

# Neural-net modeling for direct and inverse problems of shell theory

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**Abstract.** Effectiveness of the use of neural-net technology for the solving of shell theory problems is shown. Some results of neural-net interpolation and extrapolation for direct and inverse problems are discussed. Exact accuracy of neural-net solving opens wide latitude for shell constructions engineering design and optimization.

## 1. Introduction

Neural network technologies as a modern universal approximation tool are widely applied in both classic and new areas of science and technology [1, 2]. Their use in the theory of shells, shells designing and research seems relevant and useful.

The joint use of solutions of direct and inverse problems can provide substantial assistance in the process of design engineering, which can be considered as finding the optimal form in the selection of certain parameters and characteristics.

The solution of the inverse problem is usually fraught with difficulties related to the ambiguity of the decision. We have to use different classical and non-classical mathematical methods for solving. For example, the definition of the inverse function or matrix inversion is required for solutions in linear problems without restrictions. For problems with constraints and nonlinear problems it is possible to apply the methods of mathematical programming.

In cases where the problem is poorly formalized or not formalized, we have to look for special innovative ways of solving. They include neural network approaches.

In terms of a neural network, inverse problem is an identification of dependencies between the input and output data, which are interchanged with respect to the direct problem. In this regard, the solution of direct and inverse problems can be carried out by a single algorithm. The neural network solution of the inverse problem will be simplified if we can build such a neural network, for which it would be possible to get a set of ambiguous solutions.

In this case the use of neural network technology, specifically neural network modeling, is convenient and efficient, it allows high-speed seek adoption of rational and optimal solutions.

Note that in this study, we have used standard neural network simulators, that were unfortunately not assumed to be applied to the problems of mechanics while being developed. It seems promising to develop specialized neural simulators, as well as the individual neural networks, which take into

account the properties of the structure and, apparently, can significantly improve the results. However, it should be noted that this approach can restrict the class of problems.

As a neural simulator for most of the problems we used the program "Modely" written on the basis of interpolation of the solutions using Fourier series and minimization of the functional of squared deflection error using the Lagrange multipliers [3]. We consider the examples of solving direct and inverse problems [4] using neural networks.

## 2. Thin smooth shallow spherical shell on a square plan

Area of moments is shown for a smooth spherical shell under the load uniformly distributed over the band of width  $\lambda$  along the  $Y$  axis. It shows a change of the bending moment in the center of band-pass load, depending on the load bandwidths (Figure 1).

**Material for training the neural network:** the value of the bending moment  $M_X$  in the local center of the site, depending on the width of the band-pass load  $\lambda$ .

$M_X$ , kg·cm	11.5	10.3	10	9.4	8	6.2	4.2
$\lambda$ , mm	16.67	27	33.33	37	50	66.67	100

**Direct problem:** to investigate the influence of bandwidth  $\lambda$  of uniformly distributed load on the value of the bending moment.

*The input parameters of the network* – the bandwidth of load  $\lambda$ .

*The output network parameters* – values of the moment  $M_X$ .

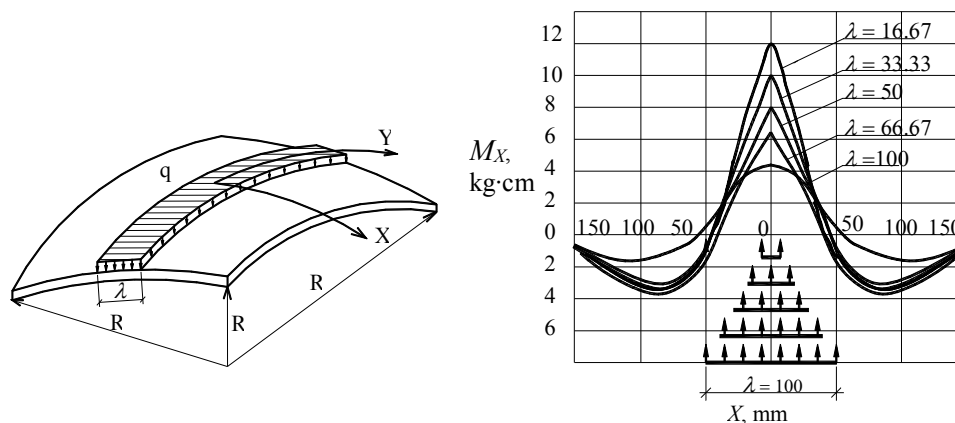


Figure 1. The dependence of the bending moment in the center of band-pass load, depending on the load bandwidths for a spherical shell

*Features of the neural network solving the problem.* In the case where  $M_X$  is determined for  $\lambda$ , located within the training pattern interval (interpolation problem – training pattern: moments for  $\lambda = 16.67; 33.33; 50; 66.67; 100$ ; tests:  $\lambda = 27; 37$ ), the neural network training takes instantaneously. In the training pattern, consisting of just five examples with the minimum number of neurons, equal to 2, and unit spectral density, training error is 1.4%. If  $\lambda$  extends beyond training pattern interval, for example to decrease the bandwidth load (extrapolation problem – training pattern: moments for  $\lambda = 33.33; 37; 50; 66.67; 100$ , tests:  $\lambda = 16.67; 27$ ), to achieve a similar accuracy of the solution the number of neutrons should be increased to 9 (with unchanged other network parameters). Adding just one neuron reduces training error to 0.5%. The process of training a neural network in the case of interpolation and in the case of extrapolation takes seconds.

This problem is neural forecasting problem of design type. With the help of the graph of dependence of the moment  $M_X$  of the load bandwidth  $\lambda$ , it is possible to determine the maximum possible bending moment with the minimum allowable load bandwidth (Figure 2).

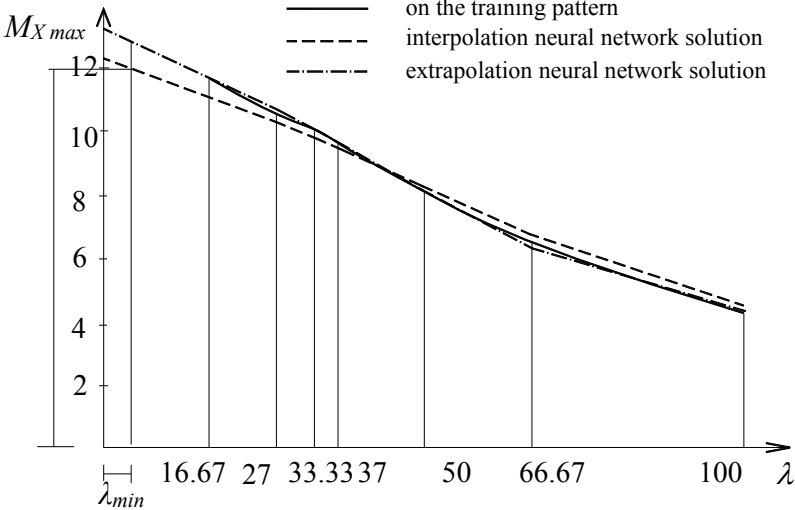


Figure 2. Changing the maximum bending moment in the shell depending on the width of the band-pass load

**Inverse problem:** to find  $\lambda$  (width of band-pass load), corresponding to the desired moment ( $M_X$ ) in the center of the local area.

This problem can be formulated differently: “what should the bandwidth of the current distributed load be to produce a bending moment that does not exceed a predetermined value?” In this case, we are dealing with the search for optimal solutions.

*The input parameters of the network* – values of the moment  $M_X$ .

*The output network parameters* – the bandwidth of load  $\lambda$ .

Similarly to the direct problem, we solved the problem of interpolation and extrapolation type. Interpolation problem was converted to determining load bandwidth  $\lambda$  for a given value of the bending moment  $M_X$  within the training pattern interval (training pattern  $M_X = 4.2; 6.2; 8; 10; 11.5$ ; tests  $M_X = 9.4; 10.4$ ). The training pattern consisted of 5 problems. Compared with the direct problem the neural network training process for the inverse problem (due to the complexity of the reversal function) is more time-consuming and complicated.

*Features of the neural network solving the problem.* The best results that have been achieved are within 4% error and obtained by means of a neural network with 15 neurons and spectral density equal to 2.6. The process of network training, the selection of its architecture has a singularity. In the process of training the spectral density should be increased gradually, starting with 1.0 in increments of 0.1. Otherwise, training error increases greatly to 15% or higher. Changing the size of the network (number of neurons) and spectrum as downwards and upwards does not give positive results (Fig. 3).

If given moment  $M_X$ , for which you want to determine the appropriate load bandwidth  $\lambda$ , is outside the training pattern interval (extrapolation problem – training pattern:  $M_X = 4.2; 6.2; 8; 9.4; 10$ ; tests:  $M_X = 10.3; 11.5$ ), the network training is very long and painstaking. It requires the use of step method neural network forecasting [3, 5]. At each step of training the magnitude of the spectral density must be increased from 1, not more than 0.01 to 1.3. In this case, two-neuron network allows to achieve error on the test cases within 6%.

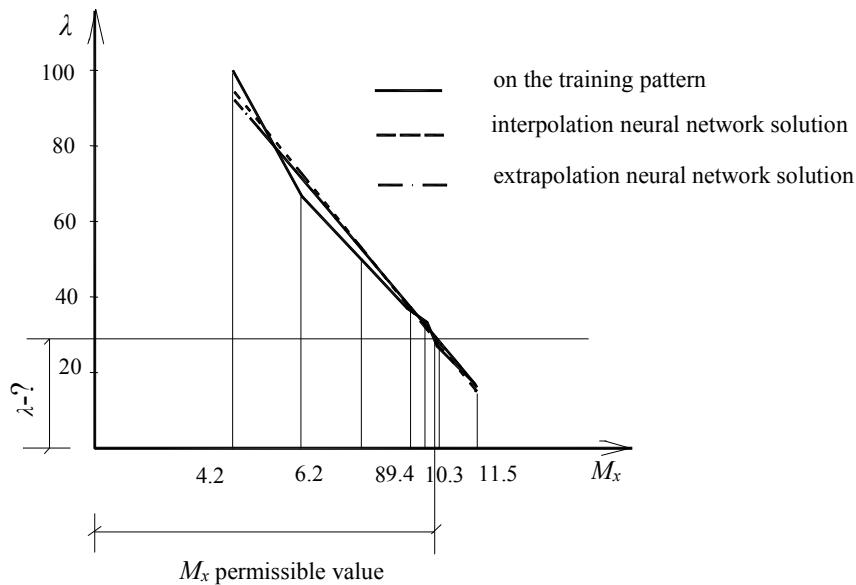


Figure 3. Dependence of  $\lambda$  on  $M_x$  for determining the lowest possible band width of the load at which the bending moment does not exceed a predetermined value

### 3. The thin shallow spherical shell on a square plan, backed by edges on a contour

In this example, the load is distributed evenly over the entire surface of the shell.

**Material for training the neural network:** values of deflections  $w/q$  and bending moments  $M_x/q$  in grid nodes on the coordinate axes for some pliability parameter values of contour arches  $\nu$  (Figures 4, 5).

**Direct problem:** to determine the impact of pliability parameter  $\nu=h/t$  of contour arch ( $h$  – section height of contour arch,  $t$  – shells thickness) on the deflections and bending moments.

*The input parameters of the network* – the value of pliability parameter  $\nu$ .

*The output network parameters* – values of deflections  $w/q$  and values of moments  $M_x/q$  in shell grid nodes.

If deflections and moments are determined for pliability parameter  $\nu$ , without departing from the training pattern interval (interpolation problem), learning is fast, does not cause difficulties.

*Features of the neural network solving the problem.* In determining the deflections (training pattern: data for  $\nu=2, 3, 4, 6, 8$ ; test:  $\nu=5$ ) the best result of training have been achieved for 6- neuron network with the spectral density 1.2. Errors of deflections calculating  $\delta$  are in the interval  $0.05\% \leq \delta \leq 5.72\%$ . The maximum error is noted in the contour arch. In determining the moments (training pattern: data for  $\nu=2, 3, 4, 6, 9$ , test:  $\nu=5$ ) the most effective is an 8-neuron network with the spectral density 0.8. Here network training error does not exceed 2%. The maximum errors are in the area of moments values close to zero.

When deflections and moments are determined for pliability parameter extending beyond the training pattern interval (extrapolation problem – training pattern: data for  $\nu=2, 3, 4, 5, 6$ , test:  $\nu=8$ ) the problem is complicated. The number of examples in the training pattern has a significant impact on the results of the network functioning.

For instance, for the training pattern of five examples (4- neuron network with the spectral density 2.5) the maximum error in the tests for deflections is 5.5%. Reducing the pattern by a single example leads to an increase in errors to 8%.

The accuracy of neural network solutions can be significantly improved if instead of one-step (traditional) neural network extrapolation the step method of neural network forecasting is used [3, 5].

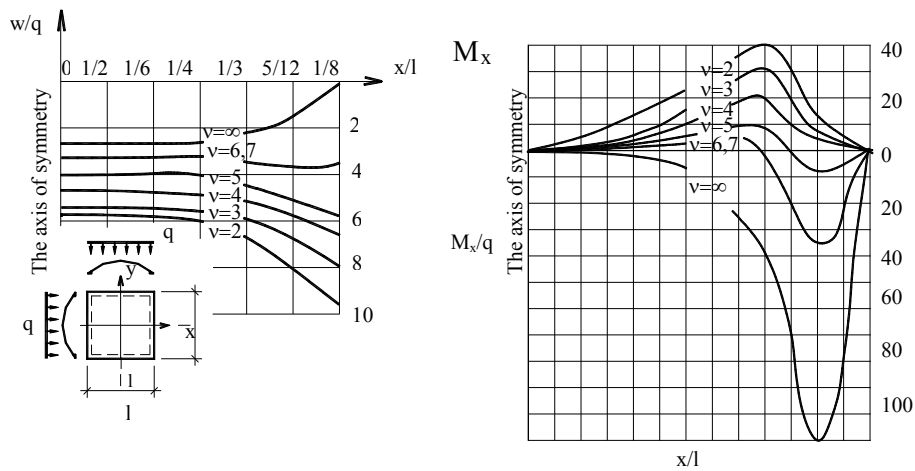


Figure 4. Epures of deflections and bending moments in the cross section  $y=0$  ( $\nu=h/t$ , where  $h$  – section height of contour arch,  $t$  – shells thickness)

Figures 4 and 5 show that with increasing pliability, quality change (even in sign) of moments epures near the contour and significant growth of deflections throughout the area occurs. The value  $\nu = 5$  can be taken as conventional limit, below which a reinforcement shell contour can be called a very pliable. It may be recommended for design purposes to take  $\nu \geq 5$ .

**Inverse problem:** what should be the parameter  $\nu$  (pliability of contour rib), to the deflections  $w/q$  and moments  $M_x/q$  do not exceed the specified values?

*The input parameters of the network* – values of deflections  $w/q$  and the values of the bending moments  $M_x/q$  at the nodes of the shell mesh.

*The output network parameters* – pliability ribs parameter  $\nu$ .

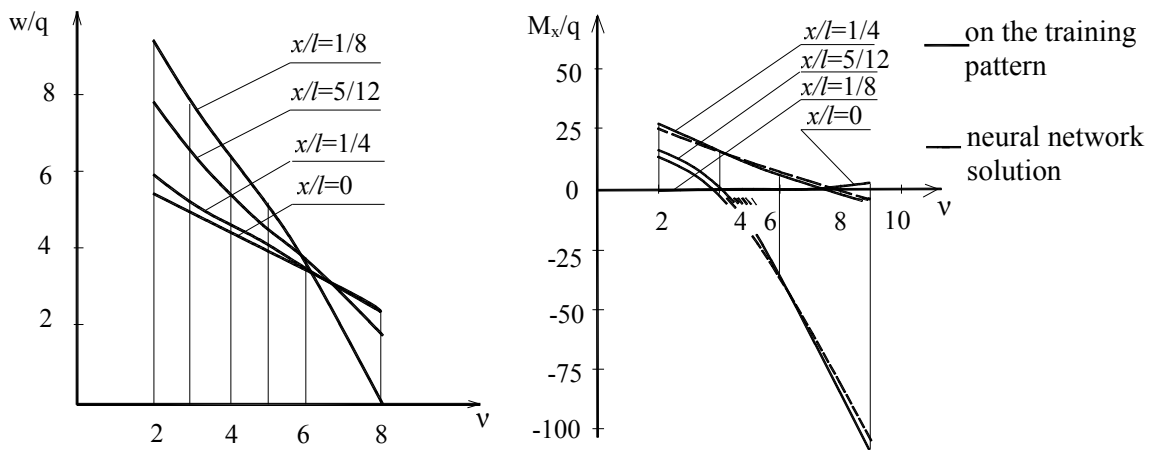


Figure 5. Dependence of deflections functions and moments on pliability of contour ribs  $\nu=h/t$  for the various points of the shell region

*Feature of the neural network solving the problem* is the fact, that the inverse problem is solved much more accurate than direct both for the interpolation problem when the network functioning is carried out within the training pattern interval, and for extrapolation problem, when network functioning is carried out outside the training pattern interval.

For example, in the problem of extrapolation (training pattern: five variants of deflection for  $\nu = 2, 3, 4, 5, 6$ , test: deflection for  $\nu = 8$ ) with the network of the two neurons with spectral density 0.8 deflection values are determined with an error not exceeding 1.7%.

To determine the moments (training pattern: five variants of moments for  $\nu = 2, 3, 4, 5, 6$ , test: moments for  $\nu = 9$ ) the most effective is the network of three neurons with spectral density 0.8. Maximum network training error on the test examples is 1.95%.

In both cases, the training pattern consists of only 5 examples. But even with such a small number of training examples the neural network gives a fairly accurate results. The error of the neural network solution does not exceed 2%. However, reducing the pattern by even one example in the problem of deflections forecast causes a significant increase in error.

It confirms the need for reasonably select training examples, their number, as well as network architecture and its characteristics for each specific problem in view of its features.

In this problem, there is no need to use step method of neural forecasting as "traditional" (one-step) neural forecasting provides acceptable accuracy of the solution. However, the use of this method could significantly improve the results

This problem can be considered as an optimization problem. Plotting of dependence of pliability parameter  $\nu$  on the value of the deflections and moments and their analysis can help in the selection of such pliability parameter of contour ribs, which would provide the desired (required) shell behavior, its stress-strain state (SSS).

#### 4. Conclusion

Thus, the results of solving the problems under consideration show, that the use of neural network models allows to solve with sufficient accuracy (within 5% error) as a direct problem of calculating shells (to receive SSS parameters according to known geometric characteristics of shell and its load) and inverse problems (to select the design dimensions according to known moments and deflections). This opens up opportunities for the design of shell structures and their optimization. Note that the neural network solution of the inverse problem will be simplified, if we can build such neural network model, by which it would be possible to get a set of ambiguous solutions.

The examples discussed herein are illustrative a greater extent for the traditional method of using neural network technology (interpolation, extrapolation). It should be noted that the development and application of stepwise technology to these problems, set out in [3, 5], using the completion of the training pattern by interpolation, additional training the neural network model and the step process, can significantly improve the results.

Neural network technologies have much more applications, and can include not only the static problem, but also the dynamics, to take into account the experimental data in order to bring the calculated results to the actual design.

#### References

- [1] Sidorov M Yu, Zablotskiy S G, Semenkin E S and Minker W 2012 Evolutionary design of networks for forecasting of financial time series *J. SibSAU Vestnik* (Krasnoyarsk) **4** (44) pp 106-110
- [2] Popov E A and Semenkina M E 2013 Self-configuring genetic programming algorithm for medical diagnostic problems *J. SibSAU Vestnik* (Krasnoyarsk) **4** (50) pp 116-123
- [3] Abovskiy N P, Deruga A P, Maximova O M and Svetashkov P A 2003 *Neuro-Controlling Structures and Systems (Scientist series "Neuro-computers and their application", book 13)* ed A I Galushkin (Moskva: Radiotekhnica) p 368
- [4] Endzhievskiy L V 1982 *Nonlinear Deformation of Ribbed Shells* (Krasnoyarsk: Krasnoyarsk State University) p 296
- [5] Abovskiy N P and Maximova O M 2007 Neuro-prognosis based on step model with teaching for natural tests results of building structures *J. Optical Memory & Neural Networks (Information Optics)*, SPRINGER **16** chapter 1 pp 40-46