

Singular Spectral Analysis in Filtration of Noise-contaminated Signals of Pseudolite Navigation

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Abstract

Background: The relevance of this research is determined by the necessity to evaluate precision specifications of ground equipment for satellite radio navigation system consumers, in particular regarding signal filtration and recovery tasks with view to the type of signal and nature of interferences. **Method:** A Pseudolite signal filtration and recovery algorithm has been developed, using singular spectral analysis, which allows for successful Pseudolite signal processing in noise-contaminated conditions and may be considered as an effective program implementation of a filtering unit as a part of the receiving equipment. **Findings:** Comparative analysis of noise components of signals used in Pseudolite navigation has been done, possibilities for enhancing interference immunity of Pseudolite systems, based on filtering such signals, have been discussed in this research. Numerical experiments were conducted to prove that the presented algorithm allows for successful Pseudolite signal processing in noise-contaminated conditions and may be considered as an effective program implementation of a filtering unit being a part of the receiving equipment. **Applications/Improvements:** Materials of this research may be useful for satellite radio navigation system consumers, while enhancing accuracy of attitude sensing of various mobile objects.

Keywords: Noise-Contaminated Signal, Pseudolites, Singular Spectral Analysis

1. Introduction

The use of consumer equipment of satellite radio navigation systems now enhances significantly accuracy of attitude sensing of a variety of mobile objects (see, for example, [1-4](#) and the related links). To enhance reliability and accuracy of coordinate setting of ground objects consumer equipment of satellite radio navigation systems that operates in a differential mode is widely used today, which minimizes positioning errors down to several meters.¹⁻⁴ The differential mode is now available in military and also civil consumer equipment. Moreover, to improve reliability and interference immunity of consumer equipment of satellite radio navigation systems, networks of ground transmitters (so called Pseudolites)

may be rolled out to emit signals that are similar in their structure and format to navigation satellite signals. A lot of publications are now dedicated to the use of Pseudolites as effective navigation devices (see, for example⁴⁻¹², and the related links). Usually, Pseudolite signal strength at 40...50 dB does not exceed that of navigation satellite signals at the input of consumer equipment of satellite radio navigation systems. Today, a task of evaluating precision specifications of ground equipment for consumers of satellite radio navigation systems dealing with Pseudolite signals, and especially under interference conditions, has not been studied comprehensively, and seems relevant⁹⁻¹². Therefore, a Pseudolite signal filtration and recovery task with view to the type of such signal and nature of interferences is also relevant.

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A Pseudolite signal filtration and recovery algorithm, based on singular spectral analysis, is presented in this research¹³⁻¹⁵. Numeric experiments done demonstrate operating efficiency of the presented algorithm in recovering a noise-contaminated Pseudolite signal.

2. Concept Headings - pseudolites: Basic Provisions and Application in Navigation Systems

Modern GLONASS/GPS satellite radio navigation systems allow for determining coordinates of a consumer with 10-15 m accuracy. Accuracy of navigational determinations may be enhanced by using differential satellite navigation methods¹⁻¹⁴.

The differential mode of satellite radio navigation systems implies availability of at least two satellite receivers or Receiver-Indicators (RI). For example, RI1 (an augmentation station) and RI2 (a consumer) are located in points 1 and 2 of the space, while RI1 has an accurate geodetic tie to the approved coordinate system. Differences between pseudo-ranges that have been measured by the RI1 and calculated using the RI1, and differences between corresponding pseudo-speeds of Data Transmission Lines (DTL) are transmitted as differential corrections of the RI2, where they are deducted from the pseudo-range (PD) and pseudo-speed values that have been measured by the PD2. If PD errors do not vary greatly in time and space, they are compensated essentially by the corrections transmitted via the DTL.

A design of a differential sub-system to comprise a communication cable well and DR with various kinds of a radio channel (HF, VHF, or satellite) has been taken as the standard today and has been well developed. A standard format of the differential correction has been developed.

Introduction of the differential mode, using a Pseudolite, is a perspective application of the DR. A Pseudolite is an augmentation system that induces a signal similar to a signal of navigation spacecrafts (NSC) and has an accurate geodetic tie to the approved coordinate system. Additional pseudo-range can be measured, using this signal. So, the use of the Pseudolite provides the consumer with an additional and highly reliable navigation point, along with differential corrections.

The use of Pseudolites diminishes contribution of a geometric factor by 6-8 times vertically and by 3-4 times horizontally due to an additional radio navigation point

appearing in the radio view area and located in the lower hemisphere. This is the condition that allows for a significant reduction in errors, when using the PC.

Implementation of the differential system, using Pseudolites, is associated with a number of challenges. Signals that are induced by the Pseudolite navigation signal source interfere with operation of the GLONASS/GPS equipment, and reference navigation receiver of the Pseudolite itself. Accordingly, a solution to the issues of normal operation of Pseudolite navigation receivers and an object that is located in Pseudolite working range, which receive simultaneously navigation signals of the GLONASS radio navigation system's satellite and Pseudolite navigation signal source, must be provided. Moreover, NSCs have poor interference immunity, which may lead to failures of the system using the NSC.

Let us consider a process of receiving Pseudolite signals, taking an aircraft (AC) as our object⁴. Pseudolites have functions of generating correcting information and transmitting signals that are required for differential correction over to the consumer. Digital messages may be transmitted within the digital frame of the Pseudolite navigation signal similar to information of the NSC. It was offered before, when elaborating systems that use Pseudolites, to receive the Pseudolite signal with the help of the same antenna as for NSC signals. Accordingly, no changes or additional developments are required for this alternative in AC on-board radio navigation equipment, associated with incorporation of a Pseudolite.

The principal disadvantage of this alternative is associated with aircraft equipment engineering. It is assumed that an antenna, which is located in the upper part of an AC, receives Pseudolite signals and simultaneously SRNS navigation satellite signals. This approach is dictated by considerations for avoidance of complicated equipment. However, the problem of Pseudolite signal receipt arises, as long as the Pseudolite is on ground (the lower hemisphere) and such receipt will be done by side lobes with their amplitude of ca. minus 20 dB (the main lobe faces upwards to receive the NSC signals). Suppression of the incoming Pseudolite signal may be even bigger due to the fact that the AC itself is a big screen. Enhancing Pseudolite transmitter power may be one way to solve this problem; however, implementation of this solution is associated with the following challenges. ACs of different types may demonstrate different signal attenuation subject to AC configuration and receiving antenna position on the AC. Attenuation may also differ depending on the type (speci-

fication) of antenna, spacial position of the AC associated with its maneuvers and distance to the Pseudolite.

It is obvious that with the increased Pseudolite transmitter power, aircraft closer to the Pseudolite, and maneuvers of the AC, when Pseudolite signals can be received not only by the side lobes, but the main lobe as well, the effective Pseudolite signal intensity will rise at the receiver input, while interfering with NSC signals receipt, and thus making receipt of NSC signals and determination of AC coordinates impossible.

Having analyzed this problem,⁴ one may conclude that the use of Pseudolites may be effective, provided two spaced antennas are installed on an aircraft, one (fixed) AVR antenna for the upper hemisphere, and the other body-fixed antenna below (for the lower hemisphere) to receive Pseudolite signals. This approach leads to aircraft equipment sophistication, however, it is absolutely necessary for the effective use of Pseudolites, and such sophistication is not big in terms of design. The need for the antenna to be installed in the lower part of an AC is also dictated by a possible decrease in Pseudolite signal intensity for this configuration, as long as the Pseudolite signal is received by the main lobe of the lower antenna.

2.1 Signal Receipt at a Substation with the Antenna Underside

A problem of processing two signals arises, when two antennas are installed on an AC. It is more reasonable to process both signals in the same AVR. For this case, signals of the upper and the lower antennas must be added.

2.1.1 Adding NSC and Pseudolite signals at a high frequency

NSC and Pseudolite signals are added at a high frequency, i.e signals are addressed directly from the outputs of a Mast Head Amplifier (MHA) to an adder and further to a Radio Link (RL), Analog to Digital Converter (ADC), Digital Signal Processing Unit (DSPU) and Computing Unit (CU). Strength of the signal that is received from a Pseudolite increases with an AC getting closer, while intensity of the signal that is received from an NSC is untouched for this pattern.⁴

A receiving antenna and (MHA) for receiving Pseudolite signals are offered the same as for receiving NSC signals. However, one should point out the problem of receiving near-field Pseudolite and NSC signals. This is a well-known effect, which is called a near-field/ far-field

problem. Additionally, the difference between the near-field and far-field is not determined solely by Pseudolite power, but also by normal working range of the AC receiver, where the signal from the Pseudolite is not an intolerably large interference towards the NSC signal. In case the above on-board equipment configuration is used, the Pseudolite signal intensity on the adder will be high and out of proportion to the NSC signal and will suppress the NSC signal in the near-field.

Baseline minimum signal/interference ratio at the navigation receiver input for normal operation is $S/N = -20$ dB. Tolerable interference strength $P_{I\text{ add}}$ is determined, using the tolerable S/N ratio and NSC signal strength at the receiver input $P_{rec} = -161.5$ dB.

$$P_{I\text{ add}} = -161.5 - (-20) = -141 \text{ dB.}$$

As evident, with the shorter distance to the Pseudolite, the signal intensity in the AC receiving antenna in the near-field will be larger than that of the NSC signal, interfering with AVR operation.

Accordingly, a simple addition of a standard antenna and MHA with adding at a HF is not enough to ensure system operation. One may offer to install an Automatic Gain Controller (AGC) after the Pseudolite signal receiving antenna, which will maintain constantly the desired signal strength at the input of the adder.

2.1.2 Adding NSC and Pseudolite Signals, while using an AGC and Converting Frequency

This option implies using a separate radio link from the antenna to the adder to receive Pseudolite signals. A radio tract comprises a Frequency Converter (FC), a heterodyne, an intermediate-frequency amplifier (i.f. amplifier) and an Automatic Gain Control (AGC), in addition to a MHA. Now, when an AC approaches a Pseudolite, and Pseudolite signal strength growth at the input of the receiving antenna, the AC can receive Pseudolite signals without any interference with NSC signals.

When this solution is implemented, there is still a probability of the pseudo-signal to be received by the main AC antenna and issues of electro-magnetic incomparability of communication cable well navigation equipment and Pseudolite Navigation Signal Transmitter (NST). As long as the receiving antenna and the transmitting antenna of the communication cable well are located close to each other and operate in the same frequency range, Pseudolite NST signals will interfere with the communication cable well navigation equipment.

To resolve this issue, spatial separation of the transmitter and receiver at the distance of about 30-50 m may be applied, directive antennas and additional absorbing screens may be installed, or special band-reject filters that suppress Pseudolite NST signals may be used in the standard Pseudolite navigation receiver.

Pseudolite signal transmission at frequencies other than that in the operating range of the GLONASS/GPS may appear to be a more perspective solution. For example, a bandwidth that is allocated for radio navigation of ≈ 9 GHz may be used.

When this range is used for Pseudolite signals, the Pseudolite receiver will not cause any interference for communication cable well navigation equipment or AC navigation equipment that receives NSC signals.

Different frequency range used will allow for enhancing Pseudolite transmitter power, if necessary, which will improve, in its turn, interference immunity of Pseudolite signals. Let us now discuss resistance to interference of radio navigation equipment that relies on GLONASS/GPS systems, with view to Pseudolites, and classify any possible interferences^{6,9-12}.

3. The Necessity to Filter Pseudolite Navigation Signals: Interference Immunity and Types of Interferences

Low interference immunity is one of the greatest weaknesses of any AVR that relies on signals of GLONASS/GPS satellite navigation systems. Therefore, intensive work has been done to develop interference-immune GPS receiving equipment.

Low intensity of the NSC desired signal at the receiving device input, which equals to minus (165...155) dBw is the main cause of low interference immunity of GLONASS/ GPS AVRs. To make a receiving device inoperable, interference strength in the bandwidth over minus 140 dBw (ratio of interference/signal plus 25 dB) is enough. This tolerable interference strength value is determined by properties (spectrum width) of a pseudo-random sequence that modulates the NSC signal. Considering that a spectral window of GLONASS/ GPS systems is known, it is easy to place an intentional interference directly in the spectral window of the receiving equipment.

When on-Board Satellite Navigation Equipment (BSNE) is affected by interferences of intensities specified below, a pseudo-range error and error-in-word indicator must not exceed the values presented in Table 1.

Table 1. Pseudo-range error and error-in-word indicator values

	GPS	GLONASS
Pseudo-range error	0.4 m	0.8 m
Error-in-word indicator	1/10 ⁴	1/10 ⁴

These pseudo-range errors do not account for a contribution determined by the signal propagation conditions (multipath, tropospheric and ionospheric effects) and faulty emperal information and clocks of GPS and GLONASS satellites.

Signal strength values that are specified in Table 1 were obtained under the condition that the amplification factor of the standard antenna at the angle of the place over 5° equals to minus 4.65 dB, relative to the isotropic circular polarization antenna. At the same time, it is assumed that maximum antenna amplification factor in the lower hemisphere is minus 10 dBik. For antennas that differ from the standard antenna and have a different minimum amplification factor at the angle of the place 5°, different interfering signal intensity values must be used, and at the same time the ratio of interference intensity and desired signal must be constant. Let us consider basic types of interferences that accompany Pseudolite navigation signals^{4,6}.

3.1 Interferences in the Form of Harmonic Oscillations

A non-modulated harmonic interference is obviously the simplest type of interference (however, this interference still needs to be eliminated by special methods, for example with the help of adaptive filters). Broadband interferences may constitute a greater threat.

When switching to the navigation determinations mode, GLONASS/ GPS receivers must satisfy the requirements in the presence of the harmonic interference at the desired signal intensity at the antenna input of minus 164.5 dBw (GPS) and minus 165.5 dBw (GLONASS).

The receivers must satisfy the established requirements in the presence of interfering signals with their

strength equaling to the values presented in the right column of Table 2.

3.2 Interference in the Form of Limited-spectrum Noise

When switching to the navigation determinations mode, GLONASS/GPS receivers must satisfy the requirements presented in Table 1 under an interference in the form of noise in the frequency bandwidth of $f_i \pm B_w/2$ and antenna input power values equaling to threshold values provided in table 5, and wanted signal intensity at the antenna input equaling to minus 164.5 dBw (GPS) and minus 165.5 dBw (GLONASS).

When the receiver operates in the determinations mode en route and search and capture modes (before switching to the navigation determinations mode), threshold values of interference in the form of noise of limited spectrum must be by 3dB lower than those specified in Table 3 at all flight stages.

3.3 Pulse Interference

When turned to the navigation determination mode, the receiver must satisfy the requirements given in table 1

under pulse interferences having the parameters given in Table 4, where threshold interference strength values are measured at the antenna input.

3.4 Ways to Enhance AVR Interference Immunity

Cause analysis of AVR's interoperability, when affected by interferences can be done, and ways to improve interference immunity can be outlined⁶. The value of minus 140 Dbw (minus 150 Dbw acc. to KT-34-01), which limits tolerable interference level of modern AVRs, corresponds to the AVR design, which is today taken as the standard. This option involves an omni-directional antenna, a wide bandwidth analog highway (all GLONASS letters are received by the same wide bandwidth highway), and no adaptive filtration in the bandwidth. Such AVR configuration ensures a rather simple hardware implementation and good metrological performance. Special methods of equipment design must apply to ensure AVR's operation against interferences.

Factors that cause equipment inoperability at different interference strengths vary. For weak interferences and until the linear mode of the radio path is maintained,

Table 2. Threshold harmonic interferences for GLONASS/ GPS receivers

Frequency range	Threshold interferences for receivers used for non-precision approach	Threshold interferences for receivers used for other flight stages
$f_i \leq 1,315$ MHz	minus 7.5 Dbw	minus 10 Dbw
1,315 MHz < $f_i \leq 1,525$ MHz	decreasing in a linear fashion from minus 7.5 Dbw to minus 45 Dbw	decreasing in a linear fashion from minus 10.5 Dbw to minus 48 Dbw
1,525 MHz < $f_i \leq 1,565.42$ MHz	decreasing in a linear fashion from minus 45 Dbw to minus 153.5 Dbw	decreasing in a linear fashion from minus 48 Dbw to minus 156.5 Dbw
1,565.42 MHz < $f_i \leq 1,609.36$ MHz	decreasing in a linear fashion from minus 153.5 Dbw to minus 152 Dbw	decreasing in a linear fashion from minus 156.5 Dbw to minus 155 Dbw
1,592.9525 MHz < $f_i \leq 1,609.36$ MHz	minus 152 Dbw	minus 155 Dbw
1,609.36 MHz < $f_i \leq 1,613.65625$ MHz	increasing in a linear fashion from minus 152 Dbw to minus 83 Dbw	increasing in a linear fashion from minus 155 Dbw to minus 86 Dbw
1,613.65625 MHz < $f_i \leq 1,635.15625$ MHz	increasing in a linear fashion from minus 83 Dbw to minus 45 Dbw	increasing in a linear fashion from minus 86 Dbw to minus 48 Dbw
1,613.65625 MHz < $f_i \leq 1,626.15625$ MHz	increasing in a linear fashion from minus 83 Dbw to minus 25 Dbw	increasing in a linear fashion from minus 86 Dbw to minus 28 Dbw
1,635.15625 MHz < $f_i \leq 2,000$ MHz	increasing in a linear fashion from minus 45 Dbw to minus 11.5 Dbw	increasing in a linear fashion from minus 48 Dbw to minus 14.5 Dbw
1,626.15625 MHz < $f_i \leq 2,000$ MHz	increasing in a linear fashion from minus 25 Dbw to minus 11.5 Dbw	increasing in a linear fashion from minus 28 Dbw to minus 14.5 Dbw
$f_i > 2,000$ MHz	minus 11 Dbw	minus 14 Dbw

individual NSC signals may be affected by the interferences, i.e. any GLONASS satellite may be affected with the interference acting at its frequency, or all GPS satellites may be affected, as long as they operate on the same frequency. For weak interferences, a variety of adaptive filtration options can be implemented in the equipment, including digital and even programming level⁹⁻¹².

Table 3. Threshold noise interferences of limited range for GLONASS/ GPS receivers

Interference spectrum width	Threshold interference
$0 \text{ Hz} < Bw_i \leq 700 \text{ Hz}$	-153.5 Dbw
$700 \text{ Hz} < Bw_i \leq 10 \text{ kHz}$	$-153.5 + 6 \log_{10}(Bw/700) \text{ Dbw}$
$10 \text{ kHz} < Bw_i \leq 100 \text{ kHz}$	$-146.5 + 3 \log_{10}(Bw/10000) \text{ Dbw}$
$100 \text{ kHz} < Bw_i \leq 1 \text{ MHz}$	-143.5 Dbw
$1 \text{ MHz} < Bw_i \leq 20 \text{ MHz}$	increasing in a linear fashion from -143.5 Dbw to -130.5 Dbw
$20 \text{ MHz} < Bw_i \leq 30 \text{ MHz}$	increasing in a linear fashion from -130.5 Dbw to -124.1 Dbw
$30 \text{ MHz} < Bw_i \leq 40 \text{ MHz}$	increasing in a linear fashion from -124.1 Dbw to -122.5 Dbw
$40 \text{ MHz} < Bw_i$	-122.5 Dbw

Table 4. Threshold pulse interferences

	GPS	GLONASS	GPS/ GLONASS
Frequency range	1,575.42±10 MHz	1,592.9525 MHz, 1,609.36 MHz	1,565.42 MHz, 1,609.36 MHz
Threshold interference (peak pulse strength)	0 Dbw	0 Dbw	0 Dbw
Pulse duration time	125 usec±1 msec	£1 msec	£1 msec
Pulse rate	£10%	£10%	£10%

If the interfering signal intensity is high, the radio highway can not work in the linear mode. The receiver is practically locked under the conditions. No filtration

may be helpful in this case, and the same applied to wavelength separation of GLONASS signals.

Decomposition of the dynamic range of the analog highway is a major step in developing of an immune GLONASS/GPS AVR. According to some evaluations, the dynamic range may be increased to »100 dB.

The use of adaptive phased array antennas is one of advanced ways of enhancing AVR's interference immunity.

From all has been said it follows that the problem of enhancement of a wanted signal (filtration and recovery) of a Pseudolite is extremely relevant. In particular, our analysis of the research performed^{6,9-12} allows us to conclude on the necessity of developing friendly and high-productive algorithms of processing incoming Pseudolite navigation signals. An effective algorithm of enhancement of a wanted signal, based on the singular spectral analysis, will be presented in the next section.¹³⁻¹⁵ Moreover, results of numerical implementation of this algorithm, using a noise-contaminated signal as an example, will be demonstrated.

4. Singular Spectral Analysis

Let us consider the real series $F = (f_0, \dots, f_{N-1}), N > 2$ of the length N .¹⁴ A basic algorithm of the spectral singular analysis is comprised of two complementary stages: decomposition (embedding and singular decomposition) and recovery (grouping and recovery). Let us explain certain specifics of the stages of this algorithm.

Step 1. Embedding. Let L be a certain integer (window length), $1 < L < N$. The embedding procedure results in $K = N - L + 1$ embedding vectors

$$X_i = (f_{i-1}, \dots, f_{i+L-2})^T, 1 \leq i \leq K, \quad (4.1)$$

having the dimension of L .

Then, the trajectory matrix \mathbf{X} is plotted and it consists of embedding vectors as its columns:

$$\mathbf{X} = \begin{pmatrix} f_0 & f_1 & f_2 & \dots & f_{K-1} \\ f_1 & f_2 & f_3 & \dots & f_K \\ f_2 & f_3 & f_4 & \dots & f_{K+1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ f_{L-1} & f_L & f_{L+1} & \dots & f_{N-1} \end{pmatrix} \quad (4.2)$$

Step 2. Singular value decomposition. Singular decomposition of the trajectory matrix takes place at this stage. Let us set $\lambda_1, \dots, \lambda_L$ own numbers of the matrix $\mathbf{S} = \mathbf{X}\mathbf{X}^T$ and U_1, \dots, U_L own vectors of the matrix \mathbf{S} . Further, if we set $V_i = \mathbf{X}^T U_i / \sqrt{\lambda_i}$, then singular decomposition of the matrix will be

$$\mathbf{X} = X_1 + \dots + X_d, i = 1, \dots, d, \tag{4.3}$$

where, $X_i = \sqrt{\lambda_i} U_i V_i^T$. We will call the set $(\sqrt{\lambda_i} U_i V_i^T)$ i -th own three of the singular value decomposition.

Step 3. Grouping. Based on the decomposition (4.3), the grouping procedure will divide the whole set of indexes $\{1, \dots, d\}$ into m x non-crossing subsets I_1, \dots, I_m , and then the resulting matrix that corresponds to the group I is

$$\mathbf{X}_I = \mathbf{X}_{i1} + \dots + \mathbf{X}_{ip}, I = \{i1, \dots, ip\}. \tag{4.4}$$

Thus, the decomposition (4.3) in the grouped form may be written as

$$\mathbf{X} = \mathbf{X}_{I_1} + \dots + \mathbf{X}_{I_m}. \tag{4.5}$$

This stage involves selection of all own threes that will be included in the series recovery out of the whole set. Actually, filtration of the original series into component parts takes place at the first two stages. In particular, these component parts are trend, cycle and noise¹⁴. Visual and analytical study of own vectors as well as principal components obtained as a result of linear filtration may provide a lot of interesting information about the structure of the study process and properties of its components. To find periodic components of large amounts of visual information, 2D diagrams should be studied that are similar to Lissajous figure, where different pairs of own vectors or main components are plotted against x and y axes. It is understood that if sinusoid values of the same frequency but different phase are lotted against the axes, we get an ellipse in a plane¹⁴.

Step 4. Recovery. At the last stage, the matrix of grouped own threes (4.5) is converted into a new series of the length N by diagonal averaging.

Let there be a certain matrix \mathbf{Y} of the size $L \times K$ and with elements y_{ij} , then diagonal averaging will convert this matrix into a series g_0, \dots, g_{N-1} by

$$g_k = \begin{cases} \frac{1}{k+1} \sum_{m=1}^{k+1} y_{m, k-m+2} & 0 \leq k < L-1, \\ \frac{1}{L} \sum_{m=1}^L y_{m, k-m+2} & L-1 \leq k < K, \\ \frac{1}{N-K} \sum_{m=k-K+2}^{N-K+1} y_{m, k-m+2} & K \leq k < N. \end{cases} \tag{4.6}$$

By applying diagonal averaging to the resulting matrices \mathbf{X}_{I_k} , we will get series $\tilde{F}^{(k)} = (\tilde{f}_0^{(k)}, \dots, \tilde{f}_{N-1}^{(k)})$, and the original series $F = (f_0, \dots, f_{N-1})$ will accordingly be expanded into a sum of series:

$$f_n = \sum_{k=1}^m \tilde{f}_n^{(k)}. \tag{4.7}$$

5. Results - numeric Findings and Conclusions

Let us determine window size first. If window size is not large enough, a less detailed series decomposition may occur (for this research, the series will mean the analyzed signal). If window length is not large enough, series components mixing may be observed, and the required own three may be missed.

Practically, if no information about signal specifics is available, window length should be taken large enough to see all the decomposition specifics.

For descending order of own numbers and evaluating

contribution of each own number $\frac{\lambda_i}{\sum_{i=1}^L \lambda_i},$

the slowly changing own three will be responsible for the signal trend. Usually, the bigger contribution of the own number, the more effect the own three that contains such number on the signal overall.

The smaller contribution of the own number, the less effect of the own three on the signal. Noise components usually contribute the least. If one compares the signal of a certain structure with a pure noise signal, the share of contribution of own numbers will decrease rather slowly for the noise signal.

Consider the recovery of two signals (we will give them dummy names signal A and signal B) that comprise different decomposition components (different spectral composition) as an exemplary implementation of the suggested algorithm.

5.1 Selecting Principal Components and Recovering Signals

First six components of the signal A are given in Figure 1.

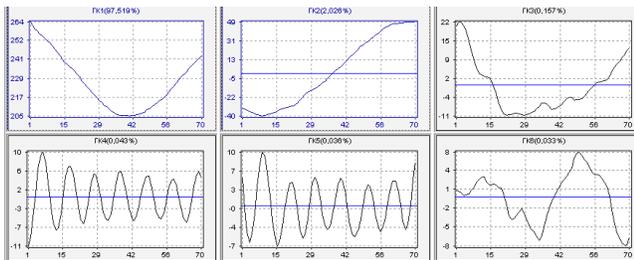


Figure 1. The first six main components of signal A.

The first two components of the signal A practically determine its behavior. Based on components 4 and 5, there is some kind of signal periodicity available. The last six components of the signal A are given in Figure 2 for comparison. From this figure, diagrams demonstrate clear noise-contaminated nature.

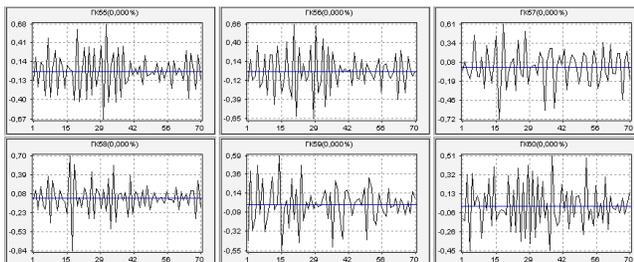


Figure 2. The last 6 main components of signal A.

Let us recover the signal A according to its first own three, using the singular spectral analysis algorithm. The recovered and original signals are presented in Figure 3.

Let us add three 4 and 5 to the recovery. According to Figure 4, the structure of the recovered signal practically matched the structure of the original signal.



Figure 3. Recovered signal A according to the first two own three (a black line indicates the original signal, and a red line indicated the recovered signal).



Figure 4. Recovered signal A according to the first, second, fourth and fifth own three (a black line indicates the original signal, and a red line indicated the recovered signal).

Non-matched elements may be positively referred to noise components with their role being minor. Let us consider another signal B of different composition vs. the signal A. The first six main components of the signal are presented in Figure 5. The first three own three contribute most to the signal. The rest three are visually similar to noise components. With view to a slow varying pattern of their diagrams, the above main components refer to the signal trend. No clear cyclic periodicity of the obtained own three is observed. Two options are possible under the circumstances: there is not any periodicity in the signal at all, or window length was not chosen correctly during signal decomposition.

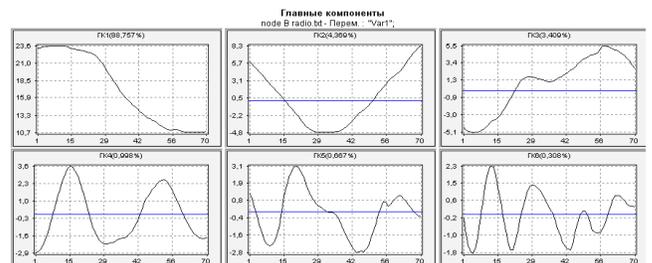


Figure 5. First six main components of signal B.

Let us recover the signal B according to the first three own three. As can be seen from Figure 6, the recovered signal repeats direction of the original signal.



Figure 6. Recovered signal B according to the first three own threes (a black line indicates the original signal, and a red line indicated the recovered signal).

Let us add threes 4,5,6 along with the first three own threes, as long as the above threes vaguely resemble some kind of periodicity and their contribution is minor, as compared to the rest threes. The recovered signal is presented in Figure 7. The structure of the recovered signal matched well the original signal.



Figure 7. Recovered signal B according to the first six own threes (a black line indicates the original signal, and a red line indicated the recovered signal).

While analyzing both signals of different structure and time, leading components of such signals were determined. All leading signals demonstrated clear weakly-manifested negative-going nature. If some kind of periodicity was observed in the signal, it was weak and did not affect the signal in overall. It should be pointed out the described behavior of the first own threes is typical for both signals. Hence it appears that if the signals are complicated, the procedure of selecting slowly varying main components will allow us to make the signal smoother and cut off unwanted noises.

6. Conclusion

Therefore, the algorithm that has been presented in this research allows for successful signal processing in noise-contaminated conditions and may be considered as an effective program implementation of a filtering unit as a part of the receiving equipment.

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