Accelerated Phase-lock-loop Frequency Control Methods of User’s Equipment in Perspective Radio Navigation Systems

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This paper investigates noise-immunity of accelerated phase-lock-loop frequency control algorithms of user equipment in perspective ground-based radio navigation systems. Three algorithms of accelerated phase-lock-loop frequency control are suggested and described. Statistic simulations of signal processing in involved system are given.

Key words: Radio navigation, spread-spectrum signal, minimum shift keying, phase-shift discriminator, phase synchronization system, accelerated phase-locked-loop frequency control, phase-tracing error, statistical modeling, quasi-optimal algorithm.

Introduction

Spread spectrum signals with minimum shift keying (MSK) are widely used in modern radio navigation systems (RNS), e.g.: GEOLOC (France). High accuracy of coordinate measuring in the whole RNS working area requires providing phase shift measurements with root-mean-square (RMS) error $\sigma_\phi \leq 3^\circ$, when signal-to-noise ratio threshold equals to $-40dB$ (in the band of MSK-signal). That is why, the meaning of phase-lock-loop frequency control pass band equals to 0,1 Hz. Thus, locking time is 600 s, and can grow by a factor of 10 under noise and jamming influence [1].

Recently, researchers have shown an increased interest in Kalman filtering, because it can provide high accuracy of phase tracing measurements. But Kalman filter has a significant disadvantage – computational complexity, therefore, in the foreseeable future it can’t be used for preprocessing algorithms.

Due to limits in computational technology, it’s necessary to investigate phase tracking algorithms with performance objectives: small values of locking time and RMS error. So, the hypothesis that will be tested is that multistage (several meanings of pass band) phase-lock-loop frequency control algorithms can provide adequate accuracy of phase-tracing measurements and greatly smaller locking time. Consequently, investigation of accelerated phase-lock-loop frequency control algorithms with invariable phase shift accuracy is a topical scientific problem.

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1. Navigation signal model of perspective RNS

Total realization of received MSK-signal and additive white Gaussian noise (AWGN) can be described as:

\[ y(t) = \text{Re}\left\{ \hat{S}(t) \exp\left[ j(2\pi(f_0 \pm F_d) t - \varphi_s) \right]\right\} + \xi(t), \]

where \( j \) is imaginary unit; \( f_0 \) is carrier frequency; \( F_d \) is Doppler frequency shift; \( \varphi_s \) is starting phase of signal; \( \xi(t) \) is AWGN; \( \hat{S}(t) \) is complex envelope of MSK-signal:

\[ \hat{S}(t) = D(t) \sqrt{2P_s} \exp\left[ j\theta(t) \right], \]

where \( P_s \) is signal’s power; \( D(t) = \pm 1 \) is information signal; \( \theta(t) = \frac{\pi}{2T} \int_0^t d(t') dt' \) is function which determines angle modulation, \( d(t) = \sum_{i=-\infty}^{\infty} d_i \text{rect}(t-iT) \), \( \{d_i\} \) is pseudorandom sequence (PRS) of \( N \)-length, \( T \) is one’s bit PRS duration, \( \text{rect}(t) \) is square pulse with \( T \) duration [2].

2. Phase synchronization system of MSK-signal receiver

Structural chart of MSK-signal receiver’s digital phase-lock-loop frequency control system (PLFS) is presented in Fig. 1. Values \( y_i = y(t_i) \) \( (t_i = i\Delta t, i = 0, 1, ..., \Delta t - \text{sampling interval}) \) are incoming observations to digital phase-shift discriminator (DPD), formed by analog-digital converter (ADC).

Reference signals of carrier frequency \( \cos \hat{\Phi}_0(k) = \cos(2\pi(f_0 \pm \hat{F}_d(k))t_i) \) and \( \sin \hat{\Phi}_0(k) = \sin(2\pi(f_0 \pm \hat{F}_d(k))t_i) \) come into supporting inputs of DPD. These signals are formed by digital synthesizer (DS) and based on Doppler frequency shift estimation \( \hat{F}_d(k) \) in each \( k \)-period of filtering. Reference signals \( \hat{Q}_i = \sin \hat{\theta}_i \) and \( \hat{L}_i = \cos \hat{\theta}_i \), which are synchronous with quadrature components of MSK-signal, are formed by delay lock system. Quadrature components of bandwidth compressing signal (after MSK-detection) are formed by summarizing of multiplications of quadrature components of realization (1) and reference signals \( \hat{L}_i, \hat{Q}_i \) and integration on intervals \( t \in [kT_p, (k+1)T_p] \), \( k = 0, 1, ..., (T_p = 40 \text{ms} - \text{MSK-signal’s period}) \). Time of one cycle radio-range beacon transmission equals \( T_p = 25T_r \). Error signal which is proportional to phase mismatch forms in compliance with quasi-optimal algorithm [3]:

\[ Z_j(k) = \text{sign}(z_j(k))z_k(k) = \hat{D}(k)z_j(k), \]

where \( \text{sign}(x) \) is sign function, \( \hat{D}(k) \) is estimation of information signal \( D(t) \) on \( k \)-period of filtering, \( z_j(k) \) and \( z_k(k) \) are quadrature components of correlation, computed on interval \( t \in [kT_p, (k+1)T_p] \). Error signal \( Z_j(k) \) comes into digital filter (DF). Output signal of DF used to control signals \( \cos \hat{\Phi}_0(k) \) and \( \sin \hat{\Phi}_0(k) \) frequencies. When there is no noise, discrimination characteristic can be described as

\[ Z_j(\varphi) = \frac{1}{2} M \text{sign}(\cos \varphi) \sin \varphi. \]

Structural chart of the DPD is presented in Fig. 2, where \( \times \) is multiplier; \( + \) is adder; \( \Sigma \) is adder accumulator (digital integrator), which interrogated in \( kT_p \) moments, \( k = 0, 1, ..., M = T_p / \Delta t - \text{integer} \).

Normalized discrimination (curves 1, 2) and fluctuation (curve 3) characteristics of DPD are presented in Fig. 3. At that, curve 1 corresponds with no-noise case, and curves 2, 3 present discrimination and fluctuation characteristics respectively. Curves 2, 3 are the statistical simulation.
Fig. 1

Fig. 2

Fig. 3

Fig. 4
data then signal-to-noise ratio equals to $-40\,dB$. Length of using PRS $N = 2^{14} - 1 = 16383$. Number of statistical examinations equals to $10^4$.

The model of PLFS is presented in Fig. 4, where $Z_d(\varphi)$ – discrimination characteristic of DPD; $T_i$ – time constant of integrator; $K = K_f K_c$ – instantaneous element, taking account of transfer constants of digital filter $K_f$ and digital synthesizer $K_c$; the meaning of another designation are clear without comments.

Doppler frequency shift on $k$-period of filtering is estimated in compliance with the following algorithm:

$$\hat{F}_d(k) = K\left(Z_d(k) + x(k-1) + \frac{T_p}{T_i} Z_d(k-1)\right). \quad (4)$$

Discriminator nonlinearity in case of using quasi-continuous analyzing method for digital synchronization systems is taking into account by it parameters, which depend on signal-to-noise ratio [4].

### 3. Accelerated phase synchronization target setting

In phase navigation systems RMS error of coordinate measuring (in meters) can be approximately determined as

$$\sigma_c \approx \frac{\lambda_0}{2\pi} \lambda \sigma_\varphi, \quad (5)$$

where $\lambda_0$ – wave-length, $\lambda$ – geometric quotient, $\sigma_\varphi$ – RMS error of phase-shift measurements [5]. In steady-state regime phase-tracking error dispersion value can be determined by using quasi-continuous analyzing method for digital synchronization systems [6]:

$$\sigma^2_e = 2\sigma^2_T F_\omega, \quad (6)$$

here $\sigma^2_e$ – phase fluctuation dispersion, which can be calculated as

$$\sigma^2_e = \frac{\sigma^2_d}{k^2_d}, \quad (7)$$

where $\sigma^2_d = \sigma^2_d(0)$ – fluctuation characteristic for algorithm (3) of phase mismatch failing;

$k_d = \frac{\partial Z_d(\varphi)}{\partial \varphi}$ – discrimination characteristic slope for algorithm (3), line from the top means statistical estimation. Noise pass band of PLFS can be written as

$$F_\omega = \frac{1}{2\pi} \int |K(\jmath \omega)|^2 \omega d\omega, \quad (8)$$

where $K(\jmath \omega)$ – complex transfer coefficient of PLFS.

Using (5) it can be shown that in case of $\Gamma = 1.5$ (rho-rho navigation), $\lambda_0 = 150\,m$ for attainment of coordinate measuring accuracy with RMS $\sigma_c \leq 2\,m$ needed RMS error of phase-shift measurements value is $\sigma_\varphi \leq 3.3' = 0.053\,rad$. Further, using results [3] for $\sigma^2_d$ and $k^2_d$, when signal-to-noise ratio threshold equals to $-40\,dB$, and using equation (6) let’s compute required noise pass band of PLFS for MSK-signal receiver:

$$F_\omega \leq \frac{\sigma^2_e}{2\sigma^2_T} \leq \frac{0.053^2}{2 \cdot 0.364 \cdot 0.04} \approx 0.1\,Hz. \quad (9)$$
Thus, PLFS must provide RMS error of phase-shift measurements value \( \sigma_\phi \leq 0,05 \text{ rad} \) in case of noise pass band value \( F_\phi \leq 0,1 \text{ Hz} \).

Functional dependences of phase-tracking and frequency estimation error average values from discrete time \( k \) in digital PLFS are presented in Fig. 5, \( a \) and 5, \( b \) respectively. Computational approach conditions are equal to discriminator modeling, except number of statistical examinations – \( 10^5 \).

Presented functional dependences are correspondent to noise pass band value \( F_\phi = 0,1 \text{ Hz} \), user’s top speed equals \( V_{\text{max}} = 100 \text{ km/h} \) (peak level of Doppler frequency shift \( |F_{\text{d max}}| = 0,2 \text{ Hz} \) ) and capture probability \( P_c \rightarrow 1 \).

Analysis of statistic simulation data of digital PLFS (Fig. 5) shows that average locking time has intolerable level for perspective RNS for special users – \( \overline{T_l} \approx 15 \cdot 10^3 \cdot T_p = 600 \text{ s} \).

4. Digital PLFS statistical simulation

Progress in locking time decrease can be realized by varying of PLFS noise pass band. Thus, using “wide” noise pass band \( F_\phi = 0,5 \text{ Hz} \) on the first time stage and “narrow” \( F_\phi = 0,1 \text{ Hz} \) on the second time stage, it is possible to attain benefit in synchronization time.

Digital PLFS statistical simulation results, namely: phase \( \overline{\phi} \) and frequency \( \overline{F} \) tracking errors average meanings (\( a, c \)), and RMS phase \( \sigma_\phi \) and frequency \( \sigma_F \) tracking errors (\( b, d \)) are presented in Fig. 6 and in Fig. 7. All curves are functional dependences on discrete time \( k \).

Curves 1, 2, and 3 are signifying Doppler frequency shifts: 0; 0,02; 0,2 Hz respectively. Noise pass bands are described by discrete time step functions (10). Function \( F_\phi^w(k) \) describes noise pass band for Fig. 6, and \( F_\phi^n(k) \) for Fig. 7.
Fig. 6
Fig. 6 (continue)
Fig. 7
Fig. 7 (continue)
It becomes clear from Fig. 6, 7 that using multistage phase-lock-loop frequency control algorithms for MSK-signals receivers, average locking time can be significantly decreased (in comparison with autonomous algorithm $F_{\psi} = 0,1 Hz$) to $T_l \approx 1000 \cdot T_p = 40 s$, with phase tracking RMS error desired value ($\sigma_{\phi} = 0,05 rad$) in case of using function $F'_{\psi}(k)$. Using function $F''_{\psi}(k)$, it can be shown that phase tracking RMS error desired value is provided in time equal to 40 s. Also, using function $F''_{\psi}(k)$ it is possible to achieve $\sigma_{\phi} = 0,03 rad$ in 120 s and in steady-state regime $\sigma_{\phi} = 0,02 rad$ ($k > 200 s$).

Number of statistical examinations for Fig. 6 and Fig. 7 equals to $10^3$. In all examinations there are no tracking losses. Described two- and three-stage phase-lock-loop frequency control algorithms with discrete time step functions (10) can be used in MSK-signal receivers of perspective frequency-limited RNS.

### Conclusions

In present paper multistage phase-lock-loop frequency control algorithms of perspective RNS user’s equipment are suggested. Statistical simulation was used to prove that a two-stage phase-lock-loop frequency control algorithm, using function $F'_{\psi}(k)$, has gain in synchronization time equal to 560 s (in comparison with autonomous algorithm) and provides steady-state RMS error values $\sigma_{\phi} \leq 3^\circ$ and $\sigma_{\phi} \leq 0,03 Hz$. It was also stated that a three-stage phase-lock-loop frequency control algorithm has two benefits: first, gain in synchronization time is not less than 560 s; second, RMS error values in steady-state regime ($k > 200 s$) is $\sigma_{\phi} \leq 1,1^\circ$ and $\sigma_{\phi} \leq 0,01 Hz$ – better than required.

This article contains specific results which can be used in digital phase synchronization systems of user’s equipment for perspective RNS with spread-spectrum MSK-signals. The investigated algorithms of accelerated phase synchronization can be easily realized on the basis of field programmable gate array technology (FPGA).

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### References


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$$F'_{\psi}(k) = \begin{cases} F_{\psi0} = 0,5 Hz, 0 \leq k \leq 300, \\ F_{\psi1} = 0,1 Hz, k > 300, \end{cases}$$

$$F''_{\psi}(k) = \begin{cases} F_{\psi0} = 0,5 Hz, 0 \leq k \leq 300, \\ F_{\psi1} = 0,1 Hz, 300 < k \leq 1000, \\ F_{\psi2} = 0,02 Hz, k > 1000. \end{cases}$$

(10)


