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Thompson Subgroups and Large Abelian Unipotent Subgroups of Lie-type Groups

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Let U be a unipotent radical of a Borel subgroup of a Lie-type group over a finite field. For the classical types the Thompson subgroups and large abelian subgroups of the group U were found to the middle 1980's. We complete a solution of well-known problem of their description for the exceptional Lie-types.

Keywords: Lie-type group, unipotent subgroup, large abelian subgroup, Thompson subgroup.

Introduction

It is well-known that similarly to A.I.Mal'cev's schema from [1] the problem of enumeration of the large abelian subgroups of a Lie-type group G over a finite field is reduced to the same problem for the unipotent radical U of the Borel subgroup of G. The problem has been under active investigation since 1970's. For classical types the sets $A(U)$ of large abelian subgroups of U were found by the middle 1980's, as well as the subsets $A_N(U)$ of normal subgroups and $A_e(U)$ of elementary abelian subgroups and, also, the Thompsons subgroups

$$J(U) = \langle A \mid A \in A(U)\rangle, \quad J_e(U) = \langle A \mid A \in A_e(U)\rangle.$$

In 1986 A.S.Kondratiev singled out in his survey [2, (1.6)] the following problem:

Problem (A): *Describe the sets $A(U), A_N(U), A_e(U)$ and the Thompson subgroups $J(U)$ and $J_e(U)$ for the remaining cases of G.*

The present paper summarizes the investigations of this problem, carried out by E.P.Vdovin [3,4], the authors [5–8] and G.S.Suleimanova [9–12].

1. Preliminaries

A Chevalley group $\Phi(K)$ over a field K, associated with a root system Φ, is generated by the root subgroups $X_r = x_r(K)$, $r \in \Phi$; the root subgroups taken for the positive roots $r \in \Phi^+$ generate the unipitent subgroup $U = U\Phi(K)$. A twisted group ${}^m\Phi(K)$ is defined as the centralizer in $\Phi(K)$ of a twisting automorphism θ of order $m = 2$ or 3. For a twisted group we

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have $U = U^m\Phi(K) := {}^m\Phi(K) \cap U\Phi(K)$. Besides, θ is a superposition of a graph automorphism $\tau \in Aut\ \Phi(K)$ and a field automorphism $\sigma : t \to \bar{t}$ $(t \in K)$, and for the only continuation $^-$ on Φ of a symmetry of a Coxeter graph of order m we have $\theta(X_r) = \tau(X_r) = X_{\bar{r}}$ $(r \in \Phi)$ [13,14].

Given a group-theoretic property $\mathcal{P}$, every $\mathcal{P}$-subgroup of the highest order is called a *large $\mathcal{P}$-subgroup*. Developing the A.I.Maltsev's approach [1], E.P.Vdovin has mainly calculated [4, Concluding table] the orders $\mathbf{a}(U)$ of large abelian subgroups of finite groups $U = UG(K)$ $(G = \Phi$ or $G = {}^m\Phi)$ and those of Thompson subgroups.

Having described the maximal normal abelian subgroups of U, the authors ([5,6,8]) also described the large normal abelian subgroups of finite groups U by showing that they form the set $A_N(U)$. (In general, a large normal $\mathcal{P}$-subgroup of a finite group is not a large $\mathcal{P}$-subgroup.) It allows us [8] to complete (for types G_2, $^3D_4$ and $^2E_6$) the calculation of the orders $\mathbf{a}(U)$. In [6] the problem (A) is reduced to the question:

Describe the groups U, in which every large abelian subgroup is $G(K)$-conjugate to a normal subgroup of U and enumerate all the exceptional large abelian subgroups of the remaining groups U ([15,16]).

See the exceptions in [6,9] and [12]. In [8] the authors proved

Theorem 1. *Either every large abelian subgroup of U is $G(K)$-conjugate to a normal subgroup of U or $G(K)$ is of type G_2, $^3D_4, F_4$ or $^2E_6$.*

G.S.Suleimanova described the exceptional large abelian subgroups for the type F_4 in [9,11] and those for the type $^2E_6$ in Section 2 and [12]. In Section 3 we complete this description for the types G_2 and $^3D_4$.

Twisted groups $^m\Phi(K)$ required further development of the methods [1]. For the types $^3D_4$ and $^2E_6$ there exists a homomorphism ζ from the lattice of the root system Φ to the lattices of the systems of types G_2 and F_4 respectively; the preimages of the elements of $\zeta(\Phi)$ being the $^-$-orbits in Φ.

Let $^m\Phi = \zeta(\Phi)$. For any $a \in \zeta(\Phi)$ is defined the root subgroup X_a of $^m\Phi(K)$. Similarly to [17] and [8], we have $X_a = x_a(K_\sigma)$, $K_\sigma := \mathrm{Ker}\,(1-\sigma)$ when $\zeta^{-1}(a)$ is an $^-$-orbit of length 1; in the remaining cases $X_a = x_\alpha(K)$. Then $U = UG(K) = \langle X_r \mid r \in G^+\rangle$, where $G = \Phi$ or $^m\Phi$. The standard central series is $U = U_1 \supseteq U_2 \supseteq \cdots$ [13]. Let $\{r\}^+$ be the set of all $s \in G^+$ for which the coefficients in the decomposition of $s - r$ in $\Pi(G)$ are all nonnegative. Set

$$T(r) := \langle X_s \mid s \in \{r\}^+\rangle, \quad Q(r) := \langle X_s \mid s \in \{r\}^+,\ s \neq r\rangle \quad (r \in G).$$

If $H \subseteq T(r_1)T(r_2)\ldots T(r_m)$ and the inclusion fails under every substitution of $T(r_i)$ by $Q(r_i)$ then $\mathcal{L}(H) = \{r_1, r_2, \cdots, r_m\}$ is said to be the *set of corners* of H. Also, $\mathcal{L}_1(H)$ denotes the set of first corners for all elements of H.

We fix a *regular order* of the roots, compatible with the root height function [13, Lemma 5.3.1]. Each element γ of U permits a unique compatible (*canonical*) decomposition into a product of root elements $x_r(\gamma_r)$ $(r \in G^+)$, [14, Lemma 18]. The coefficient γ_r is called an *r-projection* of the element γ. Obviously, the first corner of γ corresponds to the first multiplier in its canonical decomposition.

We use usual notation from [13]: $h(\chi)$ for diagonal automorphisms, n_r for monomial elements and the subgroups $U_r = \langle X_r \mid r \in G^+ \setminus \{r\}\rangle$, $r \in \Pi(G)$. For the root systems of types E_n and F_4 simple roots are denoted by $\alpha_1, \alpha_2, \cdots$, similarly to [18, Tables V-VIII].

Refining [4, Table 4], the following theorem completes the description of Thompson subgroups.

Theorem 2. *Let K be a finite field and $U = UG(K)$. Then:*

a) $J(U) = J_e(U) = U$ in $UG_2(K), |K| > 2$, and in $U^3D_4(K), |K_\sigma| > 2$;

b) $J_e(U) = 1$ and $J(U) = T(a)$ in $U^3D_4(8)$;

c) $J_e(U) = 1$, $J(U) =< \alpha > \times < \alpha^{n_b} >, \alpha = x_a(1)x_{2a+b}(1)$ in $UG_2(2)$;

d) $J(U) = J_e(U) = U_{\alpha_1}$ *in* $U^2E_6(K)$*;*
e) $J(U) = J_e(U) = U_{\alpha_7} \cap U_{\alpha_8}$ *in* $UE_8(K)$.

2. Large Abelian Subgroups of Groups U of Type $^2E_6$

In this section we suppose $U = U^2E_6(K)$. For the root system Φ of type E_6 the corresponding system ${}^m\Phi = \zeta(\Phi) = G$ is of type F_4. According to Section 1, the group U is generated by the root subgroups $X_a = x_a(K_\sigma)$ for all long roots $a \in G^+$ (the first type) and $X_a = x_a(K)$ for all short roots $a \in G^+$. For the system of type F_4 see also the diagram of the positive roots from [17] or [8]. Use the notation $abcd$ for the root $a\alpha_1 + b\alpha_2 + c\alpha_3 + d\alpha_4$ of the system of type F_4 ($\alpha_1, \alpha_2, \alpha_3, \alpha_4$ are simple roots) similarly to [18, Table VIII]. For certain maps $\tilde{} : K \to K$ and $\hat{} : K \to K$ we choose the following subgroups in the group U:

$$\{x_{1111}(t)x_{1231}(\tilde{t}) \mid t \in K\}\{x_{1121}(u)x_{1221}(\hat{u}) \mid u \in K\} \cdot T(0122), \quad (1)$$

$$X_{1111}(K_\sigma)\{x_{1121}(t)x_{1221}(\tilde{t}) \mid t \in K\}X_{1231}(K^{1-\sigma})T(0122), \quad (2)$$

$$X_{1111}(K_\sigma)X_{1121}(K_\sigma)X_{1221}(K^{1-\sigma})X_{1231}(K^{1-\sigma})T(0122), \quad (3)$$

$$\{x_{1121}(t)x_{1221}(\tilde{t}) \mid t \in K\}X_{1231}T(0122), \quad (4)$$

$$\{x_{1121}(t)x_{1221}(ct) \mid t \in K_\sigma\}X_{1221}(K^{1-\sigma})X_{1231}T(0122), \quad c \notin K_\sigma, \quad (5)$$

$$T(0122)U_6, \quad (6)$$

$$(T(0121) \cap E_6(K_\sigma))U_7. \quad (7)$$

Let $G(K) = {}^2E_6(K)$. The aim of this section is to prove that, up to $G(K)$-conjugacy, the subgroups (1) – (6) for $2K = K$ and the subgroup (7) for $2K = 0$ are all large abelian subgroups of U.

We need the following lemma [8, Lemma 4.4].

Lemma 1. *If* $[x_r(F), x_s(V)] \subseteq Q(r+s)$ *in the group* U *for* $F, V \subseteq K$, $FV \neq 0$, *then* $r+s$ *is of the first type,* r *and* s *are not of the first type, and, up to a diagonal automorphism conjugacy,* $F \subseteq K_\sigma$, $V \subseteq K^{1-\sigma}$.

According to A.I. Mal'cev [1], a subset Ψ of Φ^+ is said to be abelian if $r+s \notin \Phi$ for all $r, s \in \Psi$. A subset Ψ of Φ^+ is said to be *p-abelian* (E.P. Vdovin [4]), if for all $r, s \in \Psi$ either $r+s \in \Phi$ and the structure constant C_{11rs} in the Chevalley formula is zero in characteristic p, or $r+s \notin \Phi$. The maximal 2-abelian subsets for type F_4 are listed in [4, Table 3]. The table contains 7 rows and 13 columns, so the subsets are denoted by $\Psi_{i,j}$ (i is the row number and j is the column number). In particular, $\Psi_{2,12} = \{0121\}^+ := \Psi_1$ and also $\Psi_2 = \{1111\}^+ \cup \{0122\}^+$, $\Psi_3 = \{0011, 0111, 1111, 1231\} \cup \{0122\}^+$ are $\Psi_{2,13}, \Psi_{6,10}$, respectively. Using [4] we easily obtain the following lemma.

Lemma 2. *Each maximal 2-abelian subset of the root system of type* F_4 *either coincides with one of the sets* $\Psi_{1,j}$, $\Psi_{4,j}$ $(1 \leqslant j \leqslant 13)$ *of order* 10, *or is* W*-conjugate to one of the sets* Ψ_i, $\bar{\Psi}_i$ *of order* 11 *for* $i = 1$, 2 *or* 3.

Let $m(x) := \mathcal{L}_1(x)$ $(x \in U)$. The following three lemmas is proved in [12].

Lemma 3. *Let* A *be a large abelian subgroup of* U *of type* $^2E_6$. *Then for all* $x, y \in A$, $x, y \neq 1$, *the subset* $\{m(x), m(y)\}$ *of* Φ *of type* F_4 *is 2-abelian and, if* $m(x) + m(y) \in \Phi$, *then the* $m(x)$*-projections of all elements of* A *with the first corner* $m(x)$ *are contained in a 1-dimentional* K_σ*-module.*

Lemma 4. *Let A be a large abelian subgroup of U of type ${}^2E_6$, Ψ be a maximal 2-abelian subset of the root system of type F_4 and let $\mathcal{L}_1(A) \subseteq \Psi$. Then:*

a) if $\{r_1, r_2, r_3\} \subseteq \Psi$, $r_i + r_j \in \Phi$ and $C_{11,r_i,r_j} = \pm 2$ for all $i \neq j$ then the subset $\{r_1, r_2, r_3\}$ is contained in $\mathcal{L}_1(A)$,

b) if $r + s \in \Phi$ for the roots $r, s \in \Psi$, $C_{11,r,s} = \pm 2$, and the pair r, s is not contained in any triple from a), then $r \in \mathcal{L}_1(A)$ or $s \in \mathcal{L}_1(A)$;

c) if Ψ contains a root r such that $(r + \Psi) \cap \Phi = \emptyset$ then $r \in \mathcal{L}_1(A)$.

Lemma 5. *If Ψ is a maximal 2-abelian subset of type F_4, for which there exists a large abelian subgroup A such that $m(A) \subseteq \Psi$, then Ψ is W-conjugate to Ψ_1 when $2K = 0$ or to Ψ_2 when $2K = K$. Furthermore, all such sets Ψ are exhausted, respectively, by the sets*

$$\Psi_{2,9}, \Psi_{2,12}, \Psi_{3,1}, \Psi_{3,7}, \Psi_{3,12}, \Psi_{5,1}, \Psi_{5,3}, \Psi_{5,6}, \Psi_{5,8}, \Psi_{5,9}, \Psi_{5,13}, \Psi_{6,4}, \Psi_{6,11}, \Psi_{7,1};$$
$$\Psi_{2,10}, \Psi_{2,11}, \Psi_{2,13}, \Psi_{3,13}, \Psi_{5,2}, \Psi_{5,4}, \Psi_{5,5}, \Psi_{5,7}, \Psi_{6,7}, \Psi_{7,2}, \Psi_{7,3}.$$

Now we consider a large abelian subgroup A of $U = U^2E_6(K)$.

Lemma 6. *If A has a simple corner p then $A \subseteq T(p)$ and $p \neq \alpha_1$.*

Proof. In the canonical decomposition of elements of U we use the regular order of the system Φ, defined by the order $\alpha_1 < \alpha_2 < \alpha_3 < \alpha_4$ of the simple roots. Note that the inverse order is also regular.

Suppose that A has at least two simple corners $p < q$. None of the sets $\Psi_{i,j}$ from Lemma 5 contains α_1, so we get $p > \alpha_1$. Suppose that $q = \alpha_4$. If we replace the regular order of Φ by the inverse order then $\mathcal{L}_1(A)$ will contain α_4 and, by Lemma 5, $\mathcal{L}_1(A)$ is contained in one of the sets $\Psi_{5,4}$, $\Psi_{5,5}$, $\Psi_{5,6}$ for $2K = K$ or $\Psi_{5,3}$, $\Psi_{5,6}$, $\Psi_{5,8}$, $\Psi_{5,9}$ for $2K = 0$. Each of these sets contains the root 1242. This root is the only root in Φ^+ of height 9. By Lemma 4, c), the root 1242 is contained in $\mathcal{L}_1(A)$. It is clear, that if there exists an element in $A \cap T(r)$ with the corner r, then $r \in \mathcal{L}_1(A)$ for any regular order of Φ. Therefore, the roots α_2 and α_4 can not be corners in A simultaneously. If $q = \alpha_3$ then similarly $1242 \in \mathcal{L}_1(A)$. Therefore, the case $p = \alpha_2, q = \alpha_3$ is also impossible.

In the remaining case $p = \alpha_3$, $q = \alpha_4$, by Lemmas 5, 4 and [4, table 3], we get $1122 \in \mathcal{L}_1(A)$ for the inverse order of Φ. Therefore, for the initial order, the set $\mathcal{L}_1(A)$ contains a root of height 6. However, this root and the root α_3 can not be contained in $\mathcal{L}_1(A)$ simultaneously, by Lemma 5 and [4, table 3]. Therefore, in all cases the subgroup A can have only one simple corner.

Let A have a simple corner α_i and $A \subseteq X_{\alpha_i}U_2$. Suppose that $A \not\subseteq T(\alpha_i)$. Then A has the corner 0011 for $i = 2$, so A^{n_4} has the simple corners α_2 and α_3. If $i = 3$ then A has the corner 1100, therefore, A^{n_1} has the corners α_2 and α_3. If $i = 4$ then A has a corner $r \in \{1100, 0110, 1110, 0120, 1120, 1220\}$. In the first and second cases A^{n_2} has the corner α_1 or α_3, respectively, and the corner α_4. The third case is reduced to the second case by n_1-conjugacy. In the fourth case A^{n_3} has a corner α_2 and $A^{n_3} \not\subseteq T(\alpha_2)$; this gives a contradiction with the proved above. The fifth case is reduced to the fourth case by a n_1-conjugacy. The sixth case is reduced to the fifth case by a n_2-conjugacy. □

Similarly we obtain the following two lemmas.

Lemma 7. *If A has a corner r of height 2, then $A \subseteq T(r)$ and $r \neq 1100$.*

Lemma 8. *If A has a corner r of height 3, then:*
a) $A \subseteq T(r)$ for $r = 0111$;
b) $A \subseteq T(1110)T(0120)$ for $r = 0120$;
c) $A \subseteq T(1110)T(0122)$ for $r = 1110$ and $A \subseteq X_{1110}U_4$.

Lemma 9. *Each large abelian subgroup of U is $G(K)$-conjugate to a subgroup of U_2.*

Proof. Let A be a large abelian subgroup in U and $A \not\subseteq U_2$. By Lemma 6, there exists a simple corner $p \neq \alpha_1$ in A and $A \subseteq T(p)$. If $p = \alpha_2$ then $\mathcal{L}_1(A)$ is contained in one of the sets $\Psi_{2,9}$, $\Psi_{3,1}$ for $2K = 0$ or $\Psi_{2,10}$, $\Psi_{2,11}$ for $2K = K$, and $1100, 1222 \in \mathcal{L}_1(A)$, by lemma 5. Therefore for arbitrary non-zero elements $t, u \in K_\sigma$ and suitable $t_i, u_i \in K$ there exist elements $x, y \in A$ such that

$$\begin{gathered} x = x_{1100}(t)x_{0110}(t_1)x_{1110}(t_2)x_{0120}(t_3)x_{0111}(t_4) \mod U_4, \\ y = x_{1222}(u)x_{1232}(u_1)x_{1242}(u_2) \mod U_{10}. \end{gathered} \tag{8}$$

If $t_1 \neq 0$ for all elements of the form (8) then in the inverse order we have $0110 \in \mathcal{L}_1(A)$. However, the sets $\Psi_{2,11}$ and $\Psi_{3,1}$ do not contain this root. Since

$$[x, y] = x_{1342}(ut_3 \pm (\bar{u}_1 t_1 + u_1 \bar{t}_1)) \mod U_{11},$$

we have $t_3 = 0$ for $t_1 = 0$. So, there exists an element $x' \in A$ such that

$$x' = x_{1100}(t)x_{1110}(t_2)x_{0111}(t_4) \mod U_4$$

and hence

$$x' = x_{1100}(t)x_{1110}(t_2)x_{0111}(t_4)x_{1120}(t_5)x_{1111}(t_6)x_{0121}(t_7) \mod U_5$$

for some t_5, t_6, $t_7 \in K$. We may cancel t_2 by X_{α_3}-conjugacy. Moreover,

$$\begin{aligned} &(x')^{x_{\alpha_3}(y)} = x_{1100}(t)x_{1110}(t_2 + ty)x_{0111}(t_4)x_{1120}(t_5 \pm ty\bar{y} \pm (t_2\bar{y})^{1+\sigma}) \\ &x_{1111}(t_6)x_{0121}(t_7') = \\ &\qquad = x_{1100}(t)x_{1110}(ty)x_{0111}(t_4)x_{1120}(t_5 \pm ty\bar{y})x_{1111}(t_6)x_{0121}(t_7') \mod U_5 \quad (y \in K). \end{aligned}$$

If $y \in K$ and $K^\sharp = \langle y \rangle$ then $K^\sharp_\sigma = \langle y\bar{y} \rangle$. Therefore we can choose y such that $ty\bar{y} = \mp t_5$. So, we can transform x' by an U-conjugacy to the form

$$x' = x_{1100}(t)x_{1110}(ty)x_{0111}(t_4)x_{1111}(t_6)x_{0121}(t_7') \mod U_5 \quad (y \in K).$$

Then

$$(x')^{n_3} = x_{1110}(ty)x_{0111}(t_7')x_{1120}(t)x_{1111}(t_6')x_{0121}(t_4) \mod U_5 \quad (y \in K).$$

Suppose that $t_7' \neq 0$. Then in the inverse order we get $0111 \in \mathcal{L}_1(A^{n_3})$ and $\mathcal{L}_1(A^{n_3}) = \Psi_{3,1}$ for $2K = 0$ or $\mathcal{L}_1(A^{n_3}) \subseteq \Psi_{2,10}$ for $2K = K$. In the first case $1111 \in \mathcal{L}_1(A^{n_3})$. Therefore, in the initial order, the set $\mathcal{L}_1(A^{n_3})$ $(= \Psi_{2,9})$ must contain a root of height 4; so we get a contradiction with [4, Table 3]. In the second case, due to inclusion $0111 \in \mathcal{L}_1(A^{n_3})$ we get that the 1231-projection of the set of all elements $z \in A^{n_3}$ with $m(z) = 1231$ can not coincide with K, by lemma 1. Hence, in the initial order, $\mathcal{L}_1(A^{n_3})$, which is contained in $\Psi_{2,11}$, must contain the root 1111. Since in the inverse order $\mathcal{L}_1(A^{n_3}) \subseteq \Psi_{2,10}$ then the set $\Psi_{2,10}$ must contain a root of height 4, and we also get a contradiction with [4, Table 3]. Consequently,

$$(x')^{n_3} = x_{1110}(ty)x_{1120}(t)x_{1111}(t_6')x_{0121}(t_4) \mod U_5 \quad (y \in K).$$

By U-conjugacy, we get $t_6' = 0$ and for some $u_i \in K$ we obtain the equality

$$(x')^{n_3 n_4} = x_{0120}(u_1)x_{1120}(u_2)x_{1111}(u_3)x_{0121}(u_4) \mod U_5 \quad (u_3 \neq 0).$$

We may assume that $u_1 = 0$ because otherwise $0120 \in m(A^{n_3 n_4})$ and $\alpha_2 \notin \mathcal{L}_1(A^{n_3 n_4})$, see [4, Table 3]. Moreover, $u_2 = 0$, since otherwise $1120 \in \mathcal{L}_1(A^{n_3 n_4})$ and $\alpha_2 \notin \mathcal{L}_1(A^{n_3 n_4})$. Also, $u_4 = 0$, since otherwise in the inverse order of G we have $0121 \in \mathcal{L}_1(A^{n_3 n_4})$ and $\alpha_2 \notin \mathcal{L}_1(A^{n_3 n_4})$. Note that $1220 \in \Psi_{2,9} \cap \Psi_{2,10} \cap \Psi_{2,11} \cap \Psi_{3,1}$. By Lemma 4, $1220 \in \mathcal{L}_1(A^{n_3 n_4})$ and the 1220-projection of the elements y with $m(y) = 1220$ coincides with K_σ. Thus, if $A^{n_3 n_4}$ has a corner

α_2, then we may assume that the 1220-projection of $(x')^{n_3 n_4}$ is zero, up to a multiplication by a suitable element y. Applying the U- and n_3-conjugation to $(x')^{n_3 n_4}$, we get an element of the form

$$x_{1121}(v_1)x_{0122}(v_2) \mod U_6 \quad (v_1 \neq 0).$$

Hence $1121 \in \mathcal{L}_1(A^{n_3 n_4 n_3})$. It follows $\alpha_2 \notin \mathcal{L}_1(A^{n_3 n_4 n_3})$ and $A^{n_3 n_4 n_3} \subseteq U_2$.

If $p = \alpha_3$ then we get $1231 \in \mathcal{L}_1(A)$, by Lemma 5. The relation

$$1 = [A \cap U_7, A] = [A \cap U_7, X_{\alpha_3} \cap (AU_2)] \mod U_9$$

shows that $A \cap U_7 = A \cap T(1231)$. Up to X_{α_4}-conjugacy, $A \cap (X_{1231}U_9)$ has an element γ with the corner 1231. Since $1 = [\gamma, A] \mod U_9$, we have $A \subseteq X_{\alpha_3}T(0110)$ and $A^{n_4} \subseteq U_2$. Similarly we consider the case $p = \alpha_4$. □

Analogously we proved

Lemma 10. *Any large abelian subgroup of U is G-conjugate to a subgroup of U_3 and even to a subgroup of U_4.*

Finally we get that either $2K = K$ and the subgroup A is G-conjugate to ones from (1) - (6) or $2K = 0$ and A is G-conjugate to the normal subgroup (7).

Remark 1. Taking into account that (1) – (7) are abelian subgroups, we obtain the equalities $A(U) = A_e(U)$ and $J(U) = J_e(U) = U_{\alpha_1}$ for the group $U = U^2E_6(K)$. All large abelian subgroups of the group $UF_4(K)$ are described in [9] and [11].

3. Large Abelian Subgroups of Groups U of Type G_2 and $^3D_4$

According to § 1, the root elements $x_r(t)$ of the groups U of type G_2 and $^3D_4$ match the roots of the system G_2. Choosing its simple roots a and b such that $|a| < |b|$, we use a hypercentral automorphism ς_d $(d \in K)$ of a group U (see [17]), for which $\varsigma_d(x_b(t)) = x_b(t)x_{3a+b}(2dt)$ mod U_5 $(t \in K)$. We set

$$\alpha := x_a(1)x_{2a+b}(1), \qquad \beta_c(t) := x_{a+b}(t)x_{2a+b}(tc). \tag{9}$$

We now prove the following theorem.

Theorem 3. *Each large abelian subgroup of the group $U = UG_2(K)$ is $G_2(K)$-conjugate to one of the following subgroups:*
a) a normal large abelian subgroup of U;
b) an image under some automorphism ς_d $(d \in K)$ of a subgroup, which is $(X_a n_a)$-conjugate to U_3 or $X_{a+b}U_4$ for $6K = K$;

$$c)\ \{x_b(t)x_{3a+b}(t) \mid t \in K\}\beta_d(K)U_5 \quad (d \in K) \quad \text{for even } |K| > 2; \tag{10}$$

d) $\langle \alpha, \beta_1(1)\rangle U_4$ for $|K| = 4$.

The proof of the theorem is based on a number of lemmas.

In [5, 7, 8], the normal large abelian subgroups of U are described as large normal abelian ones. The following lemma follows from [8].

Lemma 11. *If the group U is of type G_2 then the set $A_N(U)$ consists of*

$$U_3 \text{ and } \beta_c(K)U_4\ (c \in K) \text{ for even } |K| > 2, \quad U_3 \text{ for } 6K = K,$$
$$U_2 \text{ for } 3K = 0, \quad \langle\alpha\rangle \times \langle\beta_1(1)\rangle \text{ for } |K| = 2.$$

Up to diagonal automorphisms, normal large abelian subgroups of the group $U^3D_4(K)$, are exhausted by the groups:

$$U_3 \text{ and } \beta_c(K_\sigma)\cdot x_{2a+b}(K^{1+\sigma})\cdot U_4 \ (c\in K_\sigma) \text{ for even } |K_\sigma|>2,$$
$$U_3 \text{ for } 2K=K, \quad \langle\alpha\rangle\times\langle\beta_1(1)\rangle\times x_{2a+b}(K^{1+\sigma}) \text{ for } |K_\sigma|=2.$$

Corollary 1. *The order $\mathbf{a}(U)$ of large abelian subgroups of the group $U=UG(K)$ of type G_2 or $^3D_4$ equals $|U_3|$, except the cases $|K|=2$ or $3K=0$ for the group $UG_2(K)$ where $\mathbf{a}(U)=|K|^4$ and the group $U^3D_4(8)$ where $\mathbf{a}(U)=2^6$.*

Due to [6, Theorem 2], the group U of type G_2 satisfies the following isomorphisms: $U/U_3\simeq UA_2(K)$ and $U/U_4\simeq UB_2(K)$. The following lemma is well known for the group $UA_2(K)\simeq UT(3,K)$.

Lemma 12. *Let A be a maximal abelian subgroup and Z be the center of the group $U\Phi(K)$. Then $A=\{x_a(t)x_b(ct)|t\in K\}Z$ $(c\in K)$ or $T(b)$ for the type A_2. For the type B_2 we have $A=T(b)$ or A is B-conjugate either to X_aZ or for the cases $2K=K$ and $2K=0$ to the subgroup, respectively,*

$$\{x_a(t)x_b(t)x_{a+b}((t^2-t)/2 \mid t\in K)\}Z, \quad \langle x_a(1)x_b(1)\rangle Z. \tag{11}$$

Proof. The center Z of the group U of type B_2 equals U_3 for $2K=K$ or U_2 for $2K=0$. If there exists an element $\gamma\in A$ having two corner, then up to B-conjugation we may suppose that $\gamma=x_a(1)x_b(1)$. Choosing an arbitrary element $\beta=x_a(t)x_b(t')x_{a+b}(t'')$ mod U_3 of A, we find

$$1=[\beta,\gamma]=x_{a+b}(t'-t) \mod U_3, \quad t'=t \ (t\in K);$$
$$[\beta,\gamma]=[x_a(t),x_b(1)][x_b(t),x_a(1)][x_{a+b}(t''),x_a(1)]=x_{2a+b}(2t''+t-t^2).$$

(The signs of the structural constants are chosen according to [6, Theorem 2].) If $2K=0$ then $t^2-t=0$ and $\beta\in\langle\gamma\rangle Z$ When $2K=K$ we have $t''=(t^2-t)/2$ and hence A is the first subgroup in (11). □

Setting $\pi:=1+\sigma+\sigma^2$ for the type $^3D_4$ we require the following lemma.

Lemma 13. *If $2K=K$, then* $\mathrm{Ker}\,(1+\sigma)=0$. *In the general case we have:*

$$K=K^{1+\sigma}+K_\sigma,\ K_\sigma\cap K^{1+\sigma}=2K_\sigma, \quad K^\pi=K_\sigma,\ \mathrm{Ker}\,(\pi)=K^{1-\sigma}.$$

Proof. If $\bar v=-v$, then $\bar{\bar v}=-\bar v=v$, $v=\bar v\in K_\sigma$ and $2v=0$. If $2K=K$ then $\mathrm{Ker}\,(1+\sigma)=0$. Since for any K_σ-linear transformation of the field K the sum of the rank and defect equals 3, the remaining statements of the lemma easily follow from relations

$$K\supseteq K^{1+\sigma}+K^\pi\supseteq K^{\sigma^2}=K, \qquad 0=1-\sigma^3=(1-\sigma)\pi=\pi(1-\sigma). \qquad □$$

The order of a subgroup A of a group $U=UG(K)$ of type G_2 or $^3D_4$ may be estimated using the orders of intersections of the projections A_i:

$$A\cap U_i=x_r(A_i) \ mod\ U_{i+1}, \quad 1<ht(r)=i\leqslant 5; \tag{12}$$

$$|A|=|A:A\cap U_2|\cdot|A_2|\cdot|A_3|\cdot|A_4|\cdot|A_5|. \tag{13}$$

Lemma 14. *Let A be an abelian subgroup of U. Then there exist elements $d_a,d_b\in K$ and an additive subgroup $F\subset K$ such that $d_bFA_4=0$, and*

$$A=\gamma(F)\cdot(A\cap U_2), \quad \gamma(t)=x_a(d_at)x_b(d_bt) \ mod\ U_2 \ (t\in F). \tag{14}$$

For the type $^3D_4$ and G_2 we have $(A_2A_3)^\pi=0$ and $3A_2A_3=0$, respectively. When $d_aF\ni 1$ we have $A_2^{1+\sigma}=A_3^\pi=0$ and $2A_2=3A_3=0$, respectively.

Proof. Recall that $(AU_2)/U_3$ is an abelian normal subgroup of the factor group U/U_3, which is isomorphic to a subgroup of the unitriangular group $UT(3,K)$. By Lemma 12 we obtain (14), where $\gamma(F)$ is the system of representatives of cosets of the subgroup $A\cap U_2$ in A. The equalities $[A\cap U_i, A\cap U_j]=1 \ mod \ U_{i+j+1}$ and (12) imply $d_bFA_4=0$ and

$$(A_2A_3)^\pi=0, \quad (d_aFA_3)^\pi=0, \quad (d_aFA_2)^{1+\sigma}=0 \quad \text{for the type } {}^3D_4,$$
$$3A_2A_3=0, \quad 3d_aFA_3=0, \quad 2d_aFA_2=0 \quad \text{for the type } G_2.$$

When $d_aF\ni 1$, we have $A_2^{1+\sigma}=A_3^\pi=0$ and $2A_2=3A_3=0$ respectively. □

Lemma 15. *If an abelian subgroup A of U has two corners, then $|A|<\mathbf{a}(U)$.*

Proof. Using the notation of lemma 14 and the representation (14) of the subgroup A, we have $F\ni 1$ and $d_a=d_b=1$ up to a diagonal automorphism. Furthermore, $|A:A\cap U_2|=|F|$ and $A_4=0$.

By Lemma 14, for the type G_2 we have $2A_2=3A_3=0$. Hence, $A_2=0$ when $3K=0$ and if $6K=K$ then $A_3=0$ as well. In both cases, $|A|<\mathbf{a}(U)$ due to (13) and Corollary 1. Since $(AU_4)/U_4$ is an abelian subgroup of a factor group $U/U_4\simeq UB_2(K)$, using Lemma 12 in the case $2K=0$ we have:

$$|F|=2, \quad |A|=|F|\cdot|A_2|\cdot|U_5|\leqslant 2\cdot|K|^2<\mathbf{a}(U).$$

For the type ${}^3D_4$ we have $F\subseteq K_\sigma$, and, by Lemma 14, $A_2^{1+\sigma}=A_3^\pi=(A_2A_3)^\pi=0$, and hence $A_2\subseteq \mathrm{Ker}\,(1+\sigma)$. When $2K=K$, using Lemma 13 we find:

$$A_2=0,\ |A_3|\leqslant|\mathrm{Ker}\,(\pi)|=|K_\sigma|^2, \quad |A|=|F|\cdot|A_3|\cdot|U_5|\leqslant|K_\sigma|^4<\mathbf{a}(U).$$

If $2K=0$ then by Lemma 13 we have $A_3\subseteq K^{1+\sigma}$ and $A_2\subseteq K_\sigma$. If $|A|\geqslant|U_3|$ then

$$|A|=|F|\cdot|A_2|\cdot|A_3|\cdot|K_\sigma|=|U_3|, \quad F=A_2=K_\sigma, \quad A_3=K^{1+\sigma}.$$

Thus, we may assume that a $2a{+}b$-projection of $\gamma(F)$ is contained in K_σ. Since $[\gamma(F),A\cap U_3]=1$, K_σ also contains the $a+b$-projection of $\gamma(F)$. Hence,

$$\langle\gamma(F)\rangle\subset U^3D_4(K)\cap UD_4(K_\sigma)\simeq UG_2(K_\sigma)$$

and, by Lemma 12 we have $|F|=2=|K_\sigma|$. Then $|A|=|U_3|=2^5<2^6=\mathbf{a}(U)$. The lemma is proved. □

The following lemma easily follows from the commutator relations for U.

Lemma 16. *If $\Delta_1:=X_{a+b}X_{2a+b}U_5$ and $\Delta_2:=X_bU_4$ then $T(b)=\Delta_1\Delta_2$. If U is of type G_2 and $3K=0$ then the center Z of U is $X_{2a+b}U_5$, and the centralizer $C(\Delta_1)$ is $T(b)$; otherwise, $Z=U_5$, $C(\Delta_1)=\Delta_2$ and $C(\Delta_2)=\Delta_1$. Furthermore, if U is of type G_2 and $3K=K$ then $\Delta_1\simeq\Delta_2\simeq UT(3,K)$, else if U is of type ${}^3D_4$ then $\Delta_2\simeq UT(3,K_\sigma)$.*

Lemma 17. *A large abelian subgroup A of $UG_2(K)$ is one of the following:*
a) U_2 or its $(X_an_a\cup X_bn_b)$-conjugates when $3K=0$;
b) a subgroup B-conjugate to $(\langle\alpha\rangle\times\langle\beta_1(1)\rangle)\cdot U_4$ for $|K|=2$ or 4;
c) a subgroup B-conjugate to $M_1\cdot M_2$ for $3K=K$, $|K|>2$, M_i being an arbitrary maximal abelian subgroup of Δ_i, $i=1,2$.

When $6K=K$, the subgroup $M_1\cdot M_2$ coincides with U_3 or $X_{a+b}U_4$ up to an automorphism of the form ς_d and to (X_an_a)-conjugacy, and when $2K=0$, it is $G(K)$-conjugate to U_3, $\beta_d(K)U_4$ or to

$$\{x_b(t)x_{3a+b}(t)\mid t\in K\}\beta_d(K)U_5 \quad (d\in K). \tag{15}$$

Proof. Clearly, A contains the center Z. If $A \not\subseteq U_2$, then there exists a corner $r = a$ or b of A and a representation (14) with $d_r = 1$ and $d_{\bar{r}} = 0$; furthermore, $r + w_{\bar{r}}(r) \in G^+$ and w_r induces a substitution $\tilde{w}_r$ on $G^+ \setminus \{r\}$:

$$\tilde{w}_a = (b\ 3a+b)(a+b\ 2a+b)(3a+2b), \quad \tilde{w}_b = (a\ a+b)(3a+b\ 3a+2b)(2a+b).$$

For the type G_2, when $i = ht(w_{\bar{r}}(r))$ and $3K = 0$ we have $A_i = 0$ by lemma 14. Hence, Corollary 1, Lemma 12 and (13) give

$$T(r) \supseteq A \supseteq C(T(r)) = X_{w_r(\bar{r})}Z = X_{w_r(\bar{r})}X_{2a+b}U_5;$$
$$A = \gamma(K)X_{w_r(\bar{r})}Z, \quad \gamma(K) = \{x_r(t)x_{w_{\bar{r}}(r)}(ct) \mid t \in K\}\ mod\ C(T(r)).$$

Having cancelled the scalar $c \in K$ with $X_{\bar{r}}$-conjugation, we map A into $n_{\bar{r}}^{-1}U_2n_{\bar{r}}$.

Let $3K = K$. Then $(X_aU_3)/U_5 \simeq UT(3,K)$, and if $2K = K$, then $T(a)/U_4 \simeq UT(3,K)$. By Lemma 14, either $r = a$, $A \supseteq U_4$ and $A_3 = 0 = 2A_2$, or $r = b$ and $A_4 = A_2A_3 = 0$. When two out of three projections A_2, A_3 and A_4 are zero, the remaining projection and F are both equal to K, since $|A| \geqslant |U_3|$. Hence

$$A = \gamma(K)U_4 \text{ when } r = a, \quad A = \gamma(K)\beta(K)U_5 \text{ when } r = b,$$

$\beta(t)$ being the coset representatives of U_5 in $A \cap U_2$ where $\beta(t) = x_q(t) \mod Q(q)$ for the angle q of $A \cap U_2$. When $r = b$ we define $\{q, s\} := \{a+b, 2a+b\}$. Due to Lemmas 12 and 16, there exist maps $'$, $''$ and $c, d \in K$, such that

$$\gamma(t) = x_b(t)x_s(t')x_{3a+b}(ct), \quad \beta(v) = x_q(v)x_s(dv)x_{3a+b}(v'') \in A \quad (t, v \in K),$$
$$1 = [\gamma(t), \beta(v)] = [x_b(t), x_{3a+b}(v'')][x_s(t'), x_q(v)] = x_{3a+2b}(\pm 3vt' \pm v''t),$$

and hence $t' = 1' \cdot t$ and $v'' = (\pm 3 \cdot 1')v$ for a suitable choise of the signs. If $q = 2a+b$ then $d = 0$ and $X_{\bar{r}}$-conjugacy cancels the scalar $1'$; when $q = a+b$, the scalar $1'$ is similarly defined up to addition of squares from K. Up to B-conjugacy of A we have $1' = 0$ and $A = (A \cap \Delta_1)(A \cap \Delta_2)$, $A \cap \Delta_i$ being arbitrary maximal abelian subgroups of Δ_i, $i = 1, 2$.

When $6K = K$, the exceptional automorphism from [17, Theorem 1] of the group U cancels the scalar c in $A \cap \Delta_2$, and the U-conjugacy implies either $n_a^{-1}An_a = U_3$ or $X_{a+b}U_4$. With a glance of Lemma 12, when $r = a$ we are able to cancel the $a+b$- and $2a+b$-projections in $\gamma(F)$ by means of U-conjugacy; thus we transform A to the form

$$X_aU_4 = n_b^{-1}(X_{a+b}U_4)n_b = (n_an_b)^{-1}(X_bX_{2a+b}U_5)n_an_b.$$

If $2K = 0$ then by means of diagonal $h(\chi)$-conjugacy we achieve $c = 1$ (when $\chi(a) = u \in K^*$, $\chi(a) = u \in K^*$, $\chi(b) = u^{-1}$ and $\chi(3a+b) = u^2$), obtaining A in the form (15). Similarly, when $r = a$, we obtain a subgroup

$$\{x_a(t)x_{2a+b}(t) \mid t \in K\}U_4 = n_b^{-1}\beta_1(K)U_4n_b = (n_an_b)^{-1}X_b\beta_1(K)U_5n_an_b.$$

Finally, we find the subgroups $A = \gamma(F)\beta_d(A_2)U_4$, where

$$\gamma(t) = x_a(t)x_{2a+b}(ct) \qquad (t \in F), \quad A_2 \neq 0,\ 2K = 0, \quad c,\ d \in K.$$

The relations

$$1 = [\gamma(t), \beta_d(v)] = x_{3a+b}((t^2 + td)v)x_{3a+2b}((v^2 + cv)t)$$

show that for all $t \in F$ and $v \in A_2$ we have

$$(t+d)tA_2 = 0,\ (v+c)vF = 0, \quad F = \{0, d\},\ A_2 = \{0, c\}, \quad |A| = 4|K|^2.$$

By corollary 1, we obtain $|K| = 2$ or 4. Clearly, if $|K| = 2$ then $A \lhd U$, and up to diagonal conjugacy A has the form

$$(\langle x_a(1)x_{2a+b}(1)\rangle \times \langle \beta_1(1)\rangle) \cdot U_4. \tag{16}$$

□

For the type ${}^3D_4$ the description is similar. If $A \subseteq T(a)$ and hence $T(a) \supseteq A \supseteq C(T(a)) = U_4$, then A has the form

$$\beta(A_2)x_{2a+b}(A_3)U_4, \quad \beta(v) := x_{a+b}(v)x_{2a+b}(\tilde{v})\ (v \in A_2) \tag{17}$$

for some map $\tilde{}\ : A_2 \to K$. Due to Lemmas 13 and 14 the commutativity of A is equivalent to the inclusion $A_2A_3 \subseteq \mathrm{Ker}\,(\pi) = K^{1-\sigma}$. Due to the maximality of A, the projections of A_2 and A_3 are both K_σ-modules, as well as $\mathrm{Ker}\,(\pi)$. If one of the projections are zero or equals K then we have either $A = U_3$ or $A = \beta(K)U_4$ for $\tilde{}$ from $End(K^+)$; besides,

$$[\beta(t), \beta(v)] = x_{3a+2b}(\pm(t\tilde{v} - \tilde{t}v)^\pi), \quad (t\tilde{v} - \tilde{t}v)^\pi = 0\ (t, v \in K).$$

Thus, $x_a(d)$-conjugation transforms the subgroup $X_{a+b}U_4$ into $\beta(K)U_4$, where

$$\tilde{t} = \bar{d}\bar{\bar{t}} + \bar{\bar{d}}\bar{t}, \quad (t\tilde{v} - \tilde{t}v)^\pi = [d(\bar{t}\bar{\bar{v}} - \bar{v}\bar{\bar{t}} + \bar{v}\bar{\bar{t}} - \bar{t}\bar{\bar{v}})]^\pi = (d \cdot 0)^\pi = 0\ \ (t, v \in K).$$

When both K_σ-modules A_2 and A_3 are nonzero, their dimension is 1 or 2. Up to n_a- and diagonal conjugacy, the dimension of A_2 is less or equals the dimension of A_3, and $1 \in A_2$. Therefore we may choose $s \in A_2)$ such that

$$A_3 \subseteq (K_\sigma + K_\sigma s)A_3 = A_3 + sA_3 \subseteq K^{1-\sigma}.$$

If the dimension of A_3 is 2 then the inclusions turn into equalities, and multiplication by s induces a K_σ-linear transformation of a 2-dimensional module $K^{1-\sigma}$ with a characteristic root s. Since the field K does not contain a quadratic extension of the subfield K_σ, A_2 is a 1-dimensional K_σ-module. Hence $A_2 = K_\sigma$ and $A_3 = K^{1-\sigma}$. It follows that $|A| = |U_3|$ or $|K| = 8$ and A is B-conjugated to a normal subgroup of U. Moreover we now find the Thompson subgroups.

Lemma 18. *For the group $UG_2(K), |K| > 2$, and $U^3D_4(K), |K_\sigma| > 2$, we have $J(U) = J_e(U) = U$. Besides, $J_e(U) = 1$ and $J(U) = T(a)$ in $U^3D_4(8)$ and*

$$J_e(U) = 1,\ J(U) = < \alpha > \times < \alpha^{n_b} >, \quad \alpha = x_a(1)x_{2a+b}(1) \ \ in \ \ UG_2(2).$$

Remark 1 from § 2, [10] and Lemma 18 give Theorem 2.

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Подгруппы Томпсона и большие абелевы унипотентные подгруппы групп лиева типа

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Пусть U — унипотентный радикал подгруппы Бореля группы лиева типа над конечным полем. Для классических типов подгруппы Томпсона и большие абелевы подгруппы групп U были описаны к середине 1980-х годов. Мы завершаем решение известной проблемы их описания для исключительных лиевых типов.

Ключевые слова: группа лиева типа, унипотнтная подгруппа, большая абелева подгруппа, подгруппа Томпсона.