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# Three-dimensional Model of the Evolution of the Ring Current Protons during the Magnetic Storm in the Earth's Magnetosphere

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*The non-stationary three-dimensional mathematical model describing evolution of proton distribution function is offered. The distribution of the ring current protons in the inner magnetosphere during the magnetic storm is studied. The temporal and spatial evolution of the proton phase space densities in a dipole field is calculated using a three-dimensional model, considering radial, pitch angle and energy diffusions. The loss terms are described due to charge exchange and wave–particle interactions. The simulation starts with a quiet time distribution. The model is tested by comparing calculated proton fluxes with Polar/MICS measurement during the magnetic storm on 21–22 October 1999. The good consent of the model pitch angle distributions of protons with the experimental data is received.*

*Keywords: protons, diffusion, magnetic storm, the Earth's magnetosphere.*

## Introduction

During the magnetic storm, energetic plasmas are injected on the nightside from a boundary near the geosynchronous orbit. In response to the convection electric field, these particles drift inward and are trapped by the geomagnetic field and form the storm time ring current. The purpose of the work is further development of three-dimensional models. Therefore, the distribution of the ring current protons during the main phase of a magnetic storm has been studied, using the offered non-stationary three-dimensional mathematical model.

## 1. The mathematical model

The 3D Fokker-Planck equation for the phase space density, describing radial, pitch angle and energy diffusions, charge exchange and due to wave-particle interactions losses, can be expressed [1–5] by

$$\begin{aligned} \frac{\partial f}{\partial t} = L^2 \frac{\partial}{\partial L}\left(L^{-2} D_{LL} \frac{\partial f}{\partial L}\right) + \frac{1}{\sin\alpha}\frac{\partial}{\partial\alpha}\left(\sin\alpha D_{\alpha\alpha}\frac{\partial f}{\partial\alpha} - \sin\alpha\frac{d\alpha}{dt} f\right) + \\ + \frac{1}{\sqrt{E}}\frac{\partial}{\partial E}\left(\sqrt{E} D_{EE}\frac{\partial f}{\partial E}\right) - \lambda\cdot f - \frac{f}{T_{wp}} + S_\perp \sin^2\alpha\cdot f. \end{aligned} \qquad (1)$$

Here, $f$ is the phase space density (referred to as distribution function), $t$ is the time, $L$ is the McIlwain parameter, $\alpha$ is the local pitch angle, $E$ is the kinetic energy, $D_{LL}$ is the radial diffusion coefficient, $D_{\alpha\alpha}$ is the pitch angle diffusion coefficient, $D_{EE}$ is the energy diffusion coefficient, $\frac{d\alpha}{dt}$

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is the pitch angle rate, $\lambda$ is the loss rate for protons against neutralization by charge exchange, $T_{wp}$ is the lifetime due to wave-particle interactions, $S_\perp$ is the perpendicular coefficient of the particle source function.

The full description of the equation (1) is presented in [2–5], excepting the expression with the coefficient of diffusion on energy. Therefore, the energy diffusion coefficient is offered as follows:

$$D_{EE} = \frac{E^2}{k_E \gamma_{\perp 0} \left(\gamma_{\perp 0} + 2\right) T_{wp}}, \tag{2}$$

where $k_E$ is the non-dimensional parameter, $\gamma_{\perp 0}$ is the anisotropy index of a pitch angle distribution for $\alpha = 90^\circ$ and at the initial moment of time $[1, 3, 5]$.

The lifetime due to wave-particle interactions is given by the formula $[3, 5]$

$$T_{wp} = k_T \frac{2R_E L^4 \sqrt{4L-3}\sqrt{m}}{\sqrt{2EL}} \left(1 - 0.15Kp\right), \tag{3}$$

where $k_T$ is the second non-dimensional parameter, $R_E$ is the radius of the Earth, $m$ is the mass of a charged particle and $Kp$ is the index of geomagnetic activity.

In the further we shall plot not the distribution functions of $f$ but the corresponding dependencies of differential fluxes (energy spectra) of protons $j$. It is such a representation that is more convenient for experimental data. For that reason, we shall be using the following relation between the differential flux of particles $j$ and the phase space density $f$ (or the distribution function) $j = 2mEf$, where $m$ is the mass of a proton in this paper.

Then the equation (1) with (2), (3) is solved numerically using the finite element projection method.

## 2. Calculations

The simulation starts with the quiet time conditions $[6, 7]$. The ion composition compiled by Sheldon and Hamilton [6] during the quietest days in 1985 – 1987, seen by the AMPTE/CCE/charge-energy-mass (CHEM) instrument in near-equatorial orbit at $L = 2-9R_E$, is used as initial distribution before storm onset. This data set provides the average differential ion fluxes in an energy range 1 – 300 keV.

The distribution of the ring current protons during the magnetic storm, similar to that on 21 – 22 October 1999 [8], is studied. The $Kp$ index is shown in Fig. 1 as a function of the run time RT during the magnetic storm on 21 – 22 October 1999 (0000 RT = 0613 UT on 21 October 1999, 2000 RT = 0213 UT on 22 October 1999).

For protons with $E = 100$ keV, $L = 5$, $\alpha = 90^\circ$, $Kp = 6$ the coefficient of pitch angle diffusion $D_{\alpha\alpha}$ is approximately equal $5 \cdot 10^{-6}$ 1/s [7]. Then using (3) and [7] it is received $k_T \approx 359$. Assuming the approached validity of the received value $k_T$ and for protons with energy $E = 90$ keV, we use this value in further calculations.

For definition $k_E$ in the formula (2) we use the experimental data Polar/MICS [8] which are presented in Fig. 2 squares. To receive the model pitch angle distribution of protons with energy $E = 90$ keV on $L = 5$ which will be well agreed with the experimental data, the parameter $k_E$ is defined by the numerical decision of the differential equation (1) for MLT = 2300 and the moment of time 0013 UT on 22 October 1999 = 1800 RT. It has turned out that $k_E \approx 1000$.

The calculations on offered non-stationary three-dimensional mathematical model have been executed for a full range of pitch angles from 0° up to 180° at distances $L = 2.26 - 6.6$ in the energy range of protons $E = (85 - 95)$ keV for a real magnetic storm during its 20 hours 30 minutes. In particular, Fig. 2 compares the model proton pitch angle distributions with the pitch angle distributions observed by Polar/MICS for $E = (80 - 100)$ keV, $L = 5$, MLT = 22.9

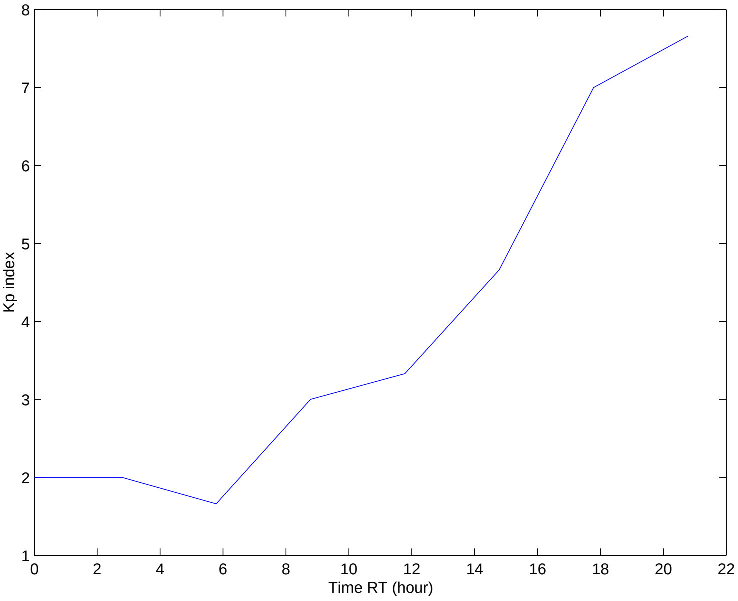

Fig. 1. $Kp$ index as a function of the run time RT during the magnetic storm on 21 – 22 October 1999 (0000 RT = 0613 UT on 21 October 1999, 2000 RT = 0213 UT on 22 October 1999)

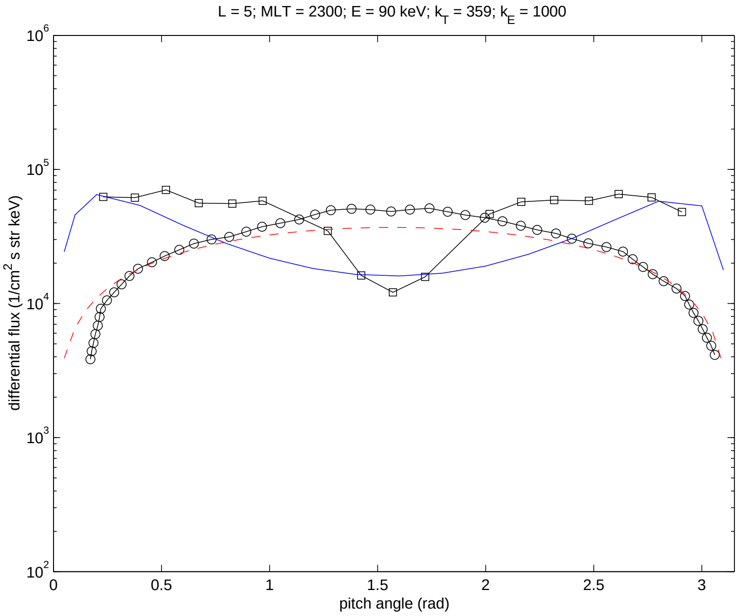

Fig. 2. Model pitch angle distributions of protons and pitch angle distributions measured by Polar/MICS for $E = (80 - 100)$ keV, $L = 5$, MLT = 22.9 – 23.2. Lines (circles and squares) indicate the differential flux at 0000 RT = 0613 UT on 21 October 1999, 23.2 MLT (prestorm condition) and 1802 RT = 0015 UT on 22 October 1999, 22.9 MLT (storm main phase), respectively. Dashed and solid lines indicate the model differential flux ($L = 5$, MLT = 2300, $E = 90$ keV) at 0000 RT and 1800 RT = 0013 UT on 22 October 1999, respectively

– 23.2. Lines (circles and squares) indicate the differential flux at 0000 RT = 0613 UT on 21 October 1999 (prestorm condition) and 1802 RT = 0015 UT on 22 October 1999 (storm main phase), respectively. Dashed and solid lines indicate the model differential flux ($L = 5$, MLT = 2300, $E = 90$ keV) at 0000 RT and 1800 RT = 0013 UT on 22 October 1999, respectively. The pitch angle distribution is pancake-like in the prestorm condition, while it becomes butterfly-like in the storm main phase. The same tendency is seen in, for example, [8].

In total the results of calculations are presented in Fig. 3 and Fig. 4.

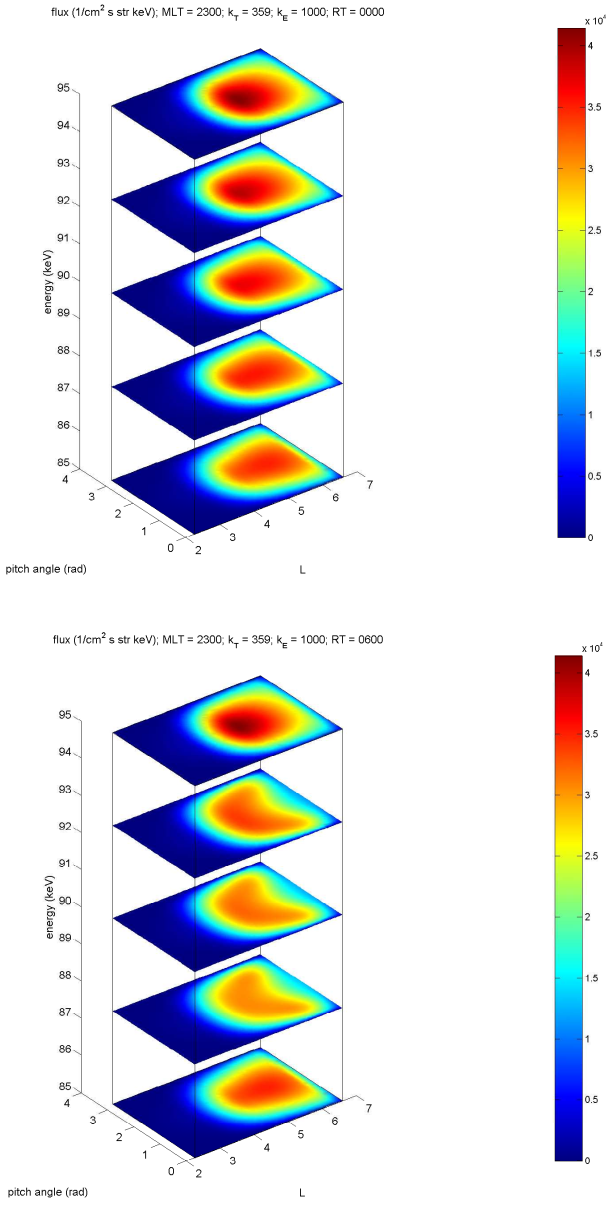

Fig. 3. Model evolution of the proton pitch angle distributions for $E = (85 - 95)$ keV, MLT = 2300, $L = 2.26 - 6.6$ at 0000 RT = 0613 UT on 21 October 1999 (prestorm condition) and 0600 RT = 1213 UT

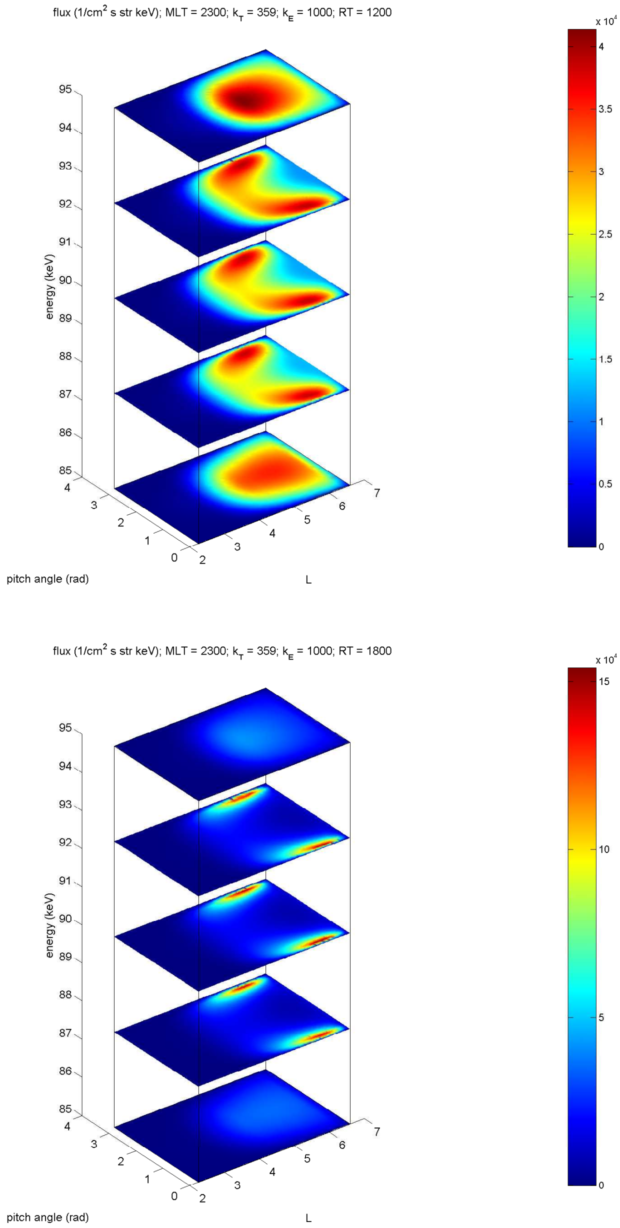

Fig. 4. Model evolution of the proton pitch angle distributions for $E = (85 - 95)$ keV, MLT = 2300, $L = 2.26 - 6.6$ at 1200 RT = 1813 UT on 21 October and 1800 RT = 0013 UT on 22 October 1999 (storm main phase)

Fig. 3 summarizes the model evolution of the proton pitch angle distributions for $E = (85-95)$ keV, MLT = 2300, $L = 2.26 - 6.6$ at 0000 RT = 0613 UT on 21 October 1999 (prestorm condition) and 0600 RT = 1213 UT.

Fig. 4 summarizes the model evolution of the proton pitch angle distributions for $E = (85-95)$ keV, MLT = 2300, $L = 2.26 - 6.6$ at 1200 RT = 1813 UT on 21 October and 1800 RT = 0013 UT on 22 October 1999 (storm main phase).

Thus, Fig. 3 and Fig. 4 show the temporal variation of the simulated pitch angle distributions of protons for four selected times. At 0000 RT = 0613 UT on 21 October 1999 (prestorm condition), the pitch angle distributions are normal or pancake because the simulation started with the normal pitch angle distributions as an initial condition. As time proceeds, the proton flux at pitch angles near 90° decreases, while that at pitch angles near 0° and 180° increases. Finally, the pitch angle distributions become butterfly-like (storm main phase). This tendency is well consistent with the observed butterfly-like pitch angle distributions for high-energy protons and electrons on the nightside of the Earth's magnetosphere [e.g., 8–10].

## 3. Conclusion

1. The non-stationary three-dimensional mathematical model describing the evolution of proton distribution function for a range of pitch angles from 0° up to 180° at distances $L = 2.26 - 6.6$ in the energy range of protons $E = (85 - 95)$ keV is offered.
2. The pitch angle distribution of the ring current protons ($E = (85 - 95)$ keV) in the inner magnetosphere ($L = 2.26 - 6.6$, MLT = 2300) during the magnetic storm on 21 – 22 October 1999 is studied.
3. The pitch angle distribution is pancake-like in the prestorm condition, while it becomes butterfly-like in the storm main phase.
4. The good consent of the model pitch angle distributions of protons with the experimental data observed by Polar/MICS satellite is received.

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# Трехмерная модель эволюции протонов кольцевого тока во время магнитной бури в магнитосфере Земли

**Сергей В. Смолин**

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*Предложена нестационарная трехмерная математическая модель, описывающая эволюцию функции распределения протонов. Исследовано распределение протонов кольцевого тока во внутренней магнитосфере во время магнитной бури. Временная и пространственная эволюция плотностей фазового пространства протонов в дипольном магнитном поле вычисляется, используя трехмерную модель, рассматривающую радиальную, по питч-углам и по энергии диффузии. Выражения потерь описываются вследствие обмена зарядами и взаимодействий волна–частица. Моделирование начинается с распределения магнитоспокойного времени. Модель тестируется сравнением вычисленных потоков протонов с измерением на спутнике Polar/MICS во время магнитной бури 21–22 октября 1999 года. Получено хорошее согласие модельных питч-угловых распределений протонов с экспериментальными данными.*

*Ключевые слова: протоны, диффузия, магнитная буря, магнитосфера Земли.*