Two-layer model of reflective ferromagnetic films in terms of magneto-optical ellipsometry studies

Olga A. Maximova, Sergey G. Ovchinnikov*
Kirensky Institute of Physics,
Federal Research Center KSC SB RAS,
Akademgorodok 50, building 38, Krasnoyarsk, 660036,
Russia
Siberian Federal University,
Svobodny 79, Krasnoyarsk, 660041

Russia

Nikolay N. Kosyrev†
Kirensky Institute of Physics,
Federal Research Center KSC SB RAS,
Akademgorodok 50, building 38, Krasnoyarsk, 660036,

Russia

Sergey A. Lyaschenko‡
Kirensky Institute of Physics,
Federal Research Center KSC SB RAS,
Akademgorodok 50, building 38, Krasnoyarsk, 660036,
Russia
Reshetnev Siberian State Aerospace University,
Krasnoyarsky Rabochy Av.31, Krasnoyarsk, 660037

Russia

An approach to analysis of magneto-optical ellipsometry measurements is presented. A two-layer model of ferromagnetic reflective films is in focus. The obtained algorithm can be used to control optical and magneto-optical properties during films growth inside vacuum chambers.

Keywords: Magneto-optical ellipsometry, Kerr effect, two-layer model, ferromagnetic metal, reflection, growth control.

Recently it has become necessary to synthesize new materials that would be applied in spintronics devices. This field of study has significantly developed and it dictates the properties that materials should have in order to be used for its purposes. It is well-known that the simplest method of generating a spin-polarised current in a metal is to pass the current through a ferromagnetic material. That is why, one of the perspective materials for spintronics is a ferromagnetic/semiconductor two-layered structure [1].

In order to synthesize them and control their properties we have to use the methods that are non-destructive, precise, easy to use, applicable for in situ investigations in the high-vacuum

*maximo.a@mail.ru, sgo@iph.krasn.ru
†kosyrev@inbox.ru
‡fenics_star@mail.ru

chambers of molecular beam epitaxy. We suggest that magneto-optical ellipsometry is a technique that reflects these requirements. Magneto-optical ellipsometry usually combines the features of conventional ellipsometry and of magneto-optical Kerr effect measurements [2–6]. Applied to the sample magnetic field changes the ellipsometric parameters, this difference can be examined and used to investigate magneto-optic properties of the sample.

In this work we give detailed explanation how to analyse magneto-ellipsometric data and obtain information on magneto-optical and optical properties of the material.

1. General approach to magneto-ellipsometric data processing

Our approach is based on the analysis of a well-known equation that relates the experimental ellipsometric parameters ψ and Δ with complex reflection coefficients corresponding to in-plane (R_p) and out-of-plane (R_S) light polarizations [7–8]. Ellipsometric parameters ψ and Δ can be presented as a sum of conventional parameters ψ_0 and Δ_0 measured without external magnetic field and additional ellipsometric parameters $\delta\psi$ and $\delta\Delta$ that are the result of magnetic field application. We suggest to consider real and imaginary parts of these coefficients, so we mark them by $'$ and $''$ respectively:

$$\tan(\psi_0 + \delta\psi)\exp(i(\Delta_0 + \delta\Delta)) = R_p R_S^{-1} = (R_p' - iR_p'')(R_S' - iR_S'')^{-1}. \tag{1}$$

We are interested in magneto-optical properties of the sample. That is why it seems to be reasonable to present reflection coefficients as a sum of magnetic (subscript 1) and non-magnetic (subscript 0) summands [9–11]:

$$R_p = R_{pp} + R_{pS} = R_{p0}' + R_{p1}' - i(R_{p0}'' + R_{p1}''), \tag{2}$$

$$R_S = R_{SS} + R_{Sp} = R_{S0} = R_{S0}' - iR_{S0}''. \tag{3}$$

This paper focuses on the case of transverse magneto-optic Kerr effect when the magnetization is perpendicular to the plane of incidence and parallel to the surface of the sample. That is why there are no magnetic summands for s-plane polarization.

From (1-3) four equations can be obtained. Two of them correspond to non-magnetic condition:

$$\tan\psi_0 = \sqrt{\frac{(R_{p0}'R_{S0}' + R_{S0}''R_{p0}'')^2 + (R_{S0}''R_{p0}' - R_{p0}''R_{S0}')^2}{R_{S0}'^2 + R_{S0}''^2}}, \tag{4}$$

$$\Delta_0 = \arctan\frac{R_{S0}''R_{p0}' - R_{p0}''R_{S0}'}{R_{p0}'R_{S0}' + R_{S0}''R_{p0}''}, \tag{5}$$

and two equations demonstrate the influence of an external magnetic field:

$$\delta\Delta = \Delta - \Delta_0 = \arctan\frac{R_{S0}''(R_{p0}' + R_{p1}') - R_{S0}'(R_{p0}'' + R_{p1}'')}{R_{S0}'(R_{p0}' + R_{p1}') + R_{S0}''(R_{p0}'' + R_{p1}'')} - \Delta_0, \tag{6}$$

$$\delta\psi = \psi - \psi_0 = \arctan\left(F\tan\left(\psi_0\right)\right) - \psi_0, \tag{7}$$

where F is a multiplier in

$$\tan\left(\psi_0 + \delta\psi\right) = F\tan\psi_0 =$$

$$= tan(\psi_0)\sqrt{1 + \frac{(R_{p1}^{'2} + R_{p1}^{''2} + 2(R_{p0}^{'}R_{p1}^{'} + R_{p0}^{''}R_{p1}^{''}))(R_{S0}^{'2} + R_{S0}^{''2})}{(R_{p0}^{'}R_{S0}^{'} + R_{S0}^{''}R_{p0}^{''})^2 + (R_{S0}^{''}R_{p0}^{'} - R_{p0}^{''}R_{S0}^{'})^2}}. \tag{8}$$

These equations do not depend on the number of layers in a sample, so can be used for every type of reflective nanostrustures models. Below, a two-layer model is presented.

2. Data processing for the case of a two-layer model

As it was mentioned above, ferromagnetic/semiconductor two-layer structures are a subject of interest nowadays. So in this chapter let us discuss a model consisting of an upper ferromagnetic layer 1 (the refraction index $N_1 = n_1 - ik_1$), a middle non-magnetic layer 2 (the refraction index $N_2 = n_2 - ik_2$) and a substrate 3 (the refraction index $N_3 = n_3 - ik_3$). The light electromagnetic wave is incident from non-magnetic dielectric medium 0 (e.g. vacuum, characterized by the refraction index $N_0 = n_0 - ik_0$) onto the upper layer. In the setup, a Cartesian coordinate system is defined with the x axis normal to the interfaces and pointing into the substrate from the sample surface. The y and x axis lie in the plane of incidence. We consider T-configuration (transverse) in which magnetization is z-axis directed, i.e perpendicular to the plane of incidence and parallel to the surface. So YX plane is a plane of incidence, YZ plane is a boundary plane.

For a two-layer model it is necessary to consider each interface (0-1, 1-2, 2-3) as each of them impacts the values of ellipsometric angles. The purpose of the data processing is to characterize a ferromagnetic layer.

The first step is carrying out ellipsometric and magneto-ellipsometric measurements. Here we do not focus on ellipsometric data analysis as there is a lot of research in this field [7, 8, 12]. So from ellipsometric measurements we can find complex refractive indices N_0, N_1, N_2, N_3, thicknesses of both layers, while magneto-ellipsometric parameters spectra are necessary for magneto-optical properties study of a ferromagnetic layer.

Fresnel coefficients that reflect magneto-optical properties can be derived from the scattering matrix:

$$\hat{S} = \hat{I}_{01}\hat{L}_1\hat{I}_{12}\hat{L}_2\hat{I}_{23}, \tag{9}$$

where $\hat{I}_{ab}$ is an interface matrix and $\hat{L}_c$ is a layer matrix [7].

$$R_S = \frac{(S_{21})_S}{(S_{11})_S}, \tag{10}$$

$$R_p = \frac{(S_{21})_p}{(S_{11})_p}, \tag{11}$$

$$R_S = \frac{r_{01S} + r_{12S}e^{-i2\beta_1} + r_{01S}r_{12S}r_{23S}e^{-i2\beta_2} + r_{23S}e^{-i2(\beta_1+\beta_2)}}{1 + r_{01S}r_{12S}e^{-i2\beta_1} + r_{12S}r_{23S}e^{-i2\beta_2} + r_{01S}r_{23S}e^{-i2(\beta_1+\beta_2)}}, \tag{12}$$

$$R_p = \frac{r_{01p} + r_{12p}\tau_{01p}e^{-i2\beta_1} - r_{01p}r_{21p}r_{23p}e^{-i2\beta_2} + r_{23p}\tau_{01p}\tau_{12p}e^{-i2(\beta_1+\beta_2)}}{1 - r_{10p}r_{12p}e^{-i2\beta_1} - r_{21p}r_{23p}e^{-i2\beta_2} - r_{10p}r_{23p}\tau_{12p}e^{-i2(\beta_1+\beta_2)}}, \tag{13}$$

where

$$\tau_{01p} = t_{10p}t_{01p} - r_{01p}r_{10p}, \tag{14}$$

$$\tau_{12p} = t_{21p}t_{12p} - r_{12p}r_{21p}. \tag{15}$$

So, in order to process magneto-ellipsometric data the following expressions are necessary:

$$r_{01p} = \frac{N_1 \cos\varphi_0 - N_0 \cos\varphi_1}{N_1 \cos\varphi_0 + N_0 \cos\varphi_1} - i\frac{2QN_0^2 \sin\varphi_0 \cos\varphi_0}{(N_1 \cos\varphi_0 + N_0 \cos\varphi_1)^2}, \tag{16}$$

$$r_{12p} = \frac{N_2 \cos \varphi_1 - N_1 \cos \varphi_2}{N_2 \cos \varphi_1 + N_1 \cos \varphi_2} - i \frac{2QN_1^2 \sin \varphi_1 \cos \varphi_1}{(N_2 \cos \varphi_1 + N_1 \cos \varphi_2)^2}, \tag{17}$$

$$r_{23p} = \frac{N_3 \cos \varphi_2 - N_2 \cos \varphi_3}{N_3 \cos \varphi_2 + N_2 \cos \varphi_3}, \tag{18}$$

$$r_{10p} = \frac{N_0 \cos \varphi_1 - N_1 \cos \varphi_0}{N_0 \cos \varphi_1 + N_1 \cos \varphi_0} + i \frac{2QN_1^2 \sin \varphi_1 \cos \varphi_1}{(N_0 \cos \varphi_1 + N_1 \cos \varphi_0)^2}, \tag{19}$$

$$r_{21p} = \frac{N_1 \cos \varphi_2 - N_2 \cos \varphi_1}{N_1 \cos \varphi_2 + N_2 \cos \varphi_1} + i \frac{2QN_2^2 \sin \varphi_2 \cos \varphi_2}{(N_1 \cos \varphi_2 + N_2 \cos \varphi_1)^2}, \tag{20}$$

$$r_{01S} = \frac{N_0 \cos \varphi_0 - N_1 \cos \varphi_1}{N_0 \cos \varphi_0 + N_1 \cos \varphi_1}, \tag{21}$$

$$r_{12S} = \frac{N_1 \cos \varphi_1 - N_2 \cos \varphi_2}{N_1 \cos \varphi_1 + N_2 \cos \varphi_2}, \tag{22}$$

$$r_{23S} = \frac{N_2 \cos \varphi_2 - N_3 \cos \varphi_3}{N_2 \cos \varphi_2 + N_3 \cos \varphi_3}, \tag{23}$$

$$t_{01p} = \frac{2N_0 \cos \varphi_0}{N_1 \cos \varphi_0 + N_0 \cos \varphi_1} + i \frac{2QN_0^3 \sin \varphi_0 \cos \varphi_0}{N_1(N_1 \cos \varphi_0 + N_0 \cos \varphi_1)^2}, \tag{24}$$

$$t_{10p} = \frac{2N_1 \cos \varphi_1}{N_1 \cos \varphi_0 + N_0 \cos \varphi_1} - i \frac{2QN_1^3 \sin \varphi_1 \cos \varphi_1}{N_0(N_1 \cos \varphi_0 + N_0 \cos \varphi_1)^2}, \tag{25}$$

$$t_{12p} = \frac{2N_1 \cos \varphi_1}{N_2 \cos \varphi_1 + N_1 \cos \varphi_2} + i \frac{2QN_1^3 \sin \varphi_1 \cos \varphi_1}{N_2(N_2 \cos \varphi_1 + N_1 \cos \varphi_2)^2}, \tag{26}$$

$$t_{21p} = \frac{2N_2 \cos \varphi_2}{N_2 \cos \varphi_1 + N_1 \cos \varphi_2} - i \frac{2QN_2^3 \sin \varphi_2 \cos \varphi_2}{N_1(N_2 \cos \varphi_1 + N_1 \cos \varphi_2)^2}, \tag{27}$$

$$\beta_1 = \frac{2\pi}{\lambda} N_1 \cos \varphi_1 d_1, \tag{28}$$

$$\beta_2 = \frac{2\pi}{\lambda} N_2 \cos \varphi_2 d_2, \tag{29}$$

where β_1 and β_2 are phase thicknesses of layer 1 and layer 2, respectively, d_1 and d_2 are thicknesses of layers 1 and 2. Subscripts 01, 12, 23 correspond to the wave propagation from medium 0 to medium 1, from 1 to 2 and from 2 to 3 respectively, while subscripts 10 and 21 correspond to the backward wave propagation. Indices r are refractive indices for the mentioned above interfaces, indices t are transmission coefficients. Angles φ_1 and φ_2 are related with φ_0 (the angle of incidence) by Snell's law. Q is a magneto-optical coupling parameter that is responsible for non-diagonal elements of dielectric tensor. It means that if we know this parameter we can fully describe the dielectric permitivity, not only diagonal elements. Hereinafter we present the formulae necessary for identifying Q from magneto-ellipsometric measurements. Let us rewrite (12-18) in the same manner as (2, 3):

$$r_{01S} = (R'_{S0})_{01} - i(R''_{S0})_{01}, \tag{30}$$

$$r_{12S} = (R'_{S0})_{12} - i(R''_{S0})_{12}, \tag{31}$$

$$r_{23S} = (R'_{S0})_{23} - i(R''_{S0})_{23}, \tag{32}$$

$$r_{23p} = (R'_{p0})_{23} - i(R''_{p0})_{23} = rr_{23} - i\, ri_{23}, \tag{33}$$

$$r_{01p} = (R'_{p0})_{01} + (R'_{p1})_{01} - i((R''_{p0})_{01} + (R''_{p1})_{01}) = rr_{01} - i\, ri_{01}, \tag{34}$$

$$r_{12p} = (R'_{p0})_{12} + (R'_{p1})_{12} - i((R''_{p0})_{12} + (R''_{p1})_{12}) = rr_{12} - i\ ri_{12}, \tag{35}$$

$$r_{23p} = (R'_{p0})_{23} + (R'_{p1})_{23} - i((R''_{p0})_{23} + (R''_{p1})_{23}) = rr_{23} - i\ ri_{23}, \tag{36}$$

$$r_{10p} = (R'_{p0})_{10} - (R'_{p1})_{10} - i((R''_{p0})_{10} - (R''_{p1})_{10}) = rr_{10} - i\ ri_{10}, \tag{37}$$

$$r_{21p} = (R'_{p0})_{21} - (R'_{p1})_{21} - i((R''_{p0})_{21} - (R''_{p1})_{21}) = rr_{21} - i\ ri_{21}, \tag{38}$$

$$t_{01p} = (T'_{p0})_{01} + (T'_{p1})_{01} - i((T''_{p0})_{01} + (T''_{p1})_{01}) = tr_{01} - i\ ti_{01}, \tag{39}$$

$$t_{12p} = (T'_{p0})_{12} + (T'_{p1})_{12} - i((T''_{p0})_{12} + (T''_{p1})_{12}) = tr_{12} - i\ ti_{12}, \tag{40}$$

$$t_{10p} = (T'_{p0})_{10} - (T'_{p1})_{10} - i((T''_{p0})_{10} - (T''_{p1})_{10}) = tr_{10} - i\ ti_{10}, \tag{41}$$

$$t_{21p} = (T'_{p0})_{21} - (T'_{p1})_{21} - i((T''_{p0})_{21} - (T''_{p1})_{21}) = tr_{21} - i\ ti_{21}, \tag{42}$$

where $(R'_{s0})_{01}, (R''_{s0})_{01}, (R'_{p0})_{01}, (R''_{p0})_{01}, (R'_{p1})_{01}, (R''_{p1})_{01}$ correspond to $R'_{s0}, R''_{s0}, R'_{p0}, R''_{p0}, R'_{p1}, R''_{p1}$ in the model of a homogeneous semi-infinite medium, respectively [11]. Subscript 01 denotes the electromagnetic wave incidence from ambient medium 0 onto layer 1. Indices $(R'_{s0})_{12}, (R''_{s0})_{12}, (R'_{p0})_{12}, (R''_{p0})_{12}, (R'_{p1})_{12}, (R''_{p1})_{12}$ are also calculated by formulae for the model of a homogeneous semi-infinite medium, the only difference is that subscript 12 denotes the electromagnetic wave incidence from layer 1 onto layer 2 that leads to the following changes in the formulae for the model of a homogeneous semi-infinite medium: $\cos\varphi_0 \to \cos\varphi_1$, $\cos\varphi_1 \to \cos\varphi_2$, $\sin\varphi_0 \to \sin\varphi_1$, $n_1 \to n_2$, $n_0 \to n_1$, $k_1 \to k_2$, $k_0 \to k_1$. Likewise, indices $(R'_{p0})_{10}, (R''_{p0})_{10}, (R'_{p1})_{10}, (R''_{p1})_{10}$ describe the electromagnetic wave propagation from layer 1 to medium 0: $\cos\varphi_0 \leftrightarrow \cos\varphi_1$, $\sin\varphi_0 \leftrightarrow \sin\varphi_1$, $n_0 \leftrightarrow n_1$, $k_0 \leftrightarrow k_1$. Indices $(R'_{p0})_{21}, (R''_{p0})_{21}, (R'_{p1})_{21}, (R''_{p1})_{21}$ correspond to the electromagnetic wave propagation from layer 2 to layer 1: $\cos\varphi_0 \to \cos\varphi_2$, $\sin\varphi_0 \to \sin\varphi_2$, $n_0 \to n_2$, $k_0 \to k_2$. Finally, indices $(R'_{s0})_{23}, (R''_{s0})_{23}, (R'_{p0})_{23}, (R''_{p0})_{23}, (R'_{p1})_{23}, (R''_{p1})_{23}$ describe the electromagnetic wave incidence from layer 2 on substrate 3: $\cos\varphi_0 \to \cos\varphi_2$, $\cos\varphi_1 \to \cos\varphi_3$, $\sin\varphi_0 \to \sin\varphi_2$, $n_1 \to n_3$, $n_0 \to n_2$, $k_1 \to k_3$, $k_0 \to k_2$.

Transmission coefficients necessary for data processing are the following:

$$(T'_{p0})_{01} = 2\frac{(n_0 n_1 + k_0 k_1)(a^2 + c^2) + (n_0^2 + k_0^2)(ab + cd)}{A_3^2 + B_3^2}, \tag{43}$$

$$(T''_{p0})_{01} = 2\frac{(n_0^2 + k_0^2)(ad - bc) + (n_1 k_0 - n_0 k_1)(a^2 + c^2)}{A_3^2 + B_3^2}, \tag{44}$$

$$(T'_{p1})_{01} = 2\frac{Q_1(pq + rs) - Q_2(pr - sq)}{(n_1^2 + k_1^2)(A_3^2 + B_3^2)^2}, \tag{45}$$

$$(T''_{p1})_{01} = 2\frac{Q_1(pr - sq) + Q_2(pq + rs)}{(n_1^2 + k_1^2)(A_3^2 + B_3^2)^2}, \tag{46}$$

where

$$A_3 = n_1 a + k_1 c + n_0 b + k_0 d, \tag{47}$$

$$B_3 = k_1 a - n_1 c + k_0 b - n_0 d, \tag{48}$$

$$p = N(3n_0^2 k_0 - k_0^3) + P(n_0^3 - 3n_0 k_0^2), \tag{49}$$

$$q = n_1(A_3^2 - B_3^2) - 2A_3 B_3 k_1, \tag{50}$$

$$r = k_1(B_3^2 - A_3^2) - 2A_3 B_3 n_1, \tag{51}$$

$$s = N(n_0^3 - 3n_0 k_0^2) - P(3n_0^2 k_0 - k_0^3), \tag{52}$$

$$a = \text{Re}(cos\varphi_0), \tag{53}$$

$$b = \text{Re}(cos\varphi_1), \tag{54}$$

$$c = \text{Im}(cos\varphi_0), \tag{55}$$

$$d = \text{Im}(cos\varphi_1), \tag{56}$$

$$N = \text{Re}(sin\varphi_0)a - \text{Im}(sin\varphi_0)c, \tag{57}$$

$$P = -\text{Re}(sin\varphi_0)c - \text{Im}(sin\varphi_0)a. \tag{58}$$

Transmission coefficients with subscripts 10, 12, 21 correspond to the electromagnetic wave propagation from layer 1 to medium 0, from layer 1 to layer 2, from layer 2 to layer 1, respectively. The changes in the formulae are the same as proposed for refractive indices.

Let us take into account $N_0 = n_0 - ik_0$, $N_1 = n_1 - ik_1$, $N_2 = n_2 - ik_2$, $Q = Q_1 - iQ_2$ and compare expressions (12, 13) with (2, 3). Thus we obtain expressions for $R'_{p0}, R''_{p0}, R'_{p1}, R''_{p1}, R'_{s0}$ and R''_{s0} in terms of numerators and denominators:

$$R_{S0} = \frac{numerator R_{S0}}{denominator R_{S0}} \equiv \frac{\text{Re}(n(R_{S0})) - i\,\text{Im}(n(R_{S0}))}{\text{Re}(d(R_{S0})) - i\,\text{Im}(d(R_{S0}))}, \tag{59}$$

$$R_{p0} = \frac{\text{Re}(n(R_{p0})) - i\,\text{Im}(n(R_{p0}))}{\text{Re}(d(R_{p0})) - i\,\text{Im}(d(R_{p0}))}, \tag{60}$$

$$R_p = \frac{\text{Re}(n(R_p)) - i\,\text{Im}(n(R_p))}{\text{Re}(d(R_p)) - i\,\text{Im}(d(R_p))}, \tag{61}$$

where n stands for numerator and d – for denominator. As a result, we have

$$R'_{p0} = \frac{\text{Re}(n(R_{p0}))\,\text{Re}(d(R_{p0})) + \text{Im}(n(R_{p0}))\,\text{Im}(d(R_{p0}))}{(\text{Re}(d(R_{p0})))^2 + (\text{Im}(d(R_{p0})))^2}, \tag{62}$$

$$R''_{p0} = \frac{\text{Im}(n(R_{p0}))\,\text{Re}(d(R_{p0})) - \text{Im}(d(R_{p0}))\,\text{Re}(n(R_{p0}))}{(\text{Re}(d(R_{p0})))^2 + (\text{Im}(d(R_{p0})))^2}, \tag{63}$$

$$R'_{p1} = \frac{\text{Re}(n(R_p))\,\text{Re}(d(R_p)) + \text{Im}(n(R_p))\,\text{Im}(d(R_p))}{(\text{Re}(d(R_p)))^2 + (\text{Im}(d(R_p)))^2} - R'_{p0}, \tag{64}$$

$$R''_{p1} = \frac{\text{Im}(n(R_p))\,\text{Re}(d(R_p)) - \text{Im}(d(R_p))\,\text{Re}(n(R_p))}{(\text{Re}(d(R_p)))^2 + (\text{Im}(d(R_p)))^2} - R''_{p0}, \tag{65}$$

$$R'_{S0} = \frac{\text{Re}(n(R_{S0}))\,\text{Re}(d(R_{S0})) + \text{Im}(n(R_{S0}))\,\text{Im}(d(R_{S0}))}{(\text{Re}(d(R_{S0})))^2 + (\text{Im}(d(R_{S0})))^2}, \tag{66}$$

$$R''_{S0} = \frac{\text{Im}(n(R_{S0}))\,\text{Re}(d(R_{S0})) - \text{Im}(d(R_{S0}))\,\text{Re}(n(R_{S0}))}{(\text{Re}(d(R_{S0})))^2 + (\text{Im}(d(R_{S0})))^2}, \tag{67}$$

where the following notations are used:

$$\text{Re}(n(R_{p0})) = (R'_{p0})_{01} + \xi_1(R'_{p0})_{12} - \eta_1(R''_{p0})_{12} + L_{0112}(\xi_2(R'_{p0})_{23} - \eta_2(R''_{p0})_{23}) -$$

$$-M_{0112}(\xi_2(R''_{p0})_{23} + \eta_2(R'_{p0})_{23}) + (R'_{p0})_{23}(\xi_1\xi_2 - \eta_1\eta_2) - (R''_{p0})_{23}(\xi_2\eta_1 + \xi_1\eta_2), \tag{68}$$

$$\text{Im}(n(R_{p0})) = (R''_{p0})_{01} + \eta_1(R'_{p0})_{12} + \xi_1(R''_{p0})_{12} + L_{0112}(\xi_2(R''_{p0})_{23} + \eta_2(R'_{p0})_{23}) +$$

$$+M_{0112}(\xi_2(R'_{p0})_{23} - \eta_2(R''_{p0})_{23}) + (R''_{p0})_{23}(\xi_1\xi_2 - \eta_1\eta_2) + (R'_{p0})_{23}(\xi_2\eta_1 + \xi_1\eta_2), \qquad (69)$$

$$\mathrm{Re}(d(R_{p0})) = 1 + L_{0112}\xi_1 - M_{0112}\eta_1 + \xi_2 L_{1223} - \eta_2 M_{1223} +$$
$$+ (\xi_1\xi_2 - \eta_1\eta_2)L_{0123} - (\xi_2\eta_1 + \xi_1\eta_2)M_{0123}, \qquad (70)$$

$$\mathrm{Im}(d(R_{p0})) = L_{0112}\eta_1 + M_{0112}\xi_1 + \xi_2 M_{1223} + \eta_2 L_{1223} +$$
$$+ (\xi_1\xi_2 - \eta_1\eta_2)M_{0123} + (\xi_2\eta_1 + \xi_1\eta_2)L_{0123}, \qquad (71)$$

$$\mathrm{Re}(n(R_p)) = rr_{01} + (\xi_1 rr_{12} - \eta_1 ri_{12})(\kappa_1)_{01} - (\xi_1 ri_{12} + \eta_1 rr_{12})(\kappa_2)_{01} -$$
$$- (rr_{01}rr_{21} - ri_{01}ri_{21})(\xi_2 rr_{23} - \eta_2 ri_{23}) + (ri_{01}rr_{21} + rr_{01}ri_{21})(\xi_2 ri_{23} + \eta_2 rr_{23}) +$$
$$+ (rr_{23}(\xi_1\xi_2 - \eta_1\eta_2) - ri_{23}(\xi_2\eta_1 + \xi_1\eta_2))((\kappa_1)_{01}(\kappa_1)_{12} - (\kappa_2)_{01}(\kappa_2)_{12}) -$$
$$- (ri_{23}(\xi_1\xi_2 - \eta_1\eta_2) + rr_{23}(\xi_1\eta_2 + \eta_1\xi_2))((\kappa_1)_{01}(\kappa_2)_{12} + (\kappa_1)_{12}(\kappa_2)_{01}), \qquad (72)$$

$$\mathrm{Im}(n(R_p)) = ri_{01} + (\xi_1 ri_{12} + \eta_1 rr_{12})(\kappa_1)_{01} + (\xi_1 rr_{12} - \eta_1 ri_{12})(\kappa_2)_{01} -$$
$$- (ri_{01}rr_{21} + rr_{01}ri_{21})(\xi_2 rr_{23} - \eta_2 ri_{23}) - (rr_{01}rr_{21} - ri_{01}ri_{21})(\xi_2 ri_{23} + \eta_2 rr_{23}) +$$
$$+ (ri_{23}(\xi_1\xi_2 - \eta_1\eta_2) + rr_{23}(\xi_2\eta_1 + \xi_1\eta_2))((\kappa_1)_{01}(\kappa_1)_{12} - (\kappa_2)_{01}(\kappa_2)_{12}) +$$
$$+ (rr_{23}(\xi_1\xi_2 - \eta_1\eta_2) - ri_{23}(\xi_1\eta_2 + \eta_1\xi_2))((\kappa_1)_{01}(\kappa_2)_{12} + (\kappa_1)_{12}(\kappa_2)_{01}), \qquad (73)$$

$$\mathrm{Re}(d(R_p)) = 1 - \xi_1(rr_{10}rr_{12} - ri_{10}ri_{12}) + \eta_1(ri_{10}rr_{12} + rr_{10}ri_{12}) -$$
$$- \xi_2(rr_{21}rr_{23} - ri_{21}ri_{23}) + \eta_2(ri_{21}rr_{23} + ri_{23}rr_{21}) -$$
$$((\kappa_1)_{12}(rr_{10}rr_{23} - ri_{10}ri_{23}) - (\kappa_2)_{12}(ri_{10}rr_{23} + ri_{23}rr_{10}))(\xi_1\xi_2 - \eta_1\eta_2) +$$
$$+ ((\kappa_1)_{12})(ri_{10}rr_{23} + ri_{23}rr_{10}) + (\kappa_2)_{12}(rr_{10}rr_{23} - ri_{10}ri_{23}))(\xi_2\eta_1 + \xi_1\eta_2), \qquad (74)$$

$$\mathrm{Im}(d(R_p)) = -\xi_1(ri_{10}rr_{12} + rr_{10}ri_{12}) - \eta_1(rr_{10}rr_{12} - ri_{10}ri_{12}) -$$
$$- \xi_2(ri_{21}rr_{23} + rr_{21}ri_{23}) - \eta_2(rr_{21}rr_{23} - ri_{23}ri_{21}) -$$
$$((\kappa_1)_{12}(ri_{10}rr_{23} + rr_{10}ri_{23}) + (\kappa_2)_{12}(rr_{10}rr_{23} - ri_{23}ri_{10}))(\xi_1\xi_2 - \eta_1\eta_2) -$$
$$- ((\kappa_1)_{12})(rr_{10}rr_{23} - ri_{23}ri_{10}) - (\kappa_2)_{12}(ri_{10}rr_{23} + rr_{10}ri_{23}))(\xi_2\eta_1 + \xi_1\eta_2), \qquad (75)$$

$$\mathrm{Re}(n(R_{S0})) = (R'_{S0})_{01} + \xi_1(R'_{S0})_{12} - \eta_1(R''_{S0})_{12} + H_{0112}(\xi_2(R'_{S0})_{23} - \eta_2(R''_{S0})_{23}) -$$
$$- J_{0112}(\xi_2(R''_{S0})_{23} + \eta_2(R'_{S0})_{23}) + (R'_{S0})_{23}(\xi_1\xi_2 - \eta_1\eta_2) - (R''_{S0})_{23}(\xi_2\eta_1 + \xi_1\eta_2), \qquad (76)$$

$$\mathrm{Im}(n(R_{S0})) = (R''_{S0})_{01} + \eta_1(R'_{S0})_{12} + \xi_1(R''_{S0})_{12} + H_{0112}(\xi_2(R''_{S0})_{23} + \eta_2(R'_{S0})_{23}) +$$
$$+ J_{0112}(\xi_2(R'_{S0})_{23} - \eta_2(R''_{S0})_{23}) + (R''_{S0})_{23}(\xi_1\xi_2 - \eta_1\eta_2) + (R'_{S0})_{23}(\xi_2\eta_1 + \xi_1\eta_2), \qquad (77)$$

$$\text{Re}(d(R_{S0})) = 1 + H_{0112}\xi_1 - J_{0112}\eta_1 + \xi_2 H_{1223} - \eta_2 J_{1223} +$$

$$+ (\xi_1\xi_2 - \eta_1\eta_2)H_{0123} - (\xi_2\eta_1 + \xi_1\eta_2)J_{0123}, \tag{78}$$

$$\text{Im}(d(R_{S0})) = H_{0112}\eta_1 + J_{0112}\xi_1 + \xi_2 J_{1223} + \eta_2 H_{1223} +$$

$$+ (\xi_1\xi_2 - \eta_1\eta_2)J_{0123} + (\xi_2\eta_1 + \xi_1\eta_2)H_{0123}, \tag{79}$$

$$\xi_1 = \text{Re}(e^{-i2\beta_1}), \tag{80}$$

$$\eta_1 = -\text{Im}(e^{-i2\beta_1}), \tag{81}$$

$$\xi_2 = \text{Re}(e^{-i2\beta_2}), \tag{82}$$

$$\eta_2 = -\text{Im}(e^{-i2\beta_2}), \tag{83}$$

$$L_{0112} = (R'_{p0})_{12}(R'_{p0})_{01} - (R''_{p0})_{12}(R''_{p0})_{01}, \tag{84}$$

$$M_{0112} = (R'_{p0})_{01}(R''_{p0})_{12} + (R''_{p0})_{01}(R'_{p0})_{12}, \tag{85}$$

$$L_{1223} = (R'_{p0})_{23}(R'_{p0})_{12} - (R''_{p0})_{23}(R''_{p0})_{12}, \tag{86}$$

$$M_{1223} = (R'_{p0})_{12}(R''_{p0})_{23} + (R''_{p0})_{12}(R'_{p0})_{23}, \tag{87}$$

$$L_{0123} = (R'_{p0})_{23}(R'_{p0})_{01} - (R''_{p0})_{23}(R''_{p0})_{01}, \tag{88}$$

$$M_{0123} = (R'_{p0})_{01}(R''_{p0})_{23} + (R''_{p0})_{01}(R'_{p0})_{23}, \tag{89}$$

$$H_{0112} = (R'_{S0})_{12}(R'_{S0})_{01} - (R''_{S0})_{12}(R''_{S0})_{01}, \tag{90}$$

$$J_{0112} = (R'_{S0})_{01}(R''_{S0})_{12} + (R''_{S0})_{01}(R'_{S0})_{12}, \tag{91}$$

$$H_{1223} = (R'_{S0})_{23}(R'_{S0})_{12} - (R''_{S0})_{23}(R''_{S0})_{12}, \tag{92}$$

$$J_{1223} = (R'_{S0})_{12}(R''_{S0})_{23} + (R''_{S0})_{12}(R'_{S0})_{23}, \tag{93}$$

$$H_{0123} = (R'_{S0})_{23}(R'_{S0})_{01} - (R''_{S0})_{23}(R''_{S0})_{01}, \tag{94}$$

$$J_{0123} = (R'_{S0})_{01}(R''_{S0})_{23} + (R''_{S0})_{01}(R'_{S0})_{23}, \tag{95}$$

$$(\kappa_1)_{01} = tr_{10}tr_{01} - ti_{10}ti_{01} - rr_{01}rr_{10} + ri_{01}ri_{10}, \tag{96}$$

$$(\kappa_2)_{01} = ti_{10}tr_{01} + tr_{10}ti_{01} - rr_{01}ri_{10} - ri_{01}rr_{10}, \tag{97}$$

$$(\kappa_1)_{12} = tr_{21}tr_{12} - ti_{21}ti_{12} - rr_{21}rr_{12} + ri_{21}ri_{12}, \tag{98}$$

$$(\kappa_2)_{12} = ti_{12}tr_{21} + tr_{12}ti_{21} - rr_{12}ri_{21} - ri_{12}rr_{21}. \tag{99}$$

So all necessary expressions that relate measured ellipsometric and magneto-ellipsometric parameters with refraction indices, coefficients of extinction, magneto-optical coupling parameter in case of a two-layer model are obtained. The final step is giving the best fit to the experimental data by the use of the wavelength-to-wavelength Nelder–Mead minimization [13] of the ellipsometric angles. It yields real and imaginary parts of magneto-optical parameter Q, thus information about all elements of the dielectric permittivity tensor can be obtained from the experiment.

3.   Conclusion

To conclude, we have proposed an approach to studying two-layer nanomaterials by means of magneto-ellipsometry. The algorithm of experimental data analysis (ψ_0, δ_0, $\psi_0 + \delta\psi$, $\Delta_0 + \delta\Delta$) is presented. As a result, optical and magneto-optical properties can be easily and reliably characterized during films growth through the presented formulae that are to be used in the software for magneto-optical ellipsometry setups.

References

[1] N.V. Volkov et al., Magnetic-field- and bias-sensitive conductivity of a hybrid $Fe/SiO_2/p-Si$ structure in planar geometry, *J. Appl. Phys.* **109**(2011), 123924 (English).

[2] K. Mok et al., Thickness independent magneto-optical coupling constant of nickel films in the visible spectral range, *Rev. Sci. Instrum.*, **82**(2011), 033112 (English).

[3] K. Mok et al., Vector-magneto-optical generalized ellipsometry, *J. Appl. Phys.* **110**(2011), 123110 (English).

[4] K. Mok et al., Magneto-optical coupling in ferromagnetic thin films investigated by vector-magneto-optical generalized ellipsometry, *Phys. Rev. B* , **84**(2011), 094413 (English).

[5] G. Neuber et al., Temperature-dependent spectral generalized magneto-optical ellipsometry for ferromagnetic compounds, *Rev. Sci. Instrum.* **76**(2005), 023910 (English).

[6] Š. Višňovský, Magneto-optical ellipsometry, *Czech. J. Phys. B* **36** (1986), 625-650 (English).

[7] R.M.A. Azzam, N.M. Bashara, Elipsometry and polirilized light , *North Holland Publishing Company, New York*, (1977), p.583 (English).

[8] H. Fujiwara, Spectroscopic Ellipsometry Principles and Applications, *Chichester: John Wiley & Sons*, (2007), p.392 (English).

[9] O.A. Maximova et al., Magnetic contribution to Fresnel coefficients at magneto-ellipsometric investigation, *J. SibSAU Vestnik* , **49**(2013), no. 3, 212-217 (English).

[10] O.A. Maximova et al., Features of the ellipsometric investigation of magnetic nanostructures, *Journal Of Structural Chemistry* **55**(1964), no. 6, 1134-1141 (English).

[11] O.A. Maximova et al., In situ magneto-optical ellipsometry data analysis for films growth control, *JMMM* (2017), accepted for publication (English).

[12] I.A. Tarasov et al., Quick ellipsometric technique for determining the thicknesses and optical constant profiles of $Fe/SiO_2/Si$(100) nanostructures during growth, *Tech. Phys.* **57**(2012), no. 6, 1225-1229 (English).

[13] J.A. Nelder, A.D. Mead, A Simplex Method for Function Minimization, *Computer Journal* **7**(1965), 308-313 (English).

Двухслойная модель отражающих ферромагнитных пленок для исследования тонких пленок методом магнитоэллипсометрии

Ольга А. Максимова, Сергей Г. Овчинников, Николай Н. Косырев, Сергей А. Лященко

Представлен метод анализа магнито-эллипсометрических измерений. Детально рассматривается двуслойная модель ферромагнитных отражающих пленок. Полученный алгоритм может использоваться для контроля оптических и магнито-оптических свойств пленок в процессе их роста в вакуумных камерах.

Ключевые слова: Магнито-оптическая эллипсометрия, эффект Керра, двухслойная модель, ферромагнетик, отражение, контроль роста.