The Evaluation of Variety of Market Structure Using the Entropy Indicator

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An effective economic system assumes the minimization of costs connected with the satisfaction of population needs and the operation of economics itself. The competition is the necessary mechanism to force companies to reduce costs. To provide such kind of pressure it is required to have a lot of companies producing similar products. Therefore, it is necessary for a competitive economics to implement a principle of maximum variety of companies operating in any branch. Reduction of the variety indicates that some forces prevent the execution of this principle.

Keywords: microeconomics, entropy of economic agent’s interaction, collation of companies concentration levels in the inter-industry comparisons

Investigations in the field of theoretical economics are aimed at the detection of common tendencies and laws stimulating the best way of using resources for the satisfaction of people needs. Nevertheless, researchers began to face difficulties when the try to coordinate the findings of various schools of economic theory, such as neoclassical microeconomics, institutional economics, not to mention the various trends of macroeconomics.

For example, the economics can be considered as the field of interaction of different homogeneous agents capable to move actively between different segments of uniform market system. Capitals mobility can be prevented only by some external barriers which impact is investigated in due sections devoted to imperfect competition. Using this approach the researcher looks like to be over the scramble and observes the common movement of the restricted resources of the society. It is a nonlocal approach to economics. From the other hand a lot of scientists take a position of some companies and investigate peculiarities taking place during their interaction. This approach can be called local.

It is essential to note that it is very hard to combine instruments and ideology of these approaches. For example, such important categories as information and money disappear in the nonlocal market system with homogenous agents. A balance is being formed out of time, but the whole information on agent resources, grounds and intentions is presented in the system, it is only to be revealed. If take a position of separate economic
subjects, interaction time and information they possesses, became very important.

Though, the interaction of economic agents in terms of imperfect market leads to the forming of different types of its organization. Depending on competition peculiarities in different branches, various structures are developed. The development of such structures assumes the execution of some extreme principles.

Theoretical description of economic systems behavior usually supposes some value to take extreme (minimum or maximum) values. The most frequently a minimization criteria is applied, for example, for evaluation of minimal level of costs, risks or time for effective realization of any value (i.e. duration).

In this case it is assumed that among all possible paths of system motion it will choose the one which will provide minimal costs for movement. Such path is the most probable. It is clear intuitively that the “right” movement of the system is incompatible with energy waste.

The natural movement must make a certain value minimal. As soon as this value is determined, forces can be found which move the system towards the direction assured the consistency of this value. Considering the dynamic system it is logic to suppose that the effective system minimizes costs within the given period of time (given duration). For example, for a given duration of the project costs tend to be minimized. From the other hand, if projects require certain investments, they are tending to be spent within minimum terms.

Stationary system investigations assume that if the system is kept silent it will sooner or later takes a configuration which will minimize costs for its operation. It is also described by theories of market organization in microeconomics. An effective economy must come to balance, which mainly is characterized by the reduction of operational costs.

So, in static position the economic system takes a configuration that minimizes the cost of its stable operation. In the other words it reduces current costs. If we consider the dynamics, the system tends to move to a new state in a natural way, minimizing the effect, i.e. either costs at a given time, or time at a given cost.

What kind of forces can minimize energy required for market system operating? Competition is considered to do so. Microeconomics demonstrates that each branch of the effective economic system (except for degenerate case) must support the existence of maximum number of companies. It helps working competitive forces which provide economic effectiveness having a tendency of each its economic agent to operate with minimal costs (i.e. operate with minimal level of power). If costs are determined correctly, the whole system operates in a right way. Consequently, the effective economics must be naturally aimed at the maximum number of market participants. Therefore, based on the theory, the principle of maximal diversity of companies operating in the branch can be introduced as the basic principle of branch-wise organization.

It should be noted that market system has two kinds of processes:

1. The process which brings companies to operate with minimal costs (with minimal level of energy)
2. The process which leads to maximal number of companies, i.e. to maximal entropy of the system

The reduction of costs as the indicator of economics effectiveness is sufficiently widespread indicator and the theory of costs- is one of the developed questions in the theory of economics. A lot of competitive companies try to reduce costs, but these companies have unequal market shares. In some branches thousands companies operate, but in the other one or two. To evaluate
the level of inequality of companies’ distribution within the branch the entropy indicator can be applied.

Even Hartley’s function can act as a simple measure of diversity sets. In this case the only aspect makes sense: elements of the set must be distinguishable. It allows giving quantitative characteristics to different sets, for example, to a lot of companies in the branch, to a lot of taxpayers, to a lot of families with different disposable yields.

It is not enough of course, because if one branch has 10 companies and the other has 12- it is evident that the second branch is more multifarious than the first. But it is interesting that the difference in the number of sets of elements can be expressed in binary format, using the information measure.

Entropy is the indicator of structure sets. Two sets having the same number of elements (for example, two branches with the same number of companies, or two countries with the similar number of customers) can be organized in a different way. Elements of the set can fill it evenly or not. The measure constructed on the basis of entropy allows evaluating not only the number of elements, but the degree of its homogeneity as well. Therefore, the entropy can be applied to data clustering techniques. For such cases some other entropy is applied, such as the Shannon entropy.

The advantage of entropy indicator is that it allows us to compare levels of concentration of investigated sets not only with the same, but also with the different number of elements. For the simplest case the entropy of a homogeneous set, i.e. Hartley entropy can be taken as the reference entropy.

\[ I = \Delta H = \ln N - \sum_{i=1}^{N} p_i \ln p_i \]

In this case we obtain the index of inequality, which shows the deviation of the distribution of market shares of companies compared to a state of complete equality. If it is not zero, it shows the presence of some structure in the market because the distribution of companies in the branch differs from its uniform state.

Presented in this form, the index of inequality allows comparisons between different sets, because in this case the factor of element number differences is suppressed.


Starting from 1980 to coordinate companies concentration in the branch antimonopoly authorities used Herfindahl-Hirschman Index to take decisions on companies merging and absorption.

\[ I_H = \sum_{i=1}^{N} p_i^2 \]

where

\( p_i \) – market share controlled by \( i \)-company;

\( N \) – number of companies in the branch.

However, it is difficult to use it for collation of concentration levels in the inter-industry comparisons, because the number of companies in different industries does not coincide usually. The most reasonable for these purposes would be the application of difference entropy indicator.

Let us assume that the branch has \( N \) companies operating and \( p_i \) – market share of \( i \)-company \((i=1,\ldots,N)\). Then the entropy of companies’ distribution within the branch by their relative shares can be evaluated s follows:
\[ H = -\sum_{i=1}^{N} p_i \cdot \log p_i \]

If market is organized by monopoly principle, than the share of one company in the branch is equal to 1 and the share of others – 0, therefore \( H = 0 \). This is the minimum value of entropy and, consequently, the maximum degree of concentration. A zero value of entropy says that we have zero information on the product producers. This is our knowledge accretion after obtaining reports on the situation in the branch.

Let us suppose now that all companies of the branch have equal shares of \( p_i = \frac{1}{N} \), than \( H = -\log N \). This is the maximum value of entropy and, consequently the maximum degree of uncertainty regarding which of the \( N \) companies issued a proposed product. If we take a randomly chosen product in the market, it can turn to be produced by any of the \( N \) companies with the equal probability. Obviously, in this case we will obtain the value of Hartley entropy.

The intermediate variant assumes that the probability that the product been chosen at the market randomly is produced by a certain company depends both on the number of companies at the market \( N \) and the company share in the market \( p_i \). Applying the difference entropy indicator,

\[ I = \Delta H = \ln N - \sum_{i=1}^{N} p_i \ln p_i \]

a suppression of the factor of companies variety within the branch occurs. Herfindahl index is more vulnerable in this respect. Therefore, the entropy indicator can be used as a competitor to Herfindahl index.

So, entropy can be used as the indicator determining the level of concentration in the branch. It will be minimal at the monopoly organization of the branch and maximal at the even distribution of companies by market shares. Maximum entropy characterizes the uniform case, which is usually considered to be the base for the investigation of micro-economic equilibrium. As the rule, it is not explained how many companies there should be nor why they should be many. It is simply postulates that it is good to have a lot of companies to provide due competition.

However, such companies’ distribution is not common practically. On the contrary, companies are distributed unevenly by their market shares. As such, it can be assumed that each market has a certain structure which is developed by some factors among which in theoretical economy usually are the demand for branch products, resources and appropriate technologies. But these reasons are not unique. Supply and demand are characteristics suitable only for the areas of distribution, but not for production and consumption.

Let us revert to the information indicator of a disparity of companies distribution within the branch introduced above

\[ I(p, N) = \ln N - \sum_x p(x) \cdot \ln p(x) \]

It tells us how the market structure of the corresponding branch is distant from its structureless, homogeneous analogue. The greater this difference is, the more powerful are forces taking the branch away from the perfect balance. There are some factors in the branch which are insistently cause it to remain in that position not allowing to switch to a structureless, homogeneous state. In this case an even distribution of companies within the branch is taken as a «normal» state, which roughly corresponds to perfect competition (when the companies are «a lot»). However, this is one of the options and as a «normal» you can select any other distribution.
To find the optimal distribution of companies in the branch by their market shares, we can construct an optimization model by which it will be possible to explain the diversity of companies’ market shares distribution (stable structure), based on the tendency of any «accurate» branch to maximize the number of companies operating in it.

To construct the objective function we introduce the principle of maximum diversity as a basic. It says that being left to its own the economic system tends spontaneously to a state in which the satisfaction of needs is provided by the maximum number of companies. For the external observer, the branch will tend to a state with maximal entropy.

It is a natural trend of any economic development. If this principle is not supported by any branch, it means that there are some forces exist which prevent its natural evolution. The general formulation of the problem states that if the objective function is of entropic nature, than the limits are of energetic nature.

\[
H = -\sum_x p(x) \ln p(x) \rightarrow \text{max} \\
\sum_x p(x) = 1 \\
\sum_x p(x) \cdot c(x) = C
\]

Any branch has its range of allowable costs \(c(x)\) and total costs (resources) \(-C\) that society can assign for the production of the given product. This is an aggregate demand of consumers for products of this branch. It should be enough to cover the full costs of the product production. Having different expenditure pattern, i.e. different shares of fixed costs and direct costs the production volume of the company will be optimal and provide the production with minimal costs. Therefore a plenty of allowable eots will make a set of options for products production providing minimum total costs. Larger production volumes due to the economy on a production scale will lead to expenditures minimization. However, as the production scale develops, indirect costs begin to grow as well; thus each enterprise of the branch has its own production concentration limits. Enterprises with the maximum production capacity will become branch leaders providing the production of mass products with minimum expenditures. Such problems are thoroughly investigated by microeconomics and conditions are identified allowing economy to use effectively the available restricted resources.

Now we can adapt the above problem on maximum entropy with the given range of costs for description of the “normal” market organization in a particular branch. It should be noticed that such market organization is not necessarily homogeneous. The distribution, which is obtained during the task solution (the distribution of Boltzmann-Gibbs), says that companies will be distributed in accordance with an exponential law. This corresponds to the practically observed companies’ distribution in the most of branches (especially in processing industry).

The assigned task demonstrates that any economic system under the Specified processing coefficients of resource consumption tends to maximum entropy of companies distribution. It is evident that in case of the lack of processing limits, maximum entropy would lead to a homogeneous distribution of companies, i.e. to the situation which is similar to one considered by the theory of perfect competition. Technological coefficients play the role of energetic levels, forming together an energetic range of distribution.

In practice, it is easier to solve an inverse task. Market shares and the total number of companies in each industry are usually known. Therefore, starting from the known distribution \(p(x)\) and the total cost of the branch \(C\), we can find its range of costs \(c(x)\).
The task in this definition is coordinated with due double “entropy” task:

\[
C = \sum_x p(x) \cdot c(x) \rightarrow \min
\]

\[
\begin{cases}
\sum_x p(x) = 1 \\
-\sum_x p(x) \cdot \ln p(x) = H
\end{cases}
\]

Double task says that for minimal consumption of resources with the specified consumption coefficients it is required to have companies’ diversity with the value not less than \(H\). It is evident that such formulation of the task shows the necessity of having a competition in the economic system.

The solution of the task of companies maximum diversity (maximum entropy) simultaneously leads to a result which proves that under this principle, the companies act in the most economical way. In other words, the branch operates with minimum costs.

Let construct the Lagrange functional

\[
L(p, \lambda_0, \lambda) = -\sum_x p(x) \ln p(x) - \\
-\lambda_0 \sum_x p(x) - \lambda \sum_x p(x)c(x)
\]

As the result we obtain the distribution of the type:

\[
p(x) = e^{-1-\lambda_0-x \cdot \lambda(x)} = \frac{e^{\lambda \cdot x(x)}}{e^{1+\lambda_0}}
\]

Where \(\lambda\) – is the indicator showing how the number of companies should increase with the increase of branch resources per unit (under the growth of the demand for its products).

If we are dealing with the balanced economy, we obtain the equilibrium distribution of companies in accordance with the existing costs limits. Moreover, this equilibrium distribution is not necessarily uniform (“perfect”). However, any equilibrium involves certain forces that are balanced with each other.

If the actual distribution of companies deviates from the balanced, it proves that some factors exist in the economy that prevents the fulfillment of maximum diversity law. They are considered in the microeconomic theory of imperfect competition, in particular, a monopoly (natural and artificial). It is often that such kinds of power are formed under the government intervention in the economy. Various kinds of government restrictions and monopolies falsify the force of diversity law. The principle of essential diversity will be carried out provided the execution of a principle of minimal compulsion.

References


Оценка разнообразия рыночной организации
с помощью показателя энтропии

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Эффективная экономическая система предполагает минимизацию затрат, с которыми связаны удовлетворение потребностей населения страны и функционирование самой экономики. Конкуренция является необходимым механизмом, заставляющим фирмы снижать затраты. Для такого давления необходимо существование множества фирм, выпускающих близкие продукты. Поэтому в конкурентной экономике должен выполняться принцип максимального разнообразия фирм, действующих в любой отрасли. Снижение разнообразия свидетельствует об определенных силах, которые препятствуют выполнению этого принципа.

Ключевые слова: микроэкономика, сравнение уровней концентрации фирм при межотраслевых сопоставлениях, энтропия взаимодействия экономических агентов.