Integral Representations and Volume Forms on Hirzebruch Surfaces

Alexey A. Kytmanov∗

Institute of Space and Information Technologies, Siberian Federal University, Kirenskogo 26, Krasnoyarsk, 660074, Russia

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We construct a class of integral representations for holomorphic functions in a polyhedron in $C^4$, associated with Hirzebruch surfaces. The kernels of the integral representations are closed differential forms in $C^4$ associated with volume forms on Hirzebruch surfaces.

Keywords: integral representation, Hirzebruch surface, toric variety.

Introduction

The kernel of the Bochner-Martinelli integral representation in $C^{n+1}$ is well known to be closely connected with the Fubini-Studi form for the projective space $P^n = C P^n$ as follows:

$$\omega(z) = \frac{1}{2\pi i} \frac{d\lambda}{\lambda} \wedge \omega_0([\xi])$$

(see, for instance, [1, Ch. 3]; [2, Ch. 4]). Here $\omega$ is the Bochner–Martinelli form,

$$\omega(z) = \frac{n!}{(2\pi i)^{n+1}} \sum_{k=1}^{n+1} (-1)^{k-1} \frac{\bar{z}_k}{|z|^{2n+2}} d\bar{z}[k] \wedge dz,$$

d$z = dz_1 \wedge \ldots \wedge dz_{n+1}$, and $d\bar{z}[k]$ results from deleting the differential $d\bar{z}$ in $d\bar{z}_k$. The form $\omega_0([\xi])$ is the volume form for the Fubini–Studi metric in $P^n$ (see [3, p. 21])

$$\omega_0([\xi]) = \frac{n!}{(2\pi i)^n} \frac{E(\xi) \wedge \bar{E}(\xi)}{[\xi]^{2(n+1)}},$$

where

$$E(\xi) = \sum_{k=1}^{n+1} (-1)^{k-1} \xi_k d\xi[k]$$

is the Euler form and $\xi = (\xi_1, \ldots, \xi_{n+1})$ are the homogeneous coordinates of a point $[\xi] \in P^n$. Moreover, $\xi, z \in C^{n+1}$ and $\lambda \in C$ are connected by the relation $z = \lambda \xi$.

The Bochner–Martinelli form is a “canonical” form of degree $2n + 1$ in $C^{n+1} \setminus \{0\}$. The latter set is a bundle over $P^n$ whose fiber is the one-dimensional torus $C^*$. In other words,
\[ \mathbb{P}^n = \left( \mathbb{C}^{n+1} \setminus \{0\} \right)/G, \]

where \( G = \{(\lambda, \ldots, \lambda) : \lambda \in \mathbb{C}_+\} \) is the transformation group of diagonal matrices. The projective space is a particular instance of a toric variety. In the general case, each \( n \)-dimensional toric variety is some quotient space (see [4, 5, 6])

\[ X = \left[ \mathbb{C}^d \setminus Z(\Sigma) \right]/G. \]

Here \( Z(\Sigma) \) is the union of some coordinate subspaces in \( \mathbb{C}^d \) constructed from a fan \( \Sigma \subset \mathbb{R}^n \) with \( d \) generators and \( G \) is a group isomorphic to the torus \((\mathbb{C}_*)^r\), \( r = d - n \), which is also constructed from \( \Sigma \).

In his report at the “Nordan” conference on complex analysis (Stockholm, April 1999) A. K. Tsikh posed the problem of calculating the volume forms \( \omega_0(\xi) \) on toric varieties \( X_k \) (the Fubini–Studi forms) and the canonical forms \( \omega(z) \) on \( \mathbb{C}^d \setminus Z(\Sigma) \) with the property

\[ \omega(z) \sim \frac{1}{(2\pi i)^r} \frac{d\lambda_1}{\lambda_1} \wedge \ldots \wedge \frac{d\lambda_r}{\lambda_r} \wedge \omega_0(\xi), \]

generalizing (1), where the sign \( \sim \) means that the forms have the same residues with respect to \( \lambda_1 = \ldots = \lambda_r = 0 \). Moreover, he noted that the forms \( \omega \) may serve as kernels of integral representations in \( \mathbb{C}^d \).

In the present work we consider a class of toric varieties of complex dimension 2 called Hirzebruch surfaces. We construct volume forms for this class and canonical forms in \( \mathbb{C}^4 \setminus Z' \) where the set \( Z' \) is, in general, not the same as the singular set \( Z(\Sigma) \). It is shown that the constructed canonical forms define an integral representation in 4-circular polyhedra \( G \subset \mathbb{C}^4 \).

In [7] author considered toric varieties, defined by convex fans. Convexity of a fan provides that the singular set of a canonical form \( \omega \) coincides with \( Z(\Sigma) \). As we will see below in the case of Hirzebruch surfaces fan fails to be convex if \( k > 2 \).

1. Hirzebruch Surfaces, Moment Maps and Integration Cycles

Hirzebruch surface \( X_k \) is the toric variety defined by the 2-dimensional fan, spanned by the vectors \( v_1 = (1, 0), \ v_2 = (0, 1), \ v_3 = (-1, 0), \ v_4 = (-k, -1) \), where \( k \in \mathbb{Z}_+ \).

\[ \text{Fig. 1. The fan of } X_2. \]

To each vector \( v_j \) we assign a complex variable \( \zeta_j \) so that \( \zeta = (\zeta_1, \zeta_2, \zeta_3, \zeta_4) \) plays role of homogeneous coordinates of Hirzebruch surfaces \( X_k \). Each pair of nonneighboring vectors
$v_i, v_j$ (i.e., those not defining a two-dimensional cone) defines a coordinate plane in $Z(\Sigma)$ (see [7]) so that

$$Z(\Sigma) = \{\zeta_1 = \zeta_3 = 0\} \cup \{\zeta_2 = \zeta_4 = 0\}.$$ 

The group $G$ is determined by the relations $\sum j \mu_j v_j = 0$ on the vectors $v_j$. The following equations

$$\begin{align*}
v_1 + v_3 &= 0, \\
k v_1 + v_2 + v_4 &= 0,
\end{align*}$$

are all linearly independent relations between the vectors $v_k$. Consequently, the vectors $\mu_1 = (1, 0, 1, 0)$, $\mu_2 = (k, 1, 0, 1)$ constitute a basis for the lattice of relations. The group $G$ is the 2-parameter surface $\{(\lambda_1, \lambda_2, \lambda_1, \lambda_2) : \lambda_j \in \mathbb{C}^*\} \subset (\mathbb{C}^*)^4$, so that $\zeta \sim \eta \iff \exists \lambda_1, \lambda_2 : \zeta = (\zeta_1, \zeta_2, \zeta_3, \zeta_4) = (\lambda_1 \lambda_2 \eta_1, \lambda_2 \eta_2, \lambda_1 \eta_3, \lambda_2 \eta_4).$

The moment map (see, for instance, [5, 8]) $\mu : \mathbb{C}^4 \to \mathbb{R}^4/\mathbb{R}^2 \simeq \mathbb{R}^2$ looks like

$$\mu(\zeta_1, \zeta_2, \zeta_3, \zeta_4) = (\rho_1, \rho_2),$$

where

$$\begin{align*}
\rho_1 &= |\zeta_1|^2 + |\zeta_4|^2, \\
\rho_2 &= k|\zeta_1|^2 + |\zeta_2|^2 + |\zeta_4|^2.
\end{align*}\quad(3)$$

For a fixed $\rho = (\rho_1, \rho_2) \in \mathbb{R}^2$, the relations (3) define the set $\Gamma^k_0(\rho) = \mu^{-1}(\rho)$.

The Kähler cone (see, for instance, [5]) for $X_k$ is defined by the following inequalities:

$$\begin{align*}
\rho_1 &> 0, \\
\rho_2 &> k \rho_1.
\end{align*}\quad(4)$$

The fact that the inequalities (4) hold provides that the integration cycle $\Gamma^k_0$ does not intersect the singular set $Z(\Sigma)$.

## 2. A Canonical Form and a Volume Form

We write down a form $\omega$ in $\mathbb{C}^d \smallsetminus Z(\Sigma)$ that is an analog of the Bochner–Martinelli form and establish its basic properties.

The sought form has bidegree $(4, 2)$ and looks like

$$\omega(\zeta) = \frac{h(\bar{\zeta}) \wedge d\zeta}{g(\zeta, \bar{\zeta})}.\quad(5)$$

The numerator is a form of type $(4, 2)$, where $d\zeta = d\zeta_1 \wedge d\zeta_2 \wedge d\zeta_3 \wedge d\zeta_4$, and

$$h(\zeta) = \zeta_3 \zeta_4 d\zeta_1 \wedge d\zeta_2 - \zeta_2 \zeta_3 d\zeta_1 \wedge d\zeta_4 + \zeta_1 \zeta_4 d\zeta_2 \wedge d\zeta_3 + k \zeta_1 \zeta_3 d\zeta_2 \wedge d\zeta_4 + \zeta_1 \zeta_2 d\zeta_3 \wedge d\zeta_4 \quad(6)$$

is an analog of the Euler form. The denominator $g$ is the function

$$g(\zeta, \bar{\zeta}) = |\zeta_1|^4 |\zeta_2|^{4-2k} + |\zeta_1|^4 |\zeta_4|^{4-2k} + |\zeta_2|^{2k+4} |\zeta_3|^4 + |\zeta_3|^4 |\zeta_4|^{2k+4}.$$
Here we have to make one important remark.

Note that \( g \) may contain negative powers of \( \zeta \). In this case we define the form \( \omega \) as in (5), whose numerator and denominator are multiplied by the least power of \( \zeta \) such that the denominator of the resulting form contains no negative powers of \( \zeta \). This procedure does not affect the transformation laws of the form \( \omega \) that we will derive below.

However, the singular set \( Z_\omega \) of the form \( \omega \) depends on \( k \). More precisely, we have the following three cases:

1. If \( k = 0 \) or \( k = 1 \) then \( Z_\omega \) coincides with \( Z(\Sigma) = \{ \zeta_1 = \zeta_3 = 0 \} \cup \{ \zeta_2 = \zeta_4 = 0 \} \);

2. If \( k = 2 \) then \( Z_\omega = Z' := \{ \zeta_1 = \zeta_3 = 0 \} \cup \{ \zeta_1 = \zeta_2 = \zeta_4 = 0 \} \);

3. If \( k > 2 \) then \( Z_\omega = Z'' := \{ \zeta_1 = \zeta_3 = 0 \} \cup \{ \zeta_2 = \zeta_4 = 0 \} \cup \{ \zeta_1 = \zeta_2 = 0 \} \cup \{ \zeta_1 = \zeta_4 = 0 \} \).

Each fixed element \( \delta = (\lambda_1, \lambda_2, \lambda_1, \lambda_2) \in G \) defines the mapping \( \delta : C^4 \setminus Z(\Sigma) \to C^4 \setminus Z(\Sigma) \) by the formula \( \zeta \to \delta \cdot \zeta \), i.e.,

\[
\begin{align*}
\zeta_1 & \to \lambda_1 \lambda_2^k \zeta_1, \\
\zeta_2 & \to \lambda_2 \zeta_2, \\
\zeta_3 & \to \lambda_1 \zeta_3, \\
\zeta_4 & \to \lambda_2 \zeta_4.
\end{align*}
\]

Proposition 1. The differential form \( \omega \) is invariant under the action of \( \delta \).

Proof. By direct substitution, we obtain the following transformation laws for \( h(\bar{\zeta}), d\zeta, \) and \( g(\zeta, \bar{\zeta}) \):

\[
\begin{align*}
h(\bar{\zeta}) & \to \bar{\lambda}_1 \bar{\lambda}_2^k h(\bar{\zeta}), \\
d\zeta & \to \bar{\lambda}_1 \bar{\lambda}_2^k d\zeta, \\
g(\zeta, \bar{\zeta}) & \to (\lambda_1 \bar{\lambda}_1)^2 (\lambda_2 \bar{\lambda}_2)^k g(\zeta, \bar{\zeta}).
\end{align*}
\]

Inserting them in \( \omega \), we arrive at the assertion of the proposition. \( \square \)

We now describe the behavior of \( \omega \) under the action of the group \( G : (C^4 \setminus Z(\Sigma)) \times C_2^* \to C^4 \setminus Z(\Sigma) \), defined by (7).

Lemma 1. The form \( d\zeta \) transforms as follows under the action of (7):

\[
d\zeta \to \lambda_1 \lambda_2^{k+1} d\lambda_1 \wedge d\lambda_2 \wedge h(\zeta) + \psi(\lambda, \zeta),
\]

where \( h \) is determined by (6), and the form \( \psi \) has higher degree in \( \zeta \) than \( h(\zeta) \).

Lemma 2. The form \( h(\bar{\zeta}) \) transforms by the following rule under the action of (7):

\[
h(\bar{\zeta}) \to \bar{\lambda}_1 \bar{\lambda}_2^k h(\bar{\zeta}).
\]

It is not hard to prove lemmas 1 and 2 by direct substitution of the action of \( G \) into the forms \( d\zeta \) and \( h(\bar{\zeta}) \).

Let us note that since the denominator \( g \) is a function (not differential form), it transforms by the same rule as in Proposition 1 under the action of (7).

We thus come to the following
Theorem 1. Under the action of (7) the form $\omega$ transforms as follows:

$$\omega \rightarrow \frac{d\lambda_1}{\lambda_1} \wedge \frac{d\lambda_2}{\lambda_2} \wedge \omega_0 + \omega_1$$  \hspace{1cm} (8)

with the positive form

$$\omega_0 = \frac{h(\zeta) \wedge h(\zeta)}{g(\zeta, \bar{\zeta})}$$

of homogeneity degree zero under the action of the group $G$ and with some form $\omega_1$, involving no conjugate differentials $d\lambda_i$ and having at most one differential $d\lambda_j$ in each summand.

The form $\omega_0$ is an analog of the Fubini–Studi form (2) for the projective space.

Recall that $\Gamma^k_0 = \Gamma^k_0(\rho)$ is the set (3). We now treat it as an integration cycle. The cycle $\Gamma^k_0$ foliates over $X_k$ with fibers isomorphic to the real tori $\mathbb{R}^k$ and having at most one differential $d\lambda_j$ in each summand.

At this point let us note that if $k \geq 2$ then the singular set $Z_\omega$ does not coincide with $Z(\Sigma)$. (This happens because the fan $\Sigma$ is not strictly convex.) If $k = 2$ then the singular set $Z_\omega$ is a subset of $Z(\Sigma)$, and therefore the cycle $\Gamma^k_0$ does not intersect $Z_\omega$. If $k > 2$ then the cycle $\Gamma^k_0$ can intersect the planes $\{\zeta_1 = \zeta_2 = 0\}$ or $\{\zeta_1 = \zeta_4 = 0\}$. In this case we need to prove the following

Proposition 2. The form $\omega$ is bounded in the neighborhood of the planes $\{\zeta_1 = \zeta_2 = 0\}$ and $\{\zeta_1 = \zeta_4 = 0\}$.

Proof. Let us show that the form $\omega$ is bounded in the neighborhood of the plane $\{\zeta_1 = \zeta_2 = 0\}$. Let $|\zeta_1| = \varepsilon_1$, and $|\zeta_2| = \varepsilon_2$. Equalities (3) imply $|\zeta_4|^2 = \rho_1 - \varepsilon_1^2 > \frac{\rho_1}{2}$ and $|\zeta_4|^2 = \rho_2 - k\varepsilon_1^2 - \varepsilon_2^2 > \frac{\rho_2}{2}$ when $\varepsilon_1$ and $\varepsilon_2$ are sufficiently small. Note that for such $|\zeta_4|$ we have that $g \geq \frac{\rho_1^2 \rho_2^{k+2}}{2k+4}$, and the numerator $h(\zeta) \wedge d\zeta$ is bounded. Therefore, the form $\omega$ is bounded in the neighborhood of $\{\zeta_1 = \zeta_2 = 0\}$. Similarly one can show that $\omega$ is bounded in the neighborhood of $\{\zeta_1 = \zeta_4 = 0\}$.

Proposition 2 implies that the form $\omega$ is integrable over the cycle $\Gamma^k_0$.

Corollary 1. The equality $\int_{\Gamma^k_0} \omega = C$ holds, where $C$ is some nonzero constant.

Proof. (8) and (9) imply

$$\int_{\Gamma^k_0} \omega = \int_{|\lambda_1|=1} \frac{d\lambda_1}{\lambda_1} \int_{|\lambda_2|=1} \frac{d\lambda_2}{\lambda_2} \int_{X_k} \omega_0 = (2\pi i)^2 \int_{X_k} \omega_0.$$  \hspace{1cm} (10)

The last integral is a positive number by positivity of the form $\omega_0$, as required. \hfill \Box

Now, we prove the following

Proposition 3. The form $\omega$ is closed.
Proof. In fact we have to demonstrate that 
\[ (g/\tilde{g})\bar{\partial}h - \partial(g/\tilde{g}) \wedge h = 0. \]
This would imply that 
\[ (g/\tilde{g})d(h \wedge d\zeta) - d(g/\tilde{g}) \wedge (h \wedge d\zeta) = (g/\tilde{g})dh \wedge d\zeta - d(g/\tilde{g}) \wedge h \wedge d\zeta = (g/\tilde{g})\bar{\partial}h - \partial(g/\tilde{g}) \wedge h \wedge d\zeta = 0, \]
i.e., the form \( \omega \) is closed. By direct calculation of \( \bar{\partial}h \) and \( \partial(g/\tilde{g}) \) we get the statement of the proposition. \( \square \)

**Proposition 4.** Let \( f(\zeta) \) be a holomorphic function in a neighborhood \( U \) about the origin and let \( \rho_1, \rho_2 \) be small enough to guarantee \( \Gamma^k_0 \subset U \). Then the following integral representation is valid:
\[ f(0) = \frac{1}{C} \int_{\Gamma^k_0} f(\zeta) \omega(\zeta), \]
where \( C \) is the normalization constant: \( \int_{\Gamma^k_0} \omega = C \neq 0. \)

Proof. Since the form \( f\omega \) is \( \bar{\partial} \)-closed, the integral in (10) is independent of \( \rho_1, \ldots, \rho_r \).

We rewrite it as
\[ \int_{\Gamma^k_0} f(\zeta) \omega(\zeta) = \int_{\Gamma^k_0} f(0) \omega(\zeta) + \int_{\Gamma^k_0} (f(\zeta) - f(0)) \omega(\zeta) = C f(0) + \int_{\Gamma^k_0} (f(\zeta) - f(0)) \omega(\zeta). \]

Let us show that the last integral vanishes. By substituting \( \zeta \to \tau \zeta \), we obtain:
\[ \begin{align*}
\zeta_1 &\to \tau^{k+1} \zeta_1, \\
\zeta_2 &\to \tau \zeta_2, \\
\zeta_3 &\to \tau \zeta_3, \\
\zeta_4 &\to \tau \zeta_4.
\end{align*} \]

Then the cycle \( \Gamma^k_0 \) goes into the cycle \( \Gamma^k_\tau \):
\[ \begin{align*}
|\tau^{k+1} \zeta_1|^2 + |\tau \zeta_3|^2 &= \rho_1, \\
|k \tau^{k+1} \zeta_1|^2 + |\tau \zeta_2|^2 + |\tau \zeta_4|^2 &= \rho_2.
\end{align*} \]

The integral goes to
\[ \int_{\Gamma^k_\tau} (f(\zeta) - f(0)) \omega(\zeta) = \lim_{\tau \to 0} \int_{\Gamma^k_\tau} (f(\zeta) - f(0)) \omega(\zeta) = \lim_{\tau \to 0} \int_{\Gamma^k_0} (f(\zeta \tau) - f(0)) \omega(\zeta \tau). \]

By Proposition 1 the form \( \omega \) is invariant under the substitution \( \omega(\zeta \tau) = \omega(\zeta) \). Since all \( s_k \) are positive, we have \( \lim_{\tau \to 0} f(\zeta \tau) = f(0) \). Thus
\[ \lim_{\tau \to 0} \int_{\Gamma^k_0} (f(\zeta \tau) - f(0)) \omega(\zeta \tau) = \lim_{\tau \to 0} \int_{\Gamma^k_0} (f(\zeta \tau) - f(0)) \omega(\zeta) = 0. \]

The proof of the proposition is now completed. \( \square \)
3. Integral Representation

We now consider the question of finding a domain $D$, such that the following integral representation is valid for every point $z \in D$

$$f(z) = \frac{1}{C} \int_{\mu^{-1}(\rho)} f(\zeta)\omega(\zeta - z).$$  \hspace{1cm} (11)

Consider the domain $D = D_\rho$:

$$\left\{ \begin{array}{l}
|\zeta_1|^2 + |\zeta_3|^2 < \rho_1, \\
|\zeta_2|^2 + |\zeta_4|^2 < \rho_2 - k\rho_1. 
\end{array} \right.$$  \hspace{1cm} (12)

We will show that it is the required domain. Note that $D$ is nonempty if the Kähler conditions (4) are satisfied.

Denote by $Z_z(\Sigma)$ the translate $z + Z(\Sigma)$:

$$Z_z(\Sigma) = \{ \zeta_1 - z_1 = \zeta_3 - z_3 = 0 \} \cup \{ \zeta_2 - z_2 = \zeta_4 - z_4 = 0 \},$$

and let $\Gamma^k_z$ be the translate $z + \Gamma^k_0$:

$$\Gamma^k_z : \left\{ \begin{array}{l}
|\zeta_1 - z_1|^2 + |\zeta_3 - z_3|^2 = \rho_1, \\
k|\zeta_1 - z_1|^2 + |\zeta_2 - z_2|^2 + |\zeta_4 - z_4|^2 = \rho_2.
\end{array} \right.$$  \hspace{1cm} (13)

Denote by $W = W_\rho$ 2-circular polyhedron defined by the system

$$\left\{ \begin{array}{l}
|\zeta_1|^2 + |\zeta_3|^2 < \rho_1, \\
k|\zeta_1|^2 + |\zeta_2|^2 + |\zeta_4|^2 < \rho_2.
\end{array} \right.$$  \hspace{1cm} (13)

By $W_2\rho$ we denote the domain like (13), where the right-hand sides of the inequalities are $2\rho_1, 2\rho_2$.

**Lemma 3.** For each $z \in D$ the cycle $\Gamma^k_z$ lies in $W_2\rho$. Moreover, if the Kähler conditions (4) are satisfied then the homology $\Gamma_z \sim \Gamma^k_0$ holds in the domain $W_2\rho \setminus Z_z(\Sigma)$.

**Proof.** Consider the following homotopy of the cycles $\Gamma^k_0$ and $\Gamma^k_z$:

$$\left\{ \begin{array}{l}
|\zeta_1 - tz_1|^2 + |\zeta_3 - tz_3|^2 = \rho_1, \\
k|\zeta_1 - tz_1|^2 + |\zeta_2 - tz_2|^2 + |\zeta_4 - tz_4|^2 = \rho_2, \end{array} \right.$$  \hspace{1cm} (14)

where $0 \leq t \leq 1$. We will prove that the cycle (14) is disjoint from $Z_z(\Sigma)$ for any $t$ in the interval $[0, 1]$.

Let us show that the cycle (14) is disjoint from the plane $\{ \zeta_1 - z_1 = \zeta_3 - z_3 = 0 \}$ in $Z_z(\Sigma)$. Substituting it to (14) we get $(1 - t)^2(|\zeta_1|^2 + |\zeta_3|^2) = \rho_1$. The last equality is false since $(1 - t)^2 \leq 1$ and $|\zeta_1|^2 + |\zeta_3|^2 < \rho_1$.

Similarly we show that the cycle (14) is disjoint from the plane $\{ \zeta_2 - z_2 = \zeta_4 - z_4 = 0 \}$ in $Z_z(\Sigma)$. Substituting it to (14) we get $k|\zeta_3 - tz_3|^2 = -(\rho_2 - k\rho_1) + (1 - t)^2(|\zeta_2|^2 + |\zeta_4|^2)$ that never holds since $(1 - t)^2 \leq 1$ and $|\zeta_2|^2 + |\zeta_4|^2 < \rho_2 - k\rho_1$. This completes the proof of the lemma. \hfill \Box
We have thus proven the integral representation (11) for functions holomorphic in $W_{2\rho}$. Note that it suffices to take the holomorphy domain of the function $f(z)$ in (11) to be $W = W_\rho$, since the latter is a convex domain whose boundary contains the cycle $\Gamma^k_0$. It follows from convexity of $W$ that a function holomorphic in $W$ and continuous in the closure of $W$ can be approximated by polynomials in the closure of $W$ for which the integral representation (11) is proven. Thus, we arrive at the following

**Theorem 2.** Suppose that $f(\zeta)$ is a holomorphic function in the domain $W$ defined by (13) and $f$ is continuous in the closure of $W$. Then the integral representation (11), with the cycle $\Gamma^k_0$ defined by (3) is valid in the domain $D$ defined by (12).

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**References**


