On Varieties of Leibniz-Poisson Algebras
with the Identity \( \{x, y\} \cdot \{z, t\} = 0 \)

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Let \( K \) be an arbitrary field and let \( A \) be a \( K \)-algebra. The polynomial identities satisfied by \( A \) can be measured through the asymptotic behavior of the sequence of codimensions of \( A \). We study varieties of Leibniz-Poisson algebras, whose ideals of identities contain the identity \( \{x, y\} \cdot \{z, t\} = 0 \), we study an interrelation between such varieties and varieties of Leibniz algebras. We show that from any Leibniz algebra \( L \) one can construct the Leibniz-Poisson algebra \( A \) and the properties of \( L \) are close to the properties of \( A \). We show that if the ideal of identities of a Leibniz-Poisson variety \( V \) does not contain any Leibniz polynomial identity then \( V \) has overexponential growth of the codimensions. We construct a variety of Leibniz-Poisson algebras with almost exponential growth.

Keywords: Poisson algebra, Leibniz-Poisson algebra, variety of algebras, growth of a variety.

Introduction

Let \( A \) be an algebra over an arbitrary field. A natural and well established way of measuring the polynomial identities satisfied by \( A \) is through the study of the asymptotic behavior of its sequence of codimensions \( c_n(A) \), \( n = 1, 2, \ldots \). The first result on the asymptotic behavior of \( c_n(A) \) was proved by A.Regev in [1]. He showed that if \( A \) is an associative algebra \( c_n(A) \) is exponentially bounded. Such result was the starting point for an investigation that has given many useful and interesting results.

For associative algebras A.R.Kemer in [2] proved that the sequence \( c_n(A) \) is either polynomially bounded or grows exponentially. Then A.Giambruno and M.V.Zaicev in [3] and [4] showed that the exponential growth of \( c_n(A) \) is always an integer called the exponent of the algebra \( A \).

When \( A \) is a Lie algebra, the sequence of codimensions has a much more involved behavior. I.B.Volichenko in [5] showed that a Lie algebra can have overexponential growth of the codimensions. Starting from this, V.M.Petrogradsky in [6] exhibited a whole scale of overexponential functions providing the exponential behavior of the identities of polynilpotent Lie algebras.

In this paper we study Leibniz-Poisson algebras satisfying polynomial identities. Remark that if a Leibniz-Poisson algebra \( A \) satisfies the identity \( \{x, x\} = 0 \) then \( A \) is a Poisson algebra. Poisson algebras arise naturally in different areas of algebra, topology, theoretical physics. We study varieties of Leibniz-Poisson algebras, whose ideals of identities contain the identity \( \{x, y\} \cdot \{z, t\} = 0 \). We show that the properties of such Leibniz-Poisson algebras are close to...
the properties of Leibniz algebras. We show that Leibniz-Poisson algebra can have overexponential growth of the codimensions and construct a variety of Leibniz-Poisson algebras with almost exponential growth.

1. Preliminaries

Let $A(+,\cdot,\{\},K)$ be a $K$-algebra with two binary multiplications $\cdot$ and $\{\}$. Let the algebra $A(+,\cdot,K)$ with multiplication $\cdot$ be a commutative associative algebra with unit and let the algebra $A(+,\{\},K)$ be a Leibniz algebra under the multiplication $\{\}$. The latter means that $A(+,\{\},K)$ satisfies the Leibniz identity

$$\{\{x,y\},z\} = \{\{x,z\},y\} + \{x,\{y,z\}\}.$$ 

Assume that these two operations are connected by the relations ($a, b, c \in A$)

$$\{a \cdot b, c\} = a \cdot \{b, c\} + \{a, c\} \cdot b,$$

$$\{c, a \cdot b\} = a \cdot \{c, b\} + \{c, a\} \cdot b.$$

Then the algebra $A(+,\cdot,\{\},K)$ is called a Leibniz-Poisson algebra.

We make the convention that brackets in left-normed form arrangements will be omitted:

$$\{\ldots\{x_1,x_2\},x_3\ldots,x_n\} = \{x_1,x_2,\ldots,x_n\}.$$ 

Let $L(X)$ be a free Leibniz algebra with multiplication $[,]$ freely generated by the countable set $X = \{x_1,x_2,\ldots\}$. Let also $F(X)$ be a free Leibniz-Poisson algebra. Denote by $P^L_n$ and $P_n$ the vector spaces in $L(X)$ and $F(X)$ accordingly, consisting of the multilinear elements of degree $n$ in the variables $x_1,\ldots,x_n$.

**Proposition 1** ([7]). A basis of the vector space $P_n$ consists of the following elements:

$$x_{k_1} \cdot \ldots \cdot x_{k_r} \cdot \{x_{i_1},\ldots,x_{i_s}\} \cdot \ldots \cdot \{x_{j_1},\ldots,x_{j_t}\},$$

where:

(i) $r \geq 0$, $k_1 < \ldots < k_r$;

(ii) all elements are multilinear in the variables $x_1,\ldots,x_n$;

(iii) each factor $\{x_{i_1},\ldots,x_{i_s}\}, \ldots, \{x_{j_1},\ldots,x_{j_t}\}$ in (1) is left normed and has length $\geq 2$;

(iv) in each product (1) the shorter factor precede the longer: $s \leq \ldots \leq t$;

(v) if two consecutive factors in (1) are brackets $\{\ldots\}$ of equal length

$$\ldots \cdot \{x_{p_1},\ldots,x_{p_r}\} \cdot \{x_{q_1},\ldots,x_{q_s}\} \cdot \ldots,$$

then $p_1 < q_1$.

Denote by $\Gamma_n$ the subspace of $P_n$ spanned by the elements (1) with $r = 0$.

Denote by $L_{\geq 2}(X)$ the subspace of the free Leibniz algebra $L(X)$ spanned by the elements $[x_{i_1},\ldots,x_{i_s}]$ with $n \geq 2$. Also denote by $PL_{\geq 2}(X)$ the subspace of $F(X)$ spanned by the elements $\{x_{i_1},\ldots,x_{i_s}\}$ with $n \geq 2$. Obviously, $L_{\geq 2}(X) \cong PL_{\geq 2}(X)$ as Leibniz algebras. We will use only the notation $L_{\geq 2}(X)$ everywhere as $L_{\geq 2}(X) = PL_{\geq 2}(X)$ up to isomorphism of Leibniz algebras.
Let \( V \) be a variety of Leibniz-Poisson algebras (pertinent information on varieties of PI-algebras can be found, for instance, in [8], [9]). Let \( \operatorname{Id}(V) \) be the ideal of identities of \( V \). Denote

\[
P_n(V) = P_n / (P_n \cap \operatorname{Id}(V)), \quad c_n(V) = \dim P_n(V).
\]

For a variety of Leibniz algebras \( V_L \) denote

\[
P_n^L(V_L) = P_n^L / (P_n^L \cap \operatorname{Id}(V_L)) \quad \text{and} \quad c_n^L(V_L) = \dim P_n^L(V_L).
\]

Let \( \operatorname{Id}(A) \) be the ideal of the free algebra \( F(X) \) of polynomial identities of \( A \).

The next proposition shows how from every Leibniz algebra one can construct a Leibniz-Poisson algebra with some conditions of the source Leibniz algebra.

**Proposition 2** ([7]). Let \( A_L \) be a nonzero Leibniz algebra with multiplication \([\cdot,\cdot]\) over an infinite field \( K \) and let \( A = A_L \oplus K \) be a vector space with multiplications \( \cdot \) and \( \{\cdot,\cdot\} \) defined as

\[
(a + \alpha) \cdot (b + \beta) = (\beta a + \alpha b) + \alpha \beta, \\
\{a + \alpha, b + \beta\} = [a, b], \quad a, b \in A_L, \quad \alpha, \beta \in K.
\]

Then the algebra \( (A, +, \cdot, \{\cdot,\cdot\}, K) \) is a Leibniz-Poisson algebra and the following conditions are true:

(i) \( \operatorname{Id}(A_L) \cap L_{\geq 2}(X) \) and the algebra \( A \) satisfies the identity \( \{x_1, x_2\} \cdot \{x_3, x_4\} = 0 \);

(ii) for any \( n \geq 2 \)

\[
\Gamma_n(A) = P_n^L(A) = P_n^L(A_L)
\]

up to isomorphism of vector spaces;

(iii) for any \( n \) the following equality holds:

\[
c_n(A) = 1 + \sum_{k=2}^{n} \binom{n}{k} \cdot \dim P_k^L(A_L).
\]

## 2. Leibniz-Poisson Algebras with Identity

\[
\{x_1, x_2\} \cdot \{x_3, x_4\} = 0
\]

Denote by \( \operatorname{Id}(\{x_1, x_2\} \cdot \{x_3, x_4\}) \) the ideal of identities of the free Leibniz-Poisson algebra \( F(X) \) generated by the element \( \{x_1, x_2\} \cdot \{x_3, x_4\} \).

**Theorem 1.** Let \( V_L \) be a variety of Leibniz algebras over an infinite field \( K \) defined by a system of identities

\[
\{f_i = 0 \mid f_i \in L_{\geq 2}(X), \ i \in I\}
\]

and let \( \{g_j \in \operatorname{Id}(\{x_1, x_2\} \cdot \{x_3, x_4\}) \mid j \in J\} \), where \( |J| > 0 \), be a set of elements in the ideal \( \operatorname{Id}(\{x_1, x_2\} \cdot \{x_3, x_4\}) \). Let \( V \) be a variety of Leibniz-Poisson algebras defined by the system of identities

\[
\{f_i = 0, \ g_j = 0 \mid i \in I, \ j \in J\}.
\]

Then:
(i) \(Id(V_L) = Id(V) \cap L_{\geq 2}(X)\);
(ii) \(P_n^L(V) = P_n^L(V_L)\);
(iii) \(c_n(V) \geq 1 + \sum_{k=2}^{n} \binom{n}{k} \cdot k!\);
(iv) if \(|I| = 0\) then \(c_n(V) \geq [n! \cdot e] - n\), where \(e = 2.71..., \lfloor \cdot \rfloor\) is an integer part of a number.

Proof. (i) Let \(f \in Id(V_L)\). Then \(f\) follows from the system of identities (3). Therefore, \(f \in Id(V) \cap L_{\geq 2}(X)\) and \(Id(V_L) \subseteq Id(V) \cap L_{\geq 2}(X)\). We will show that \(Id(V) \cap L_{\geq 2}(X) \subseteq Id(V_L)\).

Let \(W\) be a Leibniz-Poisson variety defined by the system of identities (3) and the identity \(\{x_1, x_2\} \cdot \{x_3, x_4\} = 0\). Since the element \(\{x_1, x_2\} \cdot \{x_3, x_4\}\) generates the ideal \(Id(\{x_1, x_2\} \cdot \{x_3, x_4\})\) and \(|J| > 0\) then \(W \subseteq V\). \(Id(V) \subseteq Id(W)\).

Let \((X, V_L)\) be the relatively free algebra of the variety \(V_L\) of countable rank. Theorem of Birkhoff implies that the algebra \(L(X, V_L)\) generates the variety \(V_L\). Hence \(Id(V_L) = Id(L(X, V_L))\). Let \(A = L(X, V_L) \oplus K\) be a Leibniz-Poisson algebra with the multiplications (2). Proposition 2 also implies that \(A \in W\), hence \(Id(W) \subseteq Id(A)\). Proposition 2 also implies the equality

\[Id(V_L) = Id(L(X, V_L)) = Id(A) \cap L_{\geq 2}(X).\]

Since \(Id(V) \subseteq Id(W) \subseteq Id(A)\), it follows

\[Id(V) \cap L_{\geq 2}(X) \subseteq Id(W) \cap L_{\geq 2}(X) \subseteq Id(A) \cap L_{\geq 2}(X) = Id(V_L).\]

(ii) Condition (i) implies that \(Id(V) \cap P_n^L = Id(V_L) \cap P_n^L\) for any \(n \geq 2\). Therefore,

\[P_n^L(V_L) = P_n^L/(Id(V_L) \cap P_n^L) = P_n^L/(Id(V) \cap P_n^L) = P_n^L(V).

(iii) follows from (ii) and [7, Proposition 4].

(iv) Applying the formula

\[n! \cdot \sum_{k=0}^{n} \frac{1}{k!} = [n! \cdot e],\]

inequality from (iii) and \(P_n^L = n!\), we obtain that

\[c_n(V) \geq 1 + \sum_{k=2}^{n} \binom{n}{k} \cdot k! = 1 + \sum_{k=2}^{n} \frac{n!}{(n-k)!} = \frac{n}{t} = n - k = 1 + \sum_{l=0}^{n-2} \frac{n!}{l!} = n! \cdot \sum_{l=0}^{n} \frac{1}{l!} - n = [n! \cdot e] - n.\]

Define the lower and upper exponents for the codimension sequence \(\{c_n(V)\}\) as follows:

\[\text{EXP}(V) = \lim_{n \to \infty} \sqrt[n]{c_n(V)}, \quad \text{EXP}(V) = \lim_{n \to \infty} \sqrt[n]{c_n(V)}.\]

If the lower and the upper limits coincide, we use the notation \(\text{Exp}(V)\).

Theorem 2. Let \(V_L\) be a variety of Leibniz algebras over an infinite field \(K\) defined by the system of identities (3) and let \(V\) be a variety of Leibniz-Poisson algebras defined by the system of identities (3) and the identity \(\{x_1, x_2\} \cdot \{x_3, x_4\} = 0\). Then:

1) For any \(n \geq 2\)

\[\Gamma_n(V) = P_n^L(V) = P_n^L(V_L)\]

- 100 -
up to isomorphism of vector spaces.

2) Let

\[ u^c_n(x_1, \ldots, x_n), \quad s = 1, \ldots, c^L_n(V_L), \]  

be a basis of the vector space \( P^L_n(V_L), n \geq 2 \). Then \( P_n(V) \) has a basis

\[ \psi_{x_1, \ldots, x_n}, \quad x_1, \ldots, x_{n-k} : u^c_k(x_1, \ldots, x_{j_k}), \]

\[ k = 2, \ldots, n, \quad s = 1, \ldots, c^L_k(V_L), \quad i_1 < \ldots < i_{n-k}, \quad j_1 < \ldots < j_k; \]

3) For any \( n \)

\[ c_n(V) = 1 + \sum_{k=2}^{n} \binom{n}{k} \cdot \dim P^L_k(V_L). \]

4) If exponent \( EXP(V_L) \) exists, then \( EXP(V) = EXP(V_L) + 1 \), in particular if there exist constants \( d \geq 0, \alpha \) and \( \beta \) such that for all sufficiently large \( n \) the double inequality holds

\[ n^\alpha d^n \leq c^L_n(V_L) \leq n^\beta d^n, \]

then there exist constants \( \gamma \) and \( \delta \) such that for all sufficient large \( n \) the following double inequality holds

\[ n^\gamma (d + 1)^n \leq c_n(V) \leq n^\delta (d + 1)^n. \]

5) If some Leibniz algebra \( A_L \) generate the variety \( V_L \), then the Leibniz-Poisson algebra \( A = A_L \oplus K \) with multiplications (2) generates the variety \( V \).

6) If \( |I| < +\infty \) and the variety \( V_L \) has the Specht property (i.e. all subvarieties of \( V_L \), including \( V_L \) itself, are finite based), then the variety \( V \) has the Specht property.

7) Let \( W \) be a proper subvariety of \( V \). Then the ideal of identities \( Id(W) \cap L_{\geq 2}(X) \) determines the proper subvariety of \( V_L \).

8) The variety \( V_L \) is nilpotent if and only if the variety \( V \) has a polynomial growth.

Proof. 1) The equality \( P^L_n(V_L) = P^L_n(V) \) follows from Theorem 1. Since for any \( n \) holds equality

\[ \Gamma_n = P^L_n \oplus Id(\{x_1, x_2\} \cdot \{x_3, x_4\}) \cap \Gamma_n, \]

then

\[ \Gamma_n(V) = \Gamma_n/(Id(V) \cap \Gamma_n) = \]

\[ = P^L_n \oplus Id(\{x_1, x_2\} \cdot \{x_3, x_4\}) \cap \Gamma_n = \]

\[ = (Id(V) \cap P^L_n) \oplus Id(\{x_1, x_2\} \cdot \{x_3, x_4\}) \cap \Gamma_n \]

\[ \cong P^L_n/(Id(V) \cap P^L_n) = P^L_n(V). \]

2) Follows from 1) and [7, Proposition 4].

3) Follows from 2).

4) Follows from 3) and the equality \( t^n = \sum_{k=0}^{n} \binom{n}{k} \cdot t^k \).

5) Let some Leibniz algebra \( A_L \) generates the variety \( V_L \). Define the Leibniz-Poisson algebra \( A = A_L \oplus K \) with multiplications (2). Then Proposition 2 and Theorem 1 imply such equalities

\[ Id(A) \cap L_{\geq 2}(X) = Id(A_L) = Id(V_L) = Id(V) \cap L_{\geq 2}(X), \]
with $Id(\mathcal{V}) \subseteq Id(A)$. We will show that $Id(A) \subseteq Id(\mathcal{V})$.

Denote by $B$ the subspace of the free Leibniz-Poisson algebra $F(X)$ spanned by the elements

$$\{x_{i_1}, \ldots, x_{i_s}\} \cdot \ldots \cdot \{x_{j_1}, \ldots, x_{j_t}\}, \quad s \geq 2, \ldots, t \geq 2.$$    

In particular $\Gamma_n = B \cap P_n, \ n = 1, 2, \ldots$. Note that

$$B = L_{\geq 2}(X) \oplus B \cap Id(\{x_1, x_2\} \cdot \{x_3, x_4\}). \quad (7)$$

From [7] it follows that the ideal of identities $Id(A)$ is generated by the set of identities $B \cap Id(A)$. Let $f \in B \cap Id(A)$. Since

$$Id(\{x_1, x_2\} \cdot \{x_3, x_4\}) \subseteq Id(A)$$

and (7) then

$$B \cap Id(A) = L_{\geq 2}(X) \cap Id(A) \oplus B \cap Id(\{x_1, x_2\} \cdot \{x_3, x_4\}).$$

Hence there exist unique

$$g \in L_{\geq 2}(X) \cap Id(A), \quad h \in B \cap Id(\{x_1, x_2\} \cdot \{x_3, x_4\}),$$

such that $f = g + h$. (6) implies that $g \in Id(\mathcal{V})$. Obviously, $h \in Id(\mathcal{V})$, hence $f = g + h \in Id(\mathcal{V})$. Thus $Id(A) = Id(\mathcal{V})$.

6) Let $|I| < +\infty$ and the variety of Leibniz algebras $\mathcal{V}_L$ has the Specht property. Let $\mathcal{W}$ be a subvariety of the variety $\mathcal{V}$. Obviously, $Id(\mathcal{W}) \cap L_{\geq 2}(X)$ is an ideal of identities of the free Leibniz algebra $L(X)$. Theorem 1 implies that

$$Id(\mathcal{V}_L) \subseteq Id(\mathcal{W}) \cap L_{\geq 2}(X).$$

Hence the ideal of identities $Id(\mathcal{W}) \cap L_{\geq 2}(X)$ is generated by a finite number of elements $f_1, \ldots, f_k \in L_{\geq 2}(X)$.

Using the notations of 5), we have

$$B \cap Id(\mathcal{W}) = L_{\geq 2}(X) \cap Id(\mathcal{W}) \oplus B \cap Id(\{x_1, x_2\} \cdot \{x_3, x_4\}). \quad (8)$$

Since $Id(\mathcal{W})$ is generated by $B \cap Id(\mathcal{W})$ (see [7]) then the variety $\mathcal{W}$ is generated by the elements $f_1, \ldots, f_k$ and $\{x_1, x_2\} \cdot \{x_3, x_4\}$.

7) Let $\mathcal{W}$ be a proper subvariety of $\mathcal{V}$. Then the strict inclusion $Id(\mathcal{V}) \nsubseteq Id(\mathcal{W})$ holds. We will show that

$$Id(\mathcal{V}_L) \nsubseteq Id(\mathcal{W}) \cap L_{\geq 2}(X),$$

where $Id(\mathcal{W}) \cap L_{\geq 2}(X)$ is an ideal of identities of $L(X)$.

Since $Id(\mathcal{W})$ is generated by the set $B \cap Id(\mathcal{W})$ (see [7]) and $Id(\mathcal{V}) \nsubseteq Id(\mathcal{W})$, there is such element $f \in B \cap Id(\mathcal{W})$ that $f \notin Id(\mathcal{V})$. Equality (8) implies that there exist unique

$$g \in L_{\geq 2}(X) \cap Id(\mathcal{W}), \quad h \in B \cap Id(\{x_1, x_2\} \cdot \{x_3, x_4\})$$

such that $f = g + h$. Since $h \in Id(\mathcal{V})$ and $f \notin Id(\mathcal{V})$, we obtain that

$$g \notin L_{\geq 2}(X) \cap Id(\mathcal{V}) = Id(\mathcal{V}_L).$$

Therefore, $Id(\mathcal{V}_L) \nsubseteq Id(\mathcal{W}) \cap L_{\geq 2}(X)$. 

- 102 -
8) Follows from 1), 3) and [7, Theorem 1] □

**Corollary.** Let $L(X)$ be a free Leibniz algebra over infinite field $K$ and let $L(X) \oplus K$ be a Leibniz-Poisson algebra with multiplications (2). Then:

(i) $Id(L(X) \oplus K) \cap L(X) = \{0\}$.

(ii) $Id(L(X) \oplus K) = Id(\{x_1, x_2\} \cdot \{x_3, x_4\})$, i.e. the ideal of identities of the algebra $L(X) \oplus K$ is generated by the identity $\{x_1, x_2\} \cdot \{x_3, x_4\} = 0$.

Denote by $\mathring{V}_1$ the variety of Leibniz-Poisson algebras defined by the identity $\{x_1, x_2\} \cdot \{x_3, x_4\} = 0$. Theorems 1 and 2 imply that the codimension growth of $\mathring{V}_1$ is overexponential.

**Proposition 3.** For any $n \geq 1$ the codimension of the identities of $\mathring{V}_1$ satisfy

$$c_n(\mathring{V}_1) = \lfloor n! \cdot e \rfloor - n.$$

**Proposition 4.** Let $\mathring{N}$ be a Leibniz-Poisson variety, defined by the identity $\{x_1, \{x_2, \{x_3, x_4\}\}\} = 0$.

Then the variety $\mathring{V}_1 \cap \mathring{N}$ over a field $K$ of characteristic 0 has almost exponential growth of the codimension sequence.

Proof. [11] and [10] implies that the variety of Leibniz algebras $\mathring{N}$, defined by the identity $[x_1, [x_2, [x_3, x_4]]] = 0$,

has almost exponential codimension growth. Therefore, by Theorem 1, the variety of Leibniz-Poisson algebras $\mathring{V}_1 \cap \mathring{N}$ has overexponential codimension growth.

Let $W$ be a proper subvariety of $\mathring{V}_1 \cap \mathring{N}$. Condition 7) of Theorem 2 implies that the ideal of identities $Id(W) \cap L_{\geq 2}(X)$ defines the proper subvariety of $\mathring{N}$, which has exponentially bounded codimension growth. By condition 4) of Theorem 2, the sequence of codimensions of $W$ is exponentially bounded. □

Denote by $N_s^\mathring{A}$ the variety of Leibniz-Poisson algebras, defined by the identity $\{\{x_1, x_2\}, \ldots, \{x_2s+1, x_2s+2\}\} = 0$.

**Proposition 5.** Variety $\mathring{V}_1 \cap N_s^\mathring{A}$ over a field $K$ of characteristic 0 has the Specht property.

Proof. [12] implies that the variety of Leibniz algebras $N_s^\mathring{A}$, defined by the identity $[[x_1, x_2], ..., [x_{2s+1}, x_{2s+2}]] = 0$,

has the Specht property. Therefore, by 6) of Theorem 2, $\mathring{V}_1 \cap N_s^\mathring{A}$ has the Specht property. □

**References**


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On Varieties of Leibniz-Poisson Algebras with the Identity 


О многообразиях алгебр Лейбница-Пуассона с тождеством \(\{x, y\} \cdot \{z, t\} = 0\)

Сергей М. Рацеев

В данной работе исследуются многообразия алгебр Лейбница-Пуассона, идеалы которых содержат тождество \(\{x, y\} \cdot \{z, t\} = 0\), исследуется взаимосвязь таких многообразий с многообразиями алгебр Лейбница. Показано, что из любой алгебры Лейбница можно построить алгебру Лейбница-Пуассона с похожими свойствами исходной алгебры. Показано, что если идеал тождеств многообразия алгебры Лейбница-Пуассона \(V\) не содержит ни одного тождества из свободной алгебры Лейбница, то рост многообразия \(V\) является сверхэкспоненциальным. Приводится многообразие алгебр Лейбница-Пуассона почти экспоненциального роста.

Ключевые слова: алгебра Пуассона, алгебра Лейбница-Пуассона, многообразие алгебр, рост многообразия.