation absorption coefficient, α is the parameter determining the portion of short-wave radiation penetrating to a depth of ($0 \leqslant \alpha \leqslant 1$), c_p is the specific heat capacity of water. We assume that $\alpha = 0.1$, $\beta = 0.004$. Flows in a rectangular reservoir are considered.

Equations (1)–(5) are supplemented with initial and boundary conditions. Initial conditions are

$$\psi = \psi^0(x, z), \qquad \omega^0 = -\frac{\partial^2 \psi}{\partial x^2} - \frac{\partial^2 \psi}{\partial z^2}, \qquad T = T^0(x, z).$$

Boundary conditions on the water surface ($z = 0,\ 0 \leqslant x \leqslant L$) are

$$\psi = 0, \qquad \omega = \frac{\tau_x}{\rho_0 K_z}, \qquad K_{zT}\frac{\partial T}{\partial z} = -\frac{F_n}{c_p \rho_0}.$$

Boundary conditions at the bottom of the reservoir ($z = H,\ 0 \leqslant x \leqslant L$) are

$$\psi = q, \qquad \omega = \frac{\tau_x}{\rho_0 K_z}, \qquad K_{zT}\frac{\partial T}{\partial z} = \frac{F_{bt}}{c_p \rho_0},$$ (q is specific water discharge (per unit width)).

At the inflow boundary ($x = 0,\ 0 \leqslant z \leqslant H$) the following boundary conditions are set:

$$\frac{\partial \psi}{\partial x} = 0, \quad \omega = \left(\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial z^2}\right)\Big|_{x=0}, \quad \frac{\partial T}{\partial x} = 0.$$

Conditions at the outflow boundary ($x = L,\ \ h_L \leqslant z \leqslant h_L + d$) are

$$\psi = \psi_{out}(z), \qquad \omega = \omega_{out}(z), \qquad \frac{\partial T}{\partial x} = 0.$$

Here τ_x is the wind stress, F_{bt} is the heat exchange with the reservoir bed, $\psi^0(x,z)$, $T^0(x,z)$, $\psi_{out}(z), omega_{out}(z)$ are given functions. The heat exchange with the atmosphere affects the temperature of the reservoir. The total heat flux through the free surface F_n has the form (W/m^2)

$$F_n = (1-\alpha)F_I - (F_{ef} + F_{ct} + F_{evp}),$$

where F_{ef} is effective long-wave radiation, F_{evp} is the heat transfer by evaporation, F_{ct} is the convective heat transfer, F_I is incoming short-wave solar radiation. There are various ways to determine components of heat exchange through the surface of the reservoir [2, 3, 6, 13]. We use the following approach [2, 13]. Short-wave radiation is defined as

$$F_I = 0.94 \cdot Q(h_c)(1 - 0.65 N_0^2),$$

where $Q(h_c) = \left(0.66 + 0.34\dfrac{\gamma - 0.9 + 0.4\sin h_c}{0.1 + 0.4 \sin h_c}\right)\dfrac{\kappa_n \sin^2 h_c}{\rho^2(\sin h_c + 0.107)}$,

$$h_c = \arcsin(\sin\varphi_k \sin\gamma_1 + \cos\varphi_k \cos\gamma_1 \cos((t - t_n)\frac{\pi}{12})),$$

$$\gamma_1 = 0.4 + 23.4\cos(\frac{2\pi}{365}(d+192)) - 0.4\cos(\frac{2\pi}{365}(d - 192)),$$

($\kappa_n = 1.11 \div 1.23$ depends on the atmosphere moisture content, N_0 is the total fraction, h_c is the sun height in degrees, ρ is the air density, $\gamma = 0.94$, φ_k is the latitude in degrees, $t = 0, 1, \ldots, 23$ is the local astronomical time; $t_n = 12$ is local noon time; γ_1 is the solar declination, d is the day of year).

Relations for long-wave radiation, turbulent exchange between the water surface and the atmosphere, the heat flow due to evaporation can be found in [2].

Equation of state for fresh water:

$$\hat{\rho} = -0.68 \cdot 10^{-5}(T-4)^2, \qquad \rho_0 = 1\text{g/cm}^3.$$

For deep reservoirs, equation of state for fresh water takes into account the effect of pressure on density $\rho = \rho(T, p, S)$ [14]. From the hydrostatic approximation $p = 0.1\, z$ (bar, z in meter). Fresh water has a maximum density at some temperature T_{mp} that depends on pressure (depth): $T_{mp}|_{z=0} = 3.98^0 C$; at $z = 100$ m $T_{mp} = 3.79^0 C$; $z = 200$ m $T_{mp} = 3.59^0 C$; $z = 1000$ m $T_{mp} = 1.94^0 C$. It was found that for the Ladoga lake (maximum depth is 230 m) the equation of state in the form $\rho = \rho(T)$ is applicable [15]. For the Krasnoyarsk reservoir (maximum depth is 105 m) the density can be considered independent of pressure.

The wind shear stress is defined by Saimons formula

$$\vec{\tau} = 1.5 \cdot 10^{-2} \left|\vec{W}_a\right| \cdot \vec{W}_a,$$

$\vec{\tau} = (\tau_x, \tau_y)$, $\vec{W}_a = (W_{ax}, W_{ay})$ is the wind velosity (wind speed in m/s, wind stress in g/(sm·s^2)). The boundary conditions for the vorticity on solid walls are obtained from the adhesion condition. The first-order Thom condition is used.

2. Finite difference schemes

Let us introduce computational grid. The grid is uniform in the z direction, and it is and non-uniform in the x direction. Let us introduce the following notations $i = 1, 2, \dots, ii$; $j = 1, 2, \dots, jj$;

$$f(t_n, x_i, z_j) = f^n_{i,j}, \qquad \Delta x_i = x_{i+1} - x_i, \qquad \Delta z = z_{j+1} - z_j,$$

$$f_{i,j,x} = (f_{i+1,j} - f_{i,j})/\Delta x_i, \quad f_{i,j,x1} = (f_{i,j} - f_{i-1,j})/\Delta x_{i-1}, \quad f_{i,j,x0} = 0.5(f_{i,j,x} + f_{i,j,x1}),$$

$$f_{i,j,x,x1} = 2 \cdot (f_{i,j,x} - f_{i,j,x1})/(\Delta x_i + \Delta x_{i-1}),$$

$$f_{i,j,z} = (f_{i,j+1} - f_{i,j})/\Delta z, \quad f_{i,j,z1} = (f_{i,j} - f_{i,j-1})/\Delta z, \quad f_{i,j,z0} = 0.5 \cdot (f_{i,j+1} + f_{i,j-1})/\Delta z,$$

$$f_{i,j,z,z1} = (f_{i,j+1} - 2f_{i,j} + f_{i,j-1})/(\Delta z)^2,$$

$$Af_{i,j} = U^n_{i,j} \cdot f_{i,j,x0} + 0.5 \cdot |U^n_{i,j}| \cdot (f_{i,j,x1} - f_{i,j,x}) + W^n_{i,j} \cdot f_{i,j,z0} + 0.5 \cdot |W^n_{i,j}| \cdot (f_{i,j,z1} - f_{i,j,z}).$$

Here A denotes the scheme with the differences against the flow.

Equation (1) is approximated by the explicit scheme:

$$\omega^{n+1}_{i,j} = \omega^n_{i,j} - \Delta t \cdot A\omega^n_{i,j} + \Delta t \cdot K_x \omega^n_{i,j,x,x1} + \Delta t \cdot K_z \omega^n_{i,j,z,z1} + \Delta t \cdot g \cdot \left(\frac{\partial \hat{\rho}}{\partial x}\right)^n_i.$$

Boundary conditions for the vorticity on solid walls are

on the reservoir bottom ($z = H,\ 0 \leqslant x \leqslant L$): $\qquad \omega_{i,j} = 2\dfrac{q - \psi^n_{i,jj-1}}{(\Delta z)^2}$;

at the inflow boundary ($x = 0,\ 0 \leqslant z \leqslant H$):

$$\omega_{1,j} = -\frac{\psi^n_{3,j} - \psi^n_{1,j}}{\Delta x_1(\Delta x_1 + \Delta x_2)} - \frac{\psi^n_{1,j+1} - 2\psi^n_{1,j} + \psi^n_{1,j-1}}{(\Delta z)^2};$$

at the outflow boundary ($x = L$, $0 \leqslant z \leqslant h_L$ and $h_L + d \leqslant z \leqslant H$): $\omega_{i,j} = 2\dfrac{q - \psi^n_{ii-1,j}}{(\Delta x_{ii-1})^2}$

The difference analogue of equation (2) has the form:

$$2 \cdot \frac{\psi_{i+1,j} - \psi_{i,j} - \Delta x_i(\psi_{i,j} - \psi_{i-1,j})/\Delta x_{i-1}}{\Delta x_i(\Delta x_i + \Delta x_{i-1})} + \frac{\psi_{i,j+1} - 2\psi_{i,j} + \psi_{i,j-1}}{(\Delta z)^2} = -\omega^{n+1}_{i,j}.$$

Taking into account the corresponding boundary conditions for the streamfunction, this system is solved by the method of successive over relaxation:

$$\psi^{k+1}_{i,j} = (1 - \gamma) \cdot \psi^k_{i,j} + \beta 1_i \cdot \left[\beta 2_i \psi^k_{i+1,j} + \beta 3_i \psi^{k+1}_{i-1,j} + \psi^{k+1}_{i,j-1} + \psi^k_{i,j+1} + (\Delta z)^2 \omega^{n+1}_{i,j}\right],$$

where $1 \leqslant \gamma < 2$ is the relaxation parameter, $\Delta x_{i-1} = x_i - x_{i-1} = h_{i-1}\Delta z$,

$$\beta 1_i = \frac{\gamma}{2(1 + \frac{1}{h_i h_{i-1}})}, \qquad \beta 2_i = \frac{2}{h_i(h_i + h_{i-1})}, \qquad \beta 3_i = \frac{2}{h_{i-1}(h_i + h_{i-1})}.$$

The components of the flow velocity vector are zero on solid walls, and they are determined by the given functions at the inflow and outflow boundaries as $U_{i,1} = \dfrac{\psi_{i,2} - \psi_{i,1}}{\Delta z}$, $W_{i,j} = 0$ on the water surface; and $U_{i,j} = \dfrac{\psi_{i,j+1} - \psi_{i,j-1}}{2\Delta z}$, $W_{i,j} = -\psi_{i,j,x0}$ in the inside grid nodes.

In the third stage, the equation for the water temperature with corresponding initial and boundary conditions is solved. The water density is determined by the found temperature.

In the internal points of the region $2 \leqslant i \leqslant ii-1,\ 2 \leqslant j \leqslant jj-1$ we have

$$T_{i,j}^{n+1} = T_{i,j}^n - \Delta t \cdot AT_{i,j}^n + \Delta t \cdot K_{xT} \cdot T_{i,j,x,x1}^n + \\ + \Delta t \cdot \frac{K_{zT\,i,j+1/2}(T_{i,j+1}^n - T_{i,j}^n) - K_{zT\,i,j-1/2}(T_{i,j}^n - T_{i,j-1}^n)}{(\Delta z)^2}.$$

Taking into account the boundary condition on the water surface ($2 \leqslant i \leqslant ii-1,\ j=1$), we obtain

$$T_{i,1}^{n+1} = T_{i,1}^n - \Delta t \cdot AT_{i,1}^n + \Delta t \cdot K_{xT} \cdot T_{i,1,x,x1}^n + 2 \cdot \Delta t \cdot \frac{K_{zT\,i,3/2}(T_{i,2}^n - T_{i,1}^n) + \Delta z \cdot F_n/(c_p\rho_0)}{(\Delta z)^2},$$

at the inflow boundary $(i=1,\ 1 \leqslant j \leqslant jj-1)$

$$T_{1,j}^{n+1} = \begin{cases} T_{1,j}^n & \text{then } U_{1,j}^n \geqslant 0, \\ T_{2,j}^n & \text{then } U_{1,j}^n < 0, \end{cases}$$

at the outflow boundary $(i = ii,\ j1 \leqslant j \leqslant j2\ )$

$$T_{ii,j}^{n+1} = T_{ii,j}^n - \Delta t \cdot AT_{ii,j}^n - 2\,\Delta t \cdot K_{xT} \cdot \frac{T_{ii,j}^n - T_{ii-1,j}^n}{(\Delta x_{ii-1})^2} + \\ + \Delta t \cdot \frac{K_{zT\,ii,j+1/2}(T_{ii,j+1}^n - T_{ii,j}^n) - K_{zT\,ii,j-1/2}(T_{ii,j}^n - T_{ii,j-1}^n)}{(\Delta z)^2},$$

at the bottom $(2 \leqslant i \leqslant ii-1,\ j = jj)$

$$T_{i,jj}^{n+1} = T_{i,jj}^n + \Delta t \cdot K_{xT} \cdot T_{i,jj,x,x1}^n - 2\,\Delta t \frac{K_{zT\,i,jj-1/2}(T_{i,jj}^n - T_{i,jj-1}^n)}{(\Delta z)^2},$$

at the lateral boundary $(i=1,\ jh+1 \leqslant j \leqslant jj-1\ )$

$$T_{1,j}^{n+1} = T_{1,j}^n - \Delta t \cdot AT_{1,j}^n + 2\,\Delta t \cdot K_{xT} \cdot \frac{T_{2,j}^n - T_{1,j}^n}{(\Delta x_1)^2} + \\ + \Delta t \cdot \frac{K_{zT\,1,j+1/2}(T_{1,j+1}^n - T_{1,j}^n) - K_{zT\,1,j-1/2}(T_{1,j}^n - T_{1,j-1}^n)}{(\Delta z)^2},$$

at the lateral boundary $(i = ii,\ 2 \leqslant j \leqslant j1-1$ and $i = ii,\ j2+1 \leqslant j \leqslant jj-1)$

$$T_{ii,j}^{n+1} = T_{ii,j}^n - \Delta t \cdot AT_{ii,j}^n - 2\,\Delta t \cdot K_{xT} \cdot \frac{T_{ii,j}^n - T_{ii-1,j}^n}{(\Delta x_{ii-1})^2} + \\ + \Delta t \cdot \frac{K_{zT\,ii,j+1/2}(T_{ii,j+1}^n - T_{ii,j}^n) - K_{zT\,ii,j-1/2}(T_{ii,j}^n - T_{ii,j-1}^n)}{(\Delta z)^2};$$

at the corner points:

$$i=1,\ j=jj: \quad T_{1,jj}^{n+1} = T_{1,jj}^n + 2\,\Delta t \cdot K_{xT} \cdot \frac{T_{2,jj}^n - T_{1,jj}^n}{(\Delta x_1)^2} - 2\,\Delta t \cdot \frac{K_{zT\,1,jj-1/2}(T_{1,jj}^n - T_{1,jj-1}^n)}{(\Delta z)^2};$$

$i = ii,\ j = 1:$

$$T^{n+1}_{ii,1} = T^n_{ii,1} - 2\,\Delta t \cdot K_{xT} \cdot \frac{T^n_{ii,\text{‘}} - T^n_{ii-1,1}}{(\Delta x_{ii-1})^2} + 2\,\Delta t \cdot \frac{K_{zT\,ii,3/2}(T^n_{ii,2} - T^n_{ii,1}) + \Delta z\, F_n/(c_p\,\rho_0)}{(\Delta z)^2};$$

$$i = ii,\ j = jj:\ T^{n+1}_{ii,jj} = T^n_{ii,jj} - 2\,\Delta t \cdot K_{xT} \cdot \frac{T^n_{ii,jj} - T^n_{ii-1,jj}}{(\Delta x_{ii-1})^2} - 2\,\Delta t \cdot \frac{K_{zT\,ii,jj-1/2}(T^n_{ii,jj} - T^n_{ii,jj-1})}{(\Delta z)^2}.$$

3. Simplified mathematical model of flow in a long reservoir

Hydrostatic approach is applicable in areas far from the dam. A simplified mathematical model to study the hydrothermal regime in a long reservoir is obtained for slow flows with the use of the Boussinesq approximation and and boundary layer approximation [3]:

$$\frac{\partial \omega}{\partial t} = \frac{\partial^2}{\partial z^2}(K_z\,\omega) + g\frac{\partial \hat{\rho}}{\partial x}, \tag{6}$$

$$\frac{\partial^2 \psi}{\partial z^2} = -\omega, \tag{7}$$

$$U = \frac{\partial \psi}{\partial z}, \qquad W = -\frac{\partial \psi}{\partial x},$$

$$\frac{\partial T}{\partial t} + U\frac{\partial T}{\partial x} + W\frac{\partial T}{\partial z} = \frac{\partial}{\partial z}K_{zT}\frac{\partial T}{\partial z} + \alpha\beta\frac{F_I\, e^{-\beta z}}{c_p \rho_0}. \tag{8}$$

Turbulent exchange coefficient.

The averaged flow has both molecular and turbulent viscosities. There are wind mixing and dynamic mixing. They are determined by the flow rate of the reservoir. The total coefficient of vertical turbulent exchange has two terms $K_z = K_z^e + K_z^m$, where K_z^e corresponds to wind mixing, and K_z^m corresponds to dynamic mixing. The coefficient K_z^e is determined by the Prandtl–Obukhov formula with the Ekman approximation [16]:

$$K_z^e = \begin{cases} (0.05h_1)^2\sqrt{\left(\dfrac{\tau}{\rho_0 K_0}\right)^2 e^{-2\alpha z} - \dfrac{g}{\rho_0}\left(\dfrac{\partial \rho}{\partial z}\right)} + K_{min} & \text{for} \quad B \geqslant 0, \\ K_{min} & \text{for} \quad B < 0, \end{cases} \tag{9}$$

where $B = \left(\dfrac{\tau}{\rho_0 K_0}\right)^2 e^{-2\alpha z} - \dfrac{g}{\rho_0}\left(\dfrac{\partial \rho}{\partial z}\right)$, $\tau = \sqrt{\tau_x^2 + \tau_y^2}$ is the wind stress, K_{min}=0.02 cm^2/s is the minimum value of the turbulent vertical exchange coefficient (it corresponds to the molecular viscosity), f is the Coriolis parameter, $K_0 = \dfrac{(0.05\pi)^2 \tau}{2\rho_0 f}$, $\alpha = \sqrt{\dfrac{f}{2K_0}}$, $h_1 = \pi\sqrt{\dfrac{K_0}{2f}}$.

The coefficient K_z^m is determined by the formula [17]:

$$K_z^m = \frac{n\,g\,q}{48\,H^{1/6}}, \tag{10}$$

where n is the bottom surface roughness, q is the specific water discharge (m^2/s), g is acceleration of gravity (m/s^2), H is the depth (m), $K_{zT} = 0.1\,K_z$ if $\dfrac{\partial \rho}{\partial z} \geqslant 0$. If $\dfrac{\partial \rho}{\partial z} < 0$ then there is an unstable state. In this case intensive mixing restores stability, and we have $K_{zT} = K_z$.

Comment. The constant value of the vertical turbulent exchange in problem (1)–(5) is estimated by formula (10). The horizontal turbulent viscosity is determined by the Richardson law: $K_x = K_z(L/H)^{4/3}$.

Let us consider the flow problem for m-th region: $0 \leqslant x \leqslant L_m$ and introduce computational grid $\Delta x_i = x_{i+1} - x_i$, $\Delta z_i = H_i/(jj-1)$, $i = 1, 2, \dots, ii$; $j = 1, 2, \dots, jj$. For this grid

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial x} - \frac{z}{H}\left(\frac{dH}{dx}\right)\frac{\partial}{\partial z}.$$

Initial conditions are $\psi = \psi^0(x,z)$, $\omega = \omega^0(x,z)$. Boundary conditions: on the water surface $(z = 0)$ $\psi = 0$, $\omega = \dfrac{\tau_x}{\rho_0 K_z}$; on the bottom $(z = H(x_i) = H_i)$, $\psi = q_m$, $\omega = 2\dfrac{q_m - \psi_{i,jj-1}}{(\Delta z_i)^2}$ (Thom's condition), q_m is specific water discharge per unit of width. At $x = 0$ $\psi = \psi_m$, $\omega = \omega_m$ are given. At the boundaries of the adjacent regions m and $m+1$ the width and specific water discharge are changed abruptly. The matching of solutions is performed from the condition of the water discharge balance $\psi_{m+1} = \dfrac{B_m}{B_{m+1}}\psi_m$, $\omega_{m+1} = \dfrac{B_m}{B_{m+1}}\omega_m$. Equation (6) is solved with the use of the explicit scheme

$$\omega_{i,j}^{n+1} = \omega_{i,j}^n + \Delta t\,\frac{K_{z\,i,j+1/2}\left(\omega_{i,j+1}^n - \omega_{i,j}^n\right) - K_{z\,i,j-1/2}\left(\omega_{i,j}^n - \omega_{i,j-1}^n\right)}{(\Delta z_i)^2} +$$
$$+ \Delta t\, g\left[\frac{\hat\rho_{i,j}^n - \hat\rho_{i-1,j}^n}{\Delta x_m} - \frac{z_j}{H_i}\,\frac{\hat\rho_{i,j+1}^n - \hat\rho_{i,j-1}^n}{2\Delta z_i}\right], \quad i = 2, 3, \dots, ii,\ j = 2, 3, \dots, jj-1.$$

The difference approximation of equation (7) has the form

$$\frac{\psi_{i,j+1} - 2\psi_{i,j} + \psi_{i,j-1}}{(\Delta z_i)^2} = -\omega_{i,j+1}^{n+1}.$$

Difference equations for the stream function are solved by the sweep method:

$$\alpha_{j+1} = \frac{1}{2 - \alpha_j}, \quad \beta_{j+1} = \frac{\beta_j + (\Delta z)^2\omega_{i,j}^{n+1}}{2 - \alpha_j}, \quad \alpha_1 = 0, \quad \beta_1 = 0, \quad j = 2, 3, \dots, jj-1,$$

$$\psi_{i,jj} = q_m, \quad \psi_{i,j} = \alpha_{j+1}\cdot\psi_{i,j+1} + \beta_{j+1}, \quad j = jj-1, jj-2, \dots, 1.$$

The flow velocity components are determined as

$U_{i,1} = \dfrac{\psi_{i,2} - \psi_{i,1}}{\Delta z_i}$ and $W_{i,1} = 0$ on water surface,

$U_{i,jj} = 0$ and $W_{i,jj} = 0$ on the bottom, $U_{i,j} = \dfrac{\psi_{i,j+1} - \psi_{i,j-1}}{2\,\Delta z_i}$ and $W_{i,j} = -\dfrac{\psi_{i+1,j} - \psi_{i-1,j}}{2\,\Delta x_m} +$

$+\dfrac{z_j}{H_i}\left(\frac{dH}{dx}\right)_i\cdot U_{i,j}$ in the internal grid nodes,

vspace1mm $U_{m+1,j} = \dfrac{B_m}{B_{m+1}}U_{m,j}$ at the border of regions.

The water temperature in the internal nodes is determined with the use of the explicit scheme

$$T_{i,j}^{n+1} = T_{i,j}^n - \Delta t\cdot A\,T_{i,j}^n + \Delta t\left[\frac{K_{zT\,i,j+1/2}\left(T_{i,j+1}^n - T_{i,j}^n\right) - K_{zT\,i,j-1/2}\left(T_{i,j}^n - T_{i,j-1}^n\right)}{(\Delta z_i)^2} +\right.$$
$$\left.+\frac{z_j}{H_i}\left(\frac{dH}{dx}\right)_i\cdot\frac{T_{i,j+1}^n - T_{i,j-1}^n}{2\,\Delta z_i}\right].$$

Taking into account boundary conditions, we obtain on the water surface $(j = 1;\ 2 \leqslant i \leqslant ii)$

$$T_{i,1}^{n+1} = T_{i,1}^n - \Delta t\cdot A\,T_{i,1}^n + 2\,\Delta t\,\frac{K_{zT\,i,3/2}\left(T_{i,2}^n - T_{i,1}^n\right) + \Delta z\cdot F_n/(c_p\rho_0)}{(\Delta z_i)^2},$$

and on the bottom $(j = jj;\ 2 \leqslant i \leqslant ii)$

$$T^{n+1}_{i,jj} = T^{n}_{i,jj} - 2\,\Delta t\,\frac{K_{zT\,i,jj-1/2}\left(T^{n}_{i,jj} - T^{n}_{i,jj-1}\right)}{(\Delta z_i)^2}.$$

4. Numerical results

Numerical experiments were carried out to determine the temperature regime of the Krasnoyarsk reservoir using the simplified two-dimensional model (6)–(9) in the summer period. The reservoir bed scheme consists of regions with continuous depth variation and abrupt change in width. Detailed weather data were used for the relevant period. Fig. 2 shows the calculated temperature profiles (solid lines) in five sections of the Krasnoyarsk Reservoir (July 10, 1986). The results of calculations are consistent with the actual data (points). The initial conditions were taken from the field data on the 1st of June 1986. Measured data were obtained by V. A. Korenkov [3,18]. Calculations were performed for various vertical grid steps: $\Delta z_i = H_i/20$ and $\Delta z_i = H_i/40$. The difference between results was less than 3%.

For the known vertical distributions of water temperature in the near-dam area, the flow patterns for various water intake conditions (water intake positions, water discharge) were determined. In problem (1–5) a quasi-uniform grid with smaller cells near the dam was used in the horizontal direction [19]: $\Delta x_{i+1}/\Delta x_i \approx 1 - \alpha/ii$, $\Delta x_1 = \dfrac{\alpha\, L}{ii \cdot (1 - e^{-\alpha})}$, $\Delta x_{ii} \approx e^{-\alpha} \cdot \Delta x_1$, $\alpha > 0$. When a number of nodes is large the difference between adjacent steps is much less than the step size. The results of the calculations of water temperature and flow velocities on the grids $\{L = 500\ m,\ ii = 31,\ \alpha = 2.65,\ \Delta x_{max} = 46.0\ m,\ \Delta x_{min} = 3.15\ m,\ \Delta z = 3.1\ m\}$ and $\{L = 500\ m,\ ii = 21,\ \alpha = 3.1,\ \Delta x_{max} = 77.3\ m,\ \Delta x_{min} = 3.17\ m,\ \Delta z = 3.1\ m\}$ are very close.

Fig. 2 shows a vertical profile of water temperature in the near-dam section of the Krasnoyarsk reservoir [3,18]. In Figs. 3,4 the calculated distributions of the vertical flow velocity are shown for specific discharge of $3.5\,\mathrm{m}^2/\mathrm{s}$ and various wind speeds (minus sign means that the wind blows against the flow). At a distance of more than 110 m from the dam the vertical component of the flow velocity $W \leqslant 0.04\,cm/s$. At $x < (L - 110)$ the horizontal transport of water masses dominates over the vertical transport. The temperature regime in the vicinity of the dam $(L - 110)x \leqslant x < L$ depends on the vertical flow velocity. If water intake located in the vicinity of the water surface (Fig. 4a) the layers of water from the top to 30 m are involved into the intake. As a result, the temperature of water discharged into the downstream decreases due to the intake of lower water layers. In the case of deep water intake (Fig. 4b) a surface warmer water layers move down and deep water layers rise up.

The case of surface water intake was considered. The water intake was $h = 9.3$ m and $h = 15$ m beneath the surface. If $h = 9.3$ m then the layer 24.8 m thick at a temperature of $T_{nb} = 17.7\,^0C$ is involved in the water intake. When the depth of the water intake is increased to 15 m, the thickness of the water layer involved in the intake is increased to 27.9 m, and the temperature of the water is decreased to $T_{nb} = 16.2\,^0C$. In the case of a deep water intake $(25\ m \leqslant z_{out} \leqslant 34\ m)$ in the summer period water at a temperature of $T_{nb} = 6.75\,^0C$ inflows in the water intake. The depth-average water temperature in the vicinity of the dam is $T_{av} = 7.6\,^0C$. For the specific water discharge 2.8 m^2/s the water temperature flowing into the downstream is $T_{nb} = 19.3\,^0C$ for surface water intake and $T_{nb} = 7.6\,^0C$ for deep water intake.

The temperature slightly varies with depth in spring and autumn, so the temperature of the water flowing to the downstream is close to the depth-average temperature for surface and deep water intakes.

The wind component of the flow velocity calculated by the two-dimensional model is overestimated in the deep areas of the reservoir. The estimation of the wind component of the flow velocity can be obtained from the Ekman theory [20].

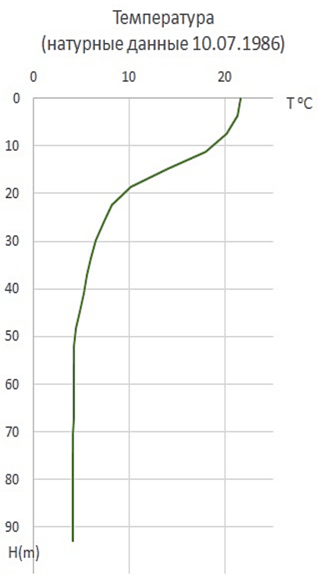

Fig. 2. Vertical profile of water temperature at the near-dam section of the Krasnoyarsk reservoir

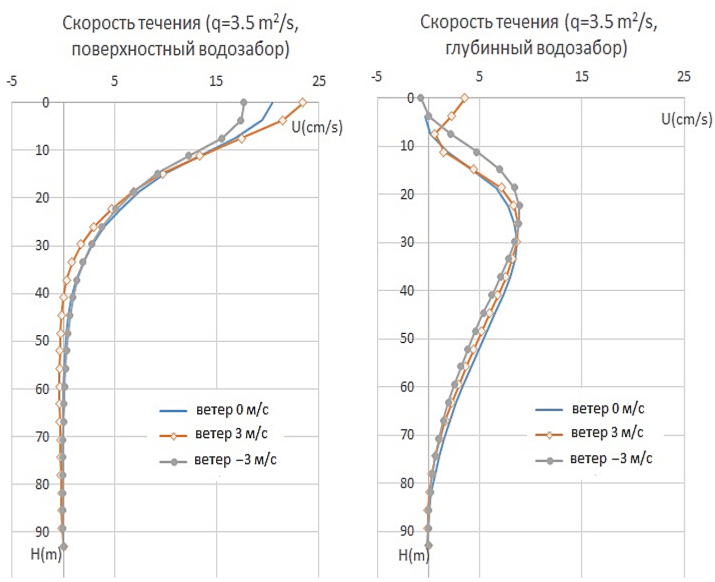

Fig. 3. The calculated distribution of the horizontal flow velocity in the near-dam section of the reservoir for various wind speeds

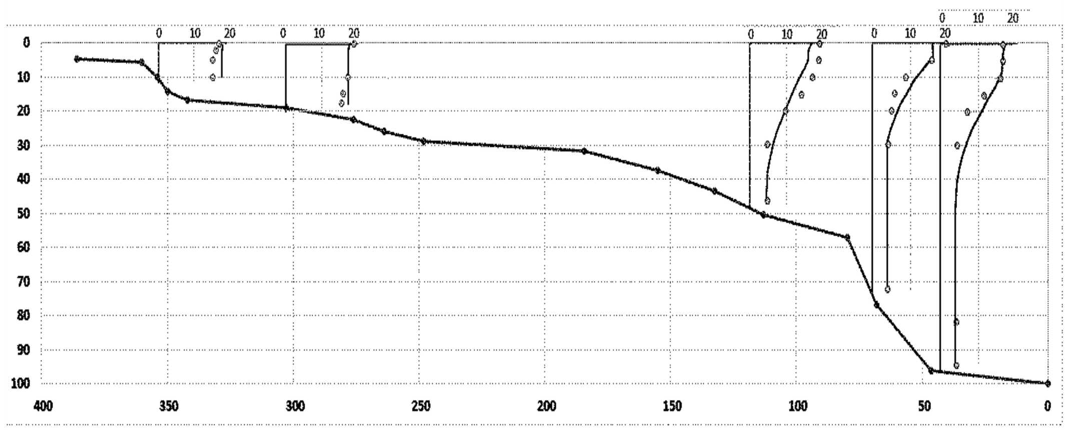

Fig. 4. The calculated distribution of the vertical flow velocity in the near-dam section of the reservoir for various wind speeds.

Calculations for the summer period showed that the surface water intake admits of discharge of layers of water at a temperature that is close to the maximum. It increases the water temperature in the lower bay of the Krasnoyarsk HPP by 6–10 ^0C.

The comparison of numerical results with the available field data showed that the proposed model adequately describes the thermal regime of the reservoir in the summer, and it allows one to assess the impact of various parameters (weather conditions, mode of operation of the hydroelectric power station, the position of water intake) on the water temperature coming from the reservoir to the lower bay of hydroelectric power station.

The work was supported by the Russian Foundation for basic research, the government of the Krasnoyarsk region, the Krasnoyarsk regional Fund for support of scientific and technical activities (project 16-41-240425).

References

[1] O.F.Vasiliev, V.I.Kwon, Y.M.Lytkin, etc., Stratified flow. Hydromechanics, *Results of science and technology. VINITI*, **8**(1975), 74–131 (in Russian).

[2] I.I.Makarov, A.S.Sokolov, S.G.Schulman, Modeling hydrothermal processes of reservoirs-coolers of thermal and nuclear power plants, Moscow, Energoatomizdat, 1986 (in Russian).

[3] V.M.Belolipetskii, S.N.Genova, V.B.Tugovikov, Yu.I.Shokin, Numerical modelling of the problems the channel ways of hydro-ice-thermics. Russian Academy of Science, Siberian Division, Institute of Computational Technologies, Krasnoyarsk Computing Center, 1993.

[4] G.I.Marchuk, V.P.Kochergin, A.S.Sarkisyan ets., Mathematical models of circulation in ocean, Nauka, Novosibirsk, 1980 (in Russian).

[5] C.S.Yih, Stratified flows, New York, Acad. Press, 1980.

[6] B.V.Arkhipov, V.V.Solbakov, D.A.Shapochkin, Two-Dimensional vertical model of the temperature regime of the cooling reservoir, *Vodyanye resursy*, **22**(1995), no. 6, 653–666 (in Russian).

[7] O.F.Vasiliev, A.F.Voevodin, V.S.Nikiforovskaya, Numerical modeling of temperature-stratified flows in systems of deep water bodies, *Computational technologies*, **10**(2005), no. 5, 29–38 (in Russian).

[8] O.B.Bocharov, A.T.Zinoviev, Effect of selective water intake on annual the thermal regime of a deep reservoir, *Water resources management*, (1992b), no. 5, 52–59 (in Russian).

[9] C.Oberkampf, Numerical study of temperature velocity fields in flowing reservoirs, *American Society of Mechanical Engineers. Heat transfer*, **98**(1976), no. 3, 10–18.

[10] H.P. Pao, T.W. Kao, Dynamics of Establishment of Selective Withdrawal from a Line Sink. Part I, *Journal of Fluid Mechanics*, **65**(1974), 657.

[11] I.J.Chen, G.M.Karadi, R.J.Lai, Surface selective withdrawal in sedimentation basins, *ZAMM*, **64**(1984), no. 3, 155–162.

[12] V.S.Kuskovsky, Yu.I.Podlipsky, V.I.Savkin, V.I.Shirokov, Formation of the banks of the Krasnoyarsk reservoir, Novosibirsk, Nauka, 1974.

[13] N.A.Timofeev, Radiation regime of the oceans. Kiev, Naukova Dumka, 1983.

[14] A.E.Gill, Atmosphere-Ocean Dynamics, International Geophysics Series, vol. 30, Academic Press, New York, London, 1982.

[15] G.P.Astrakhantsev, V.V.Menshutkin, N.A.Petrova, L.A.Rukhovets, Modeling of ecosystems of large stratified lakes, St. Petersburg, Nauka, 2003 (in Russian).

[16] V.M.Belolipetsky, S.N.Genova, Investigation of hydrothermal and ice regimes in hydropower station bays, *International Journal of Computational Fluid Dynamics*, **10**(1998), no. 2, 151–158.

[17] I.P.Spitsin, V.A.Sokolova, General and river Hydraulics–Leningrad, Hydpometeizdat, 1990 (in Russian).

[18] V.A.Korenkov, The main results of field studies of temperature regime in the Krasnoyarsk hydroelectric power station and possible ways of solving problems in the lower bief. Whether it is possible to freeze the Yenisei river, Krasnoyarsk, 1994 (in Russian).

[19] N.N.Kalitkin, Numerical methods, Moscow, Nauka, 1978 (in Russian).

[20] V.W.Ekman, On the influence of the Earths rotation on ocean currents, Arkiv Mat., Arston., Fysik, (1905), no. 11, 1–53.

Численное моделирование гидротермического режима Красноярского водохранилища

Виктор М. Белолипецкий
Павел В. Белолипецкий
Светлана Н. Генова
Институт вычислительного моделирования СО РАН
Академгородок, 50/44, Красноярск, 660036
Россия

Для описания стратифицированных течений в проточных водоемах используются математические модели гидротермических процессов в водоемах на основе уравнений механики жидкости и теплопереноса с учетом специфики рассматриваемых задач. Для численного исследования гидротермического режима Красноярского водохранилища применяется схема ложа водохранилища в виде районов с непрерывным изменением глубины и скачкообразным изменением ширины. Рассматривается численный алгоритм для исследования двумерных в вертикальной плоскости стратифицированных течений в проточных водоемах в переменных функции тока. Предложена параметризация коэффициентов вертикального турбулентного обмена. Приводятся примеры расчетов течений на приплотинном участке Красноярского водохранилища. Численная модель позволяет определить температуру воды, поступающей в водозаборные отверстия, в зависимости от стратификации, положения водозабора, расхода воды.

Ключевые слова: численное моделирование, Красноярское водохранилище, стратифицированные течения.