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On Some Maximal Clone of Partial Ultrafunctions on a Two-element Set

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Multifunctions on a two-element set are considered in this paper. Functions from finite set to set of all subsets of this set are called multifunctions. Partial functions, hyperfunctions, ultrafunctions, partial hyperfunctions and partial ultrafunctions are arised depending on the type of multifunctions and superposition. In this work the problem of description of clones (sets of function closed with respect to the operation of superposition and containing all the projections) of partial ultrafunctions is considered. We got a description of one maximal clone of partial ultrafunctions on two-element set by the predicate approach.

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Introduction

In the theory of discrete functions the classical problem is description of lattice of clones. Full description of a lattice is obtained only for Boolean functions [1, 2]. Because of difficulty of this problem lattice fragments are studied, for example, the minimum and maximum elements, different intervals. In particular, we note that the descriptions of all maximal clones are known for functions of k-valued logic, partial functions of k-valued, hyperfunctions and ultrafunctions on a two-element and partial hyperfunctions on a two-element set [3–8]. Some maximal clones of partial ultrafunctions found in [9, 10]. In this paper we got a description of one maximal clone of partial ultrafunctions on a two-element set.

1. Basic concepts and definitions

Let $E_2 = \{0, 1\}$ и $F = \{\varnothing, \{0\}, \{1\}, \{0, 1\}\}$. We define the following sets of functions:

$$P^{\bar{*}}_{2,n} = \{f | f : E_2^n \to F\},\ P^{\bar{*}}_2 = \bigcup_n P^{\bar{*}}_{2,n},$$

$$P_{2,n} = \{f | f \in P^{\bar{*}}_{2,n} \text{ end } |f(\tilde{\alpha})| = 1 \ \text{ for every } \ \tilde{\alpha} \in E_2^n\},\ P_2 = \bigcup_n P_{2,n}.$$

Functions from P_2 are called Boolean functions, and functions from $P^{\bar{*}}_2$ are called multifunctions on E_2.

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By definition [9, 10] we believe that the superposition

$$f(f_1(x_1,\dots,x_m),\dots,f_n(x_1,\dots,x_m)),$$

where $f, f_1,\dots,f_n \in P_2^{\overline{*}}$, represents some function $g(x_1,\dots,x_m)$, if for every $(\alpha_1,\dots,\alpha_m) \in E_2^m$

$$g(\alpha_1,\dots,\alpha_m) = \begin{cases} \bigcap\limits_{\beta_i \in f_i(\alpha_1,\dots,\alpha_m)} f(\beta_1,\dots,\beta_n), & \text{if the intersection is not empty;} \\ \bigcup\limits_{\beta_i \in f_i(\alpha_1,\dots,\alpha_m)} f(\beta_1,\dots,\beta_n), & \text{otherwise.} \end{cases}$$

On the tuples containing $\varnothing$, the multifunction takes the value $\varnothing$.

The multifunctions are considered with this superposition are called partial ultrafunctions. Below partial ultrafunctions are called simply functions. The tuples which contains element from E_2 are called binary tuples.

Projection is called n-ary function: $e_n^i : (\alpha_1,\dots,\alpha_i,\dots,\alpha_n) \to \{\alpha_i\}$.

For the set of functions B closure $[B]$ is defined as follows:

1. $B \cup \{e_i^n\} \subseteq [B]$.
2. If $f, f_1,\dots,f_n \in [B]$, then $f(f_1,\dots,f_n) \in [B]$.
3. In the $[B]$ other functions are not exist.

A set of functions is called closed set, if it is equal to its closure. If closure of the set of functions B coincides with the closed set M, then B is called full set in the M.

A set of function closed with respect to the operation of superposition and containing all the projections is called clone.

If for the clone K do not exist clone K_1 such that $K \subset K_1 \subset P_{2,n}^{\overline{*}}$, then the clone K is called maximal clone.

Let R^s is s-ary predicate defined on the set F. The function $f(x_1,\dots,x_n)$ preserves the predicate R^s, if for every tuples $(\alpha_{11},\dots,\alpha_{s1}),\dots,(\alpha_{1n},\dots,\alpha_{sn}) \in R^s$ the tuple

$$(f(\alpha_{11},\dots,\alpha_{1n}),\dots,f(\alpha_{s1},\dots,\alpha_{sn}))$$

also belongs to the predicate.

$Pol(R^s)$ denotes of the set of functions which preserves the predicate R^s.

For simplicity we use the following code:

$$\varnothing \leftrightarrow *, \{0\} \leftrightarrow 0, \{1\} \leftrightarrow 1, \{0,1\} \leftrightarrow -.$$

The tuple $(\tau_1,\dots,\tau_n) \in E_2^n$ is called rectification of the tuple $(\gamma_1,\dots,\gamma_n) \in F^n$, if for those i, for whom $\gamma_i \neq -$, follows that $\gamma_i = \tau_i$, and for the other i we have $\tau_i \in E_2$.

The function of $P_{2,n}^{\overline{*}}$ define its values at the binary tuples, the vector of values write in a row or column, and binary tuples assume to be given in the natural order.

If $h(x,y) = (001-)$, $f(x,y) = (10-1)$, $g(x,y) = (-00*)$, then entry $\begin{matrix}0\\0\\1\\-\end{matrix}\begin{pmatrix}1 & -\\ 0 & 0\\ - & 0\\ 1 & *\end{pmatrix} = \begin{pmatrix}1\\0\\-\\*\end{pmatrix}$, means that superposition $h(f(x,y),g(x,y))$ is equal to the function $(10-*)$.

2. Auxiliary statements

Here are some auxiliary results.

Lemma 1. *Let functions $f, f_1, \ldots, f_s$ preserve a predicate R^m defined on the set F, $g(x_1, \ldots, x_n)$ is superposition of the $f(f_1, \ldots, f_s)$ and binary tuples $(\alpha_1^1, \ldots, \alpha_1^m), \ldots, (\alpha_n^1, \ldots, \alpha_n^m)$ belong R^m. Then tuple $(g(\alpha_1^1, \ldots, \alpha_n^1), \ldots, g(\alpha_1^m, \ldots, \alpha_n^m))$ belongs R^m.*

Proof. This follo ws from the fact that for every binary tuple $(\beta_1, \ldots, \beta_n)$ is performed

$$g(\beta_1, \ldots, \beta_n) = f(f_1(\beta_1, \ldots, \beta_n), \ldots, f_s(\beta_1, \ldots, \beta_n)).$$

□

Consider the predicate $R^4 = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & - & \alpha \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & - & \beta \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & - & \gamma \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & - & \delta \end{pmatrix}$, where $(\alpha, \beta, \gamma, \delta)^t$ are all sorts of columns in which $\alpha, \beta, \gamma, \delta \in F$ are simultaneously satisfy two conditions:

- in every column $(\alpha, \beta, \gamma, \delta)^t$ among $\alpha, \beta, \gamma, \delta$ least two assume the value $*$;
- in every column $(\alpha, \beta, \gamma, \delta)^t$, if 0 or 1 are found among $\alpha, \beta, \gamma, \delta$, then all of them are not equal to $-$.

Proofs of the following Lemmas 2 and 3 are identical with the proofs of the corresponding assertions of the work [8].

Lemma 2. *Let a function $f(x_1, \ldots, x_{i-1}, x_i, x_{i+1}, \ldots, x_n)$ preserve the predicate R^m and the variable x_i is dummy. Then the function $g(x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n)$, derived from the f after removal of dummy variable x_i, preserve the predicate R^m.*

Lemma 3. *Let R^m is m-ary predicate and for every tuple $(\beta_1, \ldots, \beta_{i_1}, \ldots, \beta_{i_2}, \ldots, \beta_{i_s}, \ldots, \beta_n)$ from the R^m such that $\beta_{i_1}, \beta_{i_2}, \ldots, \beta_{i_s}$ are only equal to $*$, following condition are performed: if tuple $(\gamma_1, \ldots, \gamma_{i_1}, \ldots, \gamma_{i_2}, \ldots, \gamma_{i_s}, \ldots, \gamma_n) \in R^m$, then tuple $(\delta_1, \ldots, \delta_{i_1}, \ldots, \delta_{i_2}, \ldots, \delta_{i_s}, \ldots, \delta_n)$, where $\delta_j = *$ for $j \in \{i_1, i_2, \ldots, i_s\}$ and $\delta_j = \gamma_j$ for other j, also belongs to the predicate R^m. Then if $g(x_1, \ldots, x_{i-1}, x_i, x_{i+1}, \ldots, x_n)$ preserve the predicate, then $f(x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n)$, derived from the g after removal of dummy variable x_i, also belongs to the predicate R^m.*

Corollary 1. *$Pol(R^4)$ are closed with respect to addition and removal of dummy variables.*

The proof of the following lemma is given in the works [10, 11].

Lemma 4. *The following sets of functions coincide with $P_{2,n}^{\overline{*}}$:*
1)$[\{(1*), (1-)\}]$; 2)$[\{(*0), (-0)\}]$; 3)$[\{(0-), (-1), (0*)\}]$; 4)$[\{(0-), (-1), (*0)\}]$.

3. The main result

Theorem 3.1. *The class $Pol(R^4)$ is a maximal clone of partial ultrafunctions.*

Proof. Let us first show that $Pol(R^4)$ is a clone. By the Corollary 1, and due to the fact that the projections preserve the predicate R^4 remains to prove that R^4 is closed with respect to the operation of superposition. Proof by contradiction.

Let

$$h(x_1, \ldots, x_n) = f(g_1(x_1, \ldots, x_n), g_2(x_1, \ldots, x_n), \ldots, g_m(x_1, \ldots, x_n)),$$

where $f, g_1, \ldots, g_m$ are arbitrary functions of the class $Pol(R^4)$.

Suppose there are tuples $\tilde{\alpha}^i = (\alpha_1^i, \ldots, \alpha_n^i)$, where $i \in \{1,2,3,4\}$, such that $(\alpha_j^1, \alpha_j^2, \alpha_j^3, \alpha_j^4)^t \in R^4$ for every j, but $h\begin{pmatrix}\tilde{\alpha}^1\\ \tilde{\alpha}^2\\ \tilde{\alpha}^3\\ \tilde{\alpha}^4\end{pmatrix} \notin R^4$, i.e. $h\begin{pmatrix}\tilde{\alpha}^1\\ \tilde{\alpha}^2\\ \tilde{\alpha}^3\\ \tilde{\alpha}^4\end{pmatrix}$ coincides with one of the following columns: $\begin{pmatrix}0\\0\\0\\1\end{pmatrix}$, $\begin{pmatrix}0\\0\\1\\0\end{pmatrix}$, $\begin{pmatrix}0\\1\\0\\0\end{pmatrix}$, $\begin{pmatrix}1\\0\\0\\0\end{pmatrix}$, $\begin{pmatrix}0\\1\\1\\1\end{pmatrix}$, $\begin{pmatrix}1\\0\\1\\1\end{pmatrix}$, $\begin{pmatrix}1\\1\\0\\1\end{pmatrix}$, $\begin{pmatrix}1\\1\\1\\0\end{pmatrix}$, $\begin{pmatrix}\mu_1\\ \eta_1\\ \theta_1\\ \lambda_1\end{pmatrix}$, $\begin{pmatrix}\mu_2\\ \eta_2\\ \theta_2\\ \lambda_2\end{pmatrix}$, where among $\mu_1, \eta_1, \theta_1, \lambda_1$ only one value $*$, and among $\mu_2, \eta_2, \theta_2, \lambda_2$ least one value $-$, and also has values 0 or 1.

Since swapping rows in the predicate R^4 does not change it, it is sufficient to consider the cases: $\begin{pmatrix}0\\0\\0\\1\end{pmatrix}$, $\begin{pmatrix}0\\1\\1\\1\end{pmatrix}$, $\begin{pmatrix}\mu\\ \eta\\ \theta\\ *\end{pmatrix}$, where either $\mu, \eta, \theta \in \{0,1\}$, or $\mu = \eta = \theta = -$, and case $h\begin{pmatrix}\tilde{\alpha}^s\\ \tilde{\alpha}^l\end{pmatrix} = \begin{pmatrix}\alpha\\ -\end{pmatrix}$, where $\alpha \in \{0,1\}$ и $s, l \in \{1,2,3,4\}$.

We note that the tuples $\tilde{\alpha}^i = (\alpha_1^i, \ldots, \alpha_n^i)$ do not contain $*$, otherwise tuple $h\begin{pmatrix}\tilde{\alpha}^1\\ \tilde{\alpha}^2\\ \tilde{\alpha}^3\\ \tilde{\alpha}^4\end{pmatrix}$ belongs R^4. Also we note that for $i \in \{1,2,3,4\}$, if $h(\tilde{\alpha}^i) = \alpha \in \{0,1\}$, then among rectifications $\tilde{\alpha}^i$ are not such in which h is equal to $-$. Indeed, suppose that exists rectification $\tilde{\delta}$ of tuple $\tilde{\alpha}^i$ such that $h(\tilde{\delta}) = -$, then exists rectification $\tilde{\tau}$ of the same tuple such, that $h(\tilde{\tau}) = \alpha \in \{0,1\}$. Then $h\begin{pmatrix}\tilde{\delta}\\ \tilde{\tau}\\ \tilde{\delta}\\ \tilde{\tau}\end{pmatrix} \notin R^4$, which contradicts Lemma 1, since $(\delta_k, \tau_k, \delta_k, \tau_k)^t \in R^4$ for every $k \in \{1, \ldots, n\}$.

We consider the above four options. Everywhere we obtain a contradiction to Lemma 1.

1. $h\begin{pmatrix}\tilde{\alpha}^1\\ \tilde{\alpha}^2\\ \tilde{\alpha}^3\\ \tilde{\alpha}^4\end{pmatrix} = \begin{pmatrix}0\\0\\0\\1\end{pmatrix}$. For the first three rows we choose rectifications $\tilde{\delta}^1, \tilde{\delta}^2, \tilde{\delta}^3$ such that in which h is equal to 0, and for the fourth row we choose rectification $\tilde{\delta}^4$ such that $(\delta_k^1, \delta_k^2, \delta_k^3, \delta_k^4)^t \in R^4$ for every $k \in \{1, \ldots, n\}$. Then $h\begin{pmatrix}\tilde{\alpha}^1\\ \tilde{\alpha}^2\\ \tilde{\alpha}^3\\ \tilde{\alpha}^4\end{pmatrix} \in \left\{\begin{pmatrix}0\\0\\0\\1\end{pmatrix}, \begin{pmatrix}0\\0\\0\\ *\end{pmatrix}\right\}$.

2. $h\begin{pmatrix}\tilde{\alpha}^1\\ \tilde{\alpha}^2\\ \tilde{\alpha}^3\\ \tilde{\alpha}^4\end{pmatrix} = \begin{pmatrix}0\\1\\1\\1\end{pmatrix}$. For the second, third and fourth rows we choose rectifications $\tilde{\delta}^2, \tilde{\delta}^3, \tilde{\delta}^4$ such that in which h is equal to 1, and for the first row we choose rectification $\tilde{\delta}^1$ such that $(\delta_k^1, \delta_k^2, \delta_k^3, \delta_k^4)^t \in R^4$ for every $k \in \{1, \ldots, n\}$. Then $h\begin{pmatrix}\tilde{\alpha}^1\\ \tilde{\alpha}^2\\ \tilde{\alpha}^3\\ \tilde{\alpha}^4\end{pmatrix} \in \left\{\begin{pmatrix}0\\1\\1\\1\end{pmatrix}, \begin{pmatrix}*\\1\\1\\1\end{pmatrix}\right\}$.

3. $h\begin{pmatrix}\tilde\alpha^1\\ \tilde\alpha^2\\ \tilde\alpha^3\\ \tilde\alpha^4\end{pmatrix} = \begin{pmatrix}\mu\\ \eta\\ \theta\\ *\end{pmatrix}$, where either $\mu,\eta,\theta \in \{0,1\}$, or $\mu=\eta=\theta=-$. For the first three rows we choose rectifications $\tilde\delta^1,\tilde\delta^2,\tilde\delta^3$ such that $h(\tilde\delta^i)=\rho^i \neq *$, where $i\in\{1,2,3\}$, and for the fourth row we choose rectification $\tilde\delta^4$ such that $(\delta_k^1,\delta_k^2,\delta_k^3,\delta_k^4)^t \in R^4$ for every $k\in\{1,\ldots,n\}$. Then $h\begin{pmatrix}\tilde\alpha^1\\ \tilde\alpha^2\\ \tilde\alpha^3\\ \tilde\alpha^4\end{pmatrix} = \begin{pmatrix}\rho^1\\ \rho^2\\ \rho^3\\ *\end{pmatrix}$.

4. $h\begin{pmatrix}\tilde\alpha^s\\ \tilde\alpha^l\end{pmatrix} = \begin{pmatrix}\alpha\\ -\end{pmatrix}$, where $\alpha\in\{0,1\}$ и $s,l\in\{1,2,3,4\}$. If there is the rectification $\tilde\delta$ of tuple $\tilde\alpha^l$ in which value of h is equal to $-$, then there exists the rectification $\tilde\tau$ of tuple $\tilde\alpha^s$ such that $h(\tilde\tau)=\alpha$. Then $h\begin{pmatrix}\tilde\delta\\ \tilde\delta\\ \tilde\tau\\ \tilde\tau\end{pmatrix} = \begin{pmatrix}-\\ -\\ \alpha\\ \alpha\end{pmatrix} \notin R^4$ и $(\delta_k,\delta_k,\tau_k,\tau_k)^t\in R^4$ for every $k\in\{1,\ldots,n\}$.

If this rectification does not exist, then there are two rectifications $\tilde\delta^1,\tilde\delta^2$ of tuple $\tilde\alpha^l$ such that $h(\tilde\delta^1)=0$ and $h(\tilde\delta^2)=1$. In case, when $\alpha=0$ we have $h\begin{pmatrix}\tilde\delta^1\\ \tilde\delta^2\\ \tilde\tau^1\\ \tilde\tau^2\end{pmatrix} \in \left\{\begin{pmatrix}0\\1\\0\\0\end{pmatrix},\begin{pmatrix}0\\1\\0\\ *\end{pmatrix}\right\}$, where $\tilde\tau^1$ is rectification of tuple $\tilde\alpha^s$ in which value of h is equal to 0, and $\tilde\tau^2$ is rectification of tuple $\tilde\alpha^s$ such that $(\delta_k^1,\delta_k^2,\tau_k^1,\tau_k^2)^t\in R^4$ for every $k\in\{1,\ldots,n\}$. In case, when $\alpha=1$ we have $h\begin{pmatrix}\tilde\delta^1\\ \tilde\delta^2\\ \tilde\tau^1\\ \tilde\tau^2\end{pmatrix} \in \left\{\begin{pmatrix}0\\1\\1\\1\end{pmatrix},\begin{pmatrix}0\\1\\1\\ *\end{pmatrix}\right\}$, where $\tilde\tau^1$ is rectification of tuple $\tilde\alpha^s$ in which value of h is equal to 1, and $\tilde\tau^2$ is rectification of tuple $\tilde\alpha^s$ such that $(\delta_k^1,\delta_k^2,\tau_k^1,\tau_k^2)^t\in R^4$ for every $k\in\{1,\ldots,n\}$.

We now show that $Pol(R^4)$ is a maximal clone. It is sufficient to prove that $[Pol(R^4)\cup\{f\}] = P_2^{\overline{*}}$, where function f does not preserve the predicate R^4, i.e. there are tuples $\tilde\alpha^i=(\alpha_1^i,\ldots,\alpha_n^i)$, where $i\in\{1,2,3,4\}$, such that $(\alpha_j^1,\alpha_j^2,\alpha_j^3,\alpha_j^4)^t\in R^4$ for every j, but $f\begin{pmatrix}\tilde\alpha^1\\ \tilde\alpha^2\\ \tilde\alpha^3\\ \tilde\alpha^4\end{pmatrix}\notin R^4$.

It is easy to verify that the functions (00),(01),(11),(10),(0∗),(1∗),(∗0),(∗1),(−−),(0110),(1001) preserve the predicate R^4. Therefore, in view of Lemma 4 it is sufficient to have one of the functions (0−),(1−),(−0),(−1).

Since for every $j\in\{1,\ldots,n\}$ column $(\alpha_j^1,\alpha_j^2,\alpha_j^3,\alpha_j^4)^t$ coincides with one of the functions $\{(0000),(0011),(0101),(0110),(1111),(1100),(1010),(1001),(----)\}$, then applying operation of superposition to f and to these functions we obtain or $f_1(x,y)=(0001)$, or $f_2(x,y)=(0111)$, or $f_3(x,y)=(\mu,\eta,\theta,*)$, or immediately one of the functions (0−),(1−),(−0),(−1).

Via binary functions f_1, f_2 и f_3 we get one of the required functions $(0-),(-0),(-1)$:

$$\begin{matrix}0\\0\\0\\1\end{matrix}\begin{pmatrix}0&-\\0&-\\1&-\\1&-\end{pmatrix}=\begin{pmatrix}0\\0\\-\\-\end{pmatrix};\ \begin{matrix}0\\1\\1\\1\end{matrix}\begin{pmatrix}0&-\\0&-\\1&-\\1&-\end{pmatrix}=\begin{pmatrix}-\\-\\1\\1\end{pmatrix};\ \begin{matrix}0\\0\\0\\0\end{matrix}\begin{pmatrix}0&\mu\\0&\eta\\0&\theta\\0&*\end{pmatrix}=\begin{pmatrix}0\\0\\0\\ *\end{pmatrix};\ \begin{matrix}0\\0\\1\\1\end{matrix}\begin{pmatrix}0&0\\0&0\\1&0\\1&*\end{pmatrix}=\begin{pmatrix}0\\0\\1\\ *\end{pmatrix};\ \begin{matrix}0\\0\\1\\ *\end{matrix}\begin{pmatrix}-&0\\-&0\\-&1\\-&1\end{pmatrix}=\begin{pmatrix}-\\-\\0\\0\end{pmatrix}.$$

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Об одном максимальном клоне частичных ультрафункций на двухэлементном множестве

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Рассматриваются мультифункции на двухэлементном множестве. Под мультифункцией на конечном множестве понимается функция, определенная на данном множестве и принимающая в качестве значений его подмножества. В зависимости от вида мультифункций и соответствующей им суперпозиции возникают частичные функции, гиперфункции, ультрафункции, частичные гиперфункции и частичные ультрафункции. В заметке исследуется задача описания решетки клонов (множеств, замкнутых относительно суперпозиции и содержащих все функции-проекции) для частичных ультрафункций. С помощью предикатного подхода получено описание одного максимального клона частичных ультрафункций на двухэлементном множестве.

Ключевые слова: мультифункции, ультрафункции, максимальные клоны, решетка.