Anisotropy of DC Electric Field Influence on Acoustic Wave Propagation in Piezoelectric Plate

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Anisotropy of dc electric field influence on the different types of acoustic waves in the piezoelectric plate has been investigated by means of computer simulation. Detail calculations have made for bismuth germanium oxide crystals.

Keywords: dc electric field, Lamb wave.

Introduction

Investigation of acoustic wave propagation in the piezoelectric plates under the bias electric field leads to the possibility of the controlling of acoustoelectronic devices parameters. Detail theory of the dc electric field or mechanical stress influences on the bulk acoustic (BAW) and surface acoustic (SAW) waves propagation in piezoelectric crystals had been derived in [1–6]. The studying of the anisotropic propagation of the zero order Lamb wave in the lithium niobate piezoelectric plates has fulfilled by the authors [7, 8]. Peculiarities of Lamb waves and surface waves with the horizontal polarization (SH) propagating along the high symmetry directions of the cubic piezoelectric crystals under the dc electric field have been investigated earlier [9].

To optimize the acoustoelectronic device it is necessary to find both an appropriate crystal direction and a value of the \((h \times f)\) product for a given frequency range \((h — the\ crystalline\ plate\ thickness, f — the\ frequency)\). In the present paper the anisotropy of Lamb and \(SH\) waves parameters in the bismuth germanium oxide \((Bi_{12}GeO_{20})\) piezoelectric crystal under the influence of dc electric field has been investigated by means of computer simulation.
1. Propagation Theory of Lamb and SH-waves in Piezoelectric Plate under the Influence of Homogeneous DC Electric Field

Influence of homogeneous dc electric field $E$ on Lamb and SH-wave propagation conditions in piezoelectric crystalline plate has been considered on the basis of the theory of bulk acoustic waves propagation in piezoelectric crystals subjected to the action of a bias electric field [1].

Wave equations and electrostatics equation written in the natural state for homogeneously deformed crystals without center of symmetry have the form [2]:

$$\rho \ddot{\tilde{U}}_i = \tau_{ik,k},$$
$$\tilde{D}_{m,m} = 0.$$  \hspace{1cm} (1)

Here $\rho$ is the density of crystal taken in non-deformed (initial) state, $\tilde{U}_i$ is the vector of dynamic elastic displacements, $\tau_{ik}$ is the tensor of thermo-dynamical stresses and $D_{m}$ is the vector of the induction of electricity. Here and further the tilde sign is marked the time dependent variables. Comma after the lower index denotes a spatial derivative and Latin coordinate indexes are changed from 1 to 3. Here and further the summation on twice recurring lower index is understood.

State equations can be written as:

$$\tau_{ik} = C^{*}_{iklm} \eta_{lm} - \epsilon^{*}_{nik} E_n,$$
$$D_n = \epsilon^{*}_{nik} \eta_{nk} + \varepsilon^{*}_{nm} E_m,$$  \hspace{1cm} (2)

where $\eta_{AB}$ is the deformation tensor and effective elastic, piezoelectric, dielectric constants are defined by:

$$C^{*}_{iklm} = C^{E}_{iklm} + \left(C^{E}_{ikkmpq} d_{pq} - \epsilon^{*}_{jiklm}\right) M_j E,$$
$$\epsilon^{*}_{nik} = \epsilon_{nik} + \left(\epsilon_{nikpq} d_{pq} + H_{njpq}\right) M_j E,$$
$$\varepsilon^{*}_{nm} = \varepsilon^{*}_{nm} + \left(H_{nmik} d_{jk} + \varepsilon^{*}_{nmj}\right) M_j E.$$  \hspace{1cm} (3)

In (3) dc electric field is the value of dc electric field applied to the crystal, $M_j$ is the unit vector of $E$–direction, $C^{E}_{iklm}$, $\epsilon_{nik}$, $\varepsilon^{*}_{nm}$, $H_{njpq}$ are nonlinear elastic, piezoelectric, dielectric and electrostrictive constants (material tensors), $d_{pq}$ and $\epsilon_{nik}$ are the piezoelectric tensors, $C^{E}_{iklm}$ and $\varepsilon^{*}_{nm}$ are elastic and dielectric tensors. Substituting (2) into (1) we can obtain Green-Christoffel equation in a general form which can be used for the analysis of bulk acoustic waves propagation in the case of $E$–influence.

Let’s use coordinate system $X_3$ axis directs along the external normal to the surface of a media occupying the space $h \geq X_3 \geq 0$, and the wave propagation direction coincides with $X_1$ axis. Plane waves propagating in the piezoelectric plate are taken in the form:

$$\tilde{U}_i = \alpha_i \exp \left[i (k_j x_j - \omega t)\right],$$
$$\varphi = \alpha_4 \exp \left[i (k_j x_j - \omega t)\right],$$  \hspace{1cm} (4)

where $\alpha_i$ and $\alpha_4$ are amplitudes of elastic wave and electric potential $\varphi$ concerned closely with the wave, and $k_j$ are components of wave propagation vector. Taking into account (2) and (3) the substitution (4) into (1) gives us equation in a specific form. So if the electric field is applied to piezoelectric crystal, Green-Christoffel equation can be written as

$$[\Gamma_{pq}(E) - \rho \omega^2 \delta_{pq} ] \tilde{U}_q = 0,$$  \hspace{1cm} (5)
where Green-Christoffel tensor has the form:

\[
\begin{align*}
\Gamma_{pq} &= (C_{ipqm} + 2d_{jkq}C_{ipkm}M_jE)k_i k_m, \\
\Gamma_{q4} &= \varepsilon_{kqk} k_i k_m, \\
\Gamma_{4q} &= \Gamma_{q4} + 2\varepsilon_{ikmn}d_{jkq}M_jk_i k_m E, \\
\Gamma_{44} &= -\varepsilon_{nmn}k_n k_m.
\end{align*}
\]

(6)

Propagation of acoustic waves in the piezoelectric plate under the \( E \)–influence should satisfy to boundary conditions of zero normal components of the stress tensor on the boundaries "crystal-vacuum". Continuity of the electric field components which are tangent to the boundary surface is guaranteed by the condition of the continuity of the electrical potential and normal components of the electric displacement vector:

\[
\begin{align*}
\tau_{3k} &= 0, \quad x_3 = 0; \quad x_3 = h; \\
\varphi &= \varphi^i, \quad x_3 < 0; \\
\varphi &= \varphi^i, \quad x_3 > h; \\
D &= D^{i}, \quad x_3 < 0; \\
D &= D^{i}, \quad x_3 > h.
\end{align*}
\]

(7)

Here the upper index "I" is concerned to the half-space \( X_3 > h \) and index "II" — to the half-space \( X_3 < 0 \). Substituting the solutions (4) into equations (7) and neglecting of the terms which are proportional \( E^2 \) (and higher order ones), finally we have obtained the system of equations useful to analyze the change of the wave’s structure arising as a consequence of crystal symmetry variation and new effective constants appearance:

\[
\begin{align*}
\sum_{n=1}^{8} C_n \left[ C_{3kl} k_l^{(n)} k_k^{(n)} \alpha_k^{(n)} + \varepsilon_{3kl} k_k^{(n)} \alpha_k^{(n)} \right] \exp t \left( ik_3^{(n)} h \right) &= 0; \\
\sum_{n=1}^{8} C_n \left[ \varepsilon_{3kl} k_l^{(n)} \alpha_k^{(n)} - (\varepsilon_{3kl} k_l^{(n)} - i\varepsilon_0) \alpha_k^{(n)} \right] \exp t \left( ik_3^{(n)} h \right) &= 0; \\
\sum_{n=1}^{8} C_n \left[ C_{3jl} k_l^{(n)} \alpha_k^{(n)} + \varepsilon_{3jl} k_k^{(n)} \alpha_k^{(n)} \right] &= 0; \\
\sum_{n=1}^{8} C_n \left[ \varepsilon_{3jl} k_l^{(n)} \alpha_k^{(n)} - (\varepsilon_{3jl} k_l^{(n)} + i\varepsilon_0) \alpha_k^{(n)} \right] &= 0.
\end{align*}
\]

(8)

Index \( n = 1, \ldots, 8 \) corresponds to the number of one of eight partial waves; \( C_n \) and \( \alpha_k^{(n)} \) are its weight coefficients and amplitudes, respectively.

It can remember that the equations (8) were obtained at the assumption of homogeneity of applied dc electric field without taking into account the edge effects. But these equations take into account all changes of the crystal density and the form of crystal sample arising as a consequence of finite deformation of piezoelectric media under the action of strong dc electric field [2].

2. Anisotropy of DC Electric Field Influence on the Acoustic Wave Parameters in the Bismuth Germanium Oxide Piezoelectric Plate

As a model media the bismuth germanium oxide (23 point symmetry) has been used. Taken into account equations (8) and linear and nonlinear material properties from [2], the computer calculation of the main parameters such as phase velocity, electromechanical coupling coefficient (EMCC), and controlling coefficient of phase velocity

\[
\alpha_v = \frac{1}{v(0)} \left( \frac{\Delta v}{\Delta E} \right)_{\Delta E \to 0}
\]

(9)
has been carried out.

Analyses of the variation of waves parameters was fulfilled for (001) and (110) crystalline planes when dc electric field was directed along some kind of $X_1$, $X_2$ and $X_3$ axes.

2.1. Acoustic Wave Propagation in the (001) Crystalline Plane

If acoustic wave is propagating in the (001) crystal plane, the application of dc electric field along $X_1$ axis, i.e. along the acoustic wave propagation direction, or along $X_2$ axis, i.e. orthogonal to the sagittal plane, leads to decreasing of the crystal symmetry to triclinic one in general case except the field directions coinciding with basic axes of the crystal. Last variants have been considered in details earlier [9, 10]. Under the action of dc electric field along $X_3$ basic axis ($E \parallel [001]$), i.e. orthogonal to the free surface of the crystal plate, the crystal symmetry decreases to monoclinic one (2 point symmetry). Twofold axis coincides with the [001] direction and there are induced some effective elastic constants:

\[
\begin{align*}
C^*_{16} &= (C_{166}d_{14} - \epsilon_{1124})E; & C^*_{26} &= (C_{155}d_{14} - \epsilon_{1334})E; \\
C^*_{16} &= (C_{144}d_{14} - \epsilon_{1114})E; & C^*_{45} &= (C_{156}d_{14} - \epsilon_{1556})E; \\
\epsilon^*_{15} &= (\epsilon_{156}d_{14} + H_{55})E; & \epsilon^*_{31} &= (\epsilon_{132}d_{14} + H_{31})E; \\
\epsilon^*_{32} &= (\epsilon_{134}d_{14} + H_{32})E; & \epsilon^*_{33} &= (\epsilon_{114}d_{14} + H_{33})E.
\end{align*}
\]  

(10)

Phase velocities of the zero and first order modes of Lamb wave propagating in the (001) crystalline plane with ($h \times f$) values up 500 to 3000 m/s are shown on Fig. 1. If ($h \times f$) values are

![Fig. 1. Phase velocities of acoustic waves propagating in the (001) plane of Bi$_{12}$GeO$_{20}$ crystal at $E = 0$ and various values of $h \times f$ (m/s): a) $h \times f = 500$; b) $h \times f = 1000$; c) $h \times f = 1500$; d) $h \times f = 2000$; e) $h \times f = 2500$; f) $h \times f = 3000$. Curves for the quasi-longitudinal, fast and slow quasi-shear bulk acoustic waves are marked as $QL$, $QFS$, $QSS$ respectively](image-url)
increased, the phase velocity of antisymmetric mode $A_0$ is considerably increased, for example up to 1284.96 m/s in the [110] direction, but the square of $EMCC$ (Fig. 2) is decreased up to 2.3% to 0.8%. Values of the $EMCC$ square were calculated as in the case of $SAW$ propagation, i.e. when the metallization of one free surface of piezoelectric plate has taken into account [11]. For $A_1$ mode propagating along the [110] direction of the (001) plane there is the $EMCC$ square’s maximal value equal to 0.5% ($h \times f = 1500$ m/s). Note that the $EMCC$ square’s qualitative behavior for the first order modes is similar to the zero order ones, but its numerical values are considerably less.

If $E \parallel X_1$, dependences for $\alpha_v$ coefficients of $A_0$ mode are similar to ones for Rayleigh surface acoustic wave [12]. When the $A_0$ mode is propagating along the [010] direction, its $\alpha_v$ coefficients are considerably increased up to $-4.17 \times 10^{-11}$ to $-2.5 \times 10^{-10}$ m/V by the variation of $(h \times f)$ quantity up to 3000 m/s. For the $A_1$ mode maximal $\alpha_v$ values are reached in the [110] propagation direction, in particular $\alpha_v = -6.7 \times 10^{-11}$ m/V ($h \times f = 2500$ m/s).

Note that as a result of dc electric field application orthogonal to the sagittal plane ($E \parallel X_2$), the [010] and [001] propagation directions, undistinguished for the undisturbed crystal, become unequal ones. This effect is the consequence of the 23 point symmetry peculiarity of the given crystal, since there is a difference between components of nonlinear properties which are responsible for the dc electric field influence on the phase velocities, for example $C_{155} \neq C_{166}$, $\varepsilon_{124} \neq \varepsilon_{134}$, $H_{12} \neq H_{21}$. It can point to the fact that these components are equal in all other piezoelectric crystals of cubic symmetry. So $\alpha_v = -6.1 \times 10^{-12}$ m/V and $\alpha_v = -7.3 \times 10^{-12}$ m/V for the $A_0$ modes in the [010] and [001] directions (Fig. 2, c). In the case when both main surfaces of the plate are coated by metal, the dc electric field application along $X_3$ axis leads to increasing of $\alpha_v$ coefficients as a result of the thickness increasing. In particular $A_0$ mode ([110] propagation direction) $\alpha_v$ coefficient varies up to $-2.58 \times 10^{-10}$ to $-3.71 \times 10^{-10}$ m/V (Fig. 2).

One of the distinctive peculiarities of wave’s propagation in the (001) crystalline plane is the hybridization effect. There are some coupled modes having the energy exchange.

$$M = \frac{W_{12}^2 + W_{21}^2}{W_1 + W_2},$$

---

Fig. 2. The square of $EMCC$ and $\alpha_v$ coefficients of bulk waves and the $A_0$ modes propagating in the (001) plane of $Bi_{12}GeO_{20}$ crystal: a) the square of $EMCC$; b) $\alpha_v$ coefficients ($E \parallel X_1$); c) $\alpha_v$ coefficients ($E \parallel X_2$); d) $\alpha_v$ coefficients ($E \parallel X_3$). A number of the curve corresponds to the values $h \times f$ (m/s): 1 – $h \times f = 500$; 2 – $h \times f = 1000$; 3 – $h \times f = 1500$; 4 – $h \times f = 2000$; 5 – $h \times f = 2500$; 6 – $h \times f = 3000$. Points are marked the experimental $\alpha_v$ coefficients [2].
where $W_{12} + W_{21}$ — a complete mutual energy of two coupled modes (time average); $W_1 + W_2$ — a complete energy of acoustic wave. Without electric field the hybridization effect between the $S_0$ и $SH_0$ modes exists only in the thick plates if $h \times f \geq 2000$ m/s (Fig. 3). It should be noted that

Fig. 3. a) Hybridization coefficient $M (E \parallel X_1)$ for: 1 — $S_0$-$SH_1$ modes ($h \times f = 2000$ m/s); 2 — $S_0$-$SH_0$ ($h \times f = 2500$ m/s). b) Phase velocities of acoustic waves. Designations of curves are in accordance with Fig. 2.

Fig. 4. Square of EMCC (a) and $\alpha_v$ coefficients of the $S_0$ mode propagating in the (001) plane of $Bi_{12}GeO_{20}$ crystal: b) $E \parallel X_1$; c) $E \parallel X_2$; d) $E \parallel X_3$. A number of the curve corresponds to the values $h \times f$ (m/s): 1 — $h \times f = 500$; 2 — $h \times f = 1000$; 3 — $h \times f = 1500$; 4 — $h \times f = 2000$; 5 — $h \times f = 2500$; 6 — $h \times f = 3000$

the hybridization takes place in the point of the equality of phase velocities of the $S_0$ and $SH_0$ modes with the phase velocity of the $QFS$ bulk acoustic wave. Hybridization regions are shown
by vertical lines on insets of Fig. 3, b. The $E$–application along the $X_1$ or $X_2$ axes amplifies the hybridization effect, and $\alpha_v$ values are increased in accordance with exponential function and reach the maximal quantities (Fig. 4, 5). If the electric field is applied along the $X_1$ or $X_2$ axes the hybridization effect arises between the $S$ values for the $S_{\alpha}$ hybridization effect, and $\text{EM CC}_h$ Fig. 5. Square of $E$ by vertical lines on insets of Fig. 3, b. The $E$–application along the $X_3$ axis leads to the hybridization effect decreasing. Maximal $\alpha_v$ values for the $S_0$ mode are reached in the [110] propagation direction of the (001) plane when $E \parallel X_1$ increasing up $-1.9 \cdot 10^{-12}$ to $4.34 \cdot 10^{-10}$ m/V by the increment of $(h \times f)$ value (Fig. 4).

If $E \parallel X_3$ maximal $\alpha_v$ values take place in the [110] direction: $\alpha_v = 2.99 \cdot 10^{-10}$ m/V ($h \times f = 3000$ m/s). Square of $\text{EM CC}$ of the $S_0$ mode is considerably increased in the [110] direction if $(h \times f)$ values are increased: up 0.47 % to 0.84 % when $h \times f = 500 \sim 3000$ m/s respectively. Hybridization effect leads to exponential dependence of the $\alpha_v$ coefficient and changes the $\text{EM CC}$ coefficients in the hybridization region. When $E \parallel X_1$ anisotropy of $\alpha_v$ coefficient for the $SH_0$ mode is the similar one to the $S_0$ mode (Fig. 5). When dc electric field is applied along the $X_3$ or $X_2$ axes the $\alpha_v$ coefficient values don’t depend on $(h \times f)$ values excluding the hybridization region between the $S_0$ and $SH_0$ modes. Maximal $\text{EM CC}$ values (2.8%) take place in the [100] and [010] directions of the (001) crystalline plane.

The $\alpha_v$ coefficients of the $S_1$, $A_1$, and $SH_1$ first modes propagating in the (001) plane are shown in [14]. Distinctive peculiarity for higher order modes is the extreme behavior of $\alpha_v$ coefficient in the region when $(h \times f)$ value is close to critical one, and the appearance of acoustic modes of higher order becomes possible. In this case a small variation in the plate configuration and material properties of the crystal changes considerably the phase velocity. In particular for the $A_1$ mode propagating along the [110] direction there is maximal value $\alpha_v = -12.32 \cdot 10^{-10}$ m/V $(E \parallel X_3)$. Note that there is the influence of the hybridization effect which leads to the exponential dependence of the $\alpha_v$ coefficient.

### 2.2. Acoustic Wave Propagation in the (110) Crystalline Plane

For this plane the anisotropy of phase velocities is shown in [14] and the maximal values of the $\alpha_v$ coefficients are realized if $E \parallel X_1$. The $E$–application along the $X_3$ axis has a minimal effect on the phase velocities of the $A_0$, $S_0$ and $SH_0$ modes (Fig. 6, 7). Maximal value
\[ \alpha_v = -5.8 \cdot 10^{-10} \text{ m/V} \] for \( S_0 \) mode takes place in the direction oriented relative to the [001] axis under the angle \( \varphi = 29^\circ \) \((h \times f = 1000 \text{ m/s})\) (Fig. 7). Distinctive features of the \( \alpha_v \) coefficients in the (110) plane for the bulk acoustic waves are defined by the splitting of the tangent type acoustic axis coinciding with the [001] crystalline direction (twofold axis of symmetry) in the undisturbed state and by the displacement of the conic type acoustic axis coinciding with the [111] crystalline direction (threefold axis of symmetry) in the undisturbed state [2]. Extreme values of the \( \alpha_v \) coefficients are shown in the Table I.

Table I. Maximal and minimal values of the \( \alpha_v \) coefficients of Lamb and \( \text{SH} \) waves in the bismuth germanium oxide crystal

<table>
<thead>
<tr>
<th>Crystalline plane</th>
<th>Mode</th>
<th>DC electric field direction</th>
<th>Angle, ( \varphi^\circ )</th>
<th>( h \times f, \text{ m/s} )</th>
<th>( \alpha_v, 10^{-11} \text{ m/V} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(001)</td>
<td>( A_0 )</td>
<td>( E \parallel X_3 )</td>
<td>45</td>
<td>3000</td>
<td>39.0</td>
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<td>( E \parallel X_2 )</td>
<td>47</td>
<td>3000</td>
<td>0.126</td>
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<tr>
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<td>( SH_0 )</td>
<td>( E \parallel X_1 )</td>
<td>0</td>
<td>3000</td>
<td>19.9</td>
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<tr>
<td></td>
<td></td>
<td>( E \parallel X_2 )</td>
<td>43</td>
<td>1500</td>
<td>0.0</td>
</tr>
<tr>
<td>(001)</td>
<td>( S_0 )</td>
<td>( E \parallel X_3 )</td>
<td>45</td>
<td>3000</td>
<td>29.0</td>
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<tr>
<td></td>
<td></td>
<td>( E \parallel X_3 )</td>
<td>90</td>
<td>1500</td>
<td>0.001</td>
</tr>
</tbody>
</table>

Fig. 6. The \( \alpha_v \) coefficients of the \( A_0 \) mode propagating in the (110) plane of \( \text{Bi}_{12}\text{GeO}_{20} \) crystal: a) \( E \parallel X_1 \); b) \( E \parallel X_2 \); c) \( E \parallel X_3 \). Designations of curves are in accordance with Fig. 2.

Fig. 7. The \( \alpha_v \) coefficients of the \( \text{SH}_0 \) (a) and \( S_0 \) (b) modes propagating in the (110) plane of \( \text{Bi}_{12}\text{GeO}_{20} \) crystal. Designations of curves are in accordance with Fig. 2.
### Conclusion

Thus, the anisotropy of homogeneous dc electric field influence on the different types of acoustic waves in the bismuth germanium oxide piezoelectric crystal plate has been investigated by means of computer simulation. Detail analysis of the dispersive behavior of zero and first order Lamb and $SH$ modes has been carried out. Crystalline directions with extreme dc electric field influence have found. It was shown that the acoustic modes interaction can arise as a consequence of dc electric field action in the some directions. The obtained data can be useful to design the controlling devices of acoustoelectronics.

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### References


Анизотропия влияния постоянного электрического поля на распространение акустических волн в пьезоэлектрической пластине

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Рассмотрена анизотропия влияния однородного электрического поля $E$ на характеристики и условия распространения волн различных типов в пьезоэлектрической пластине германосильвина.

Ключевые слова: внешнее электрическое поле, волна Лэмба.