Complete Semigroups of Binary Relations Defined by Semilattices of the Class $\Sigma_7(X, 6)$

Ya. Diasamidze*, Sh. Makharadze**, N. Rokva and Il. Diasamidze

Batumi State University, 35, Ninoshvili St., Batumi 6010, Georgia
* *E-mail: diasamidze_ya@mail.ru*
** *E-mail: axiomabat@rambler.ru*

Let X be an arbitrary nonempty set, D be an X-semilattice of unions, i.e. such a nonempty set of subsets of the set X that is closed with respect to set-theoretic union operations for elements from D, f be an arbitrary mapping of the set X in the set D. To each such mapping f we assign a binary relation α_f on the set X that satisfies the condition

$$\alpha_f = \bigcup_{x \in X} (\{x\} \times f(x)).$$

The set of all such α_f $(f : X \to D)$ is denoted by $B_X(D)$. It is easy to prove that $B_X(D)$ is a semigroup with respect to the operation of multiplication of binary relations, which is called a complete semigroup of binary relations defined by the X-semilattice of unions D.

Let $\Sigma_7(X, 6)$ be such a class of all X-semillatices of unions, whose every element is isomorphic to the X-semilattice of unions $D = \{Z_5, Z_4, Z_3, Z_2, Z_1, \breve{D}\}$ and satisfies the following conditions:

$$\begin{gathered} Z_4 \subset Z_3 \subset Z_1 \subset \breve{D}, \quad Z_4 \subset Z_3 \subset Z_2 \subset \breve{D}, \quad Z_5 \subset Z_3 \subset Z_1 \subset \breve{D}, \quad Z_5 \subset Z_3 \subset Z_2 \subset \breve{D}, \\ Z_1 \setminus Z_2 \neq \varnothing, \quad Z_2 \setminus Z_1 \neq \varnothing, \quad Z_4 \setminus Z_5 \neq \varnothing, \quad Z_5 \setminus Z_4 \neq \varnothing, \quad Z_4 \cup Z_5 = Z_3, \quad Z_1 \cup Z_2 = \breve{D}. \end{gathered} \tag{1}$$

The diagram of a semilattice satisfying condition (1) is shown in Fig. 1.

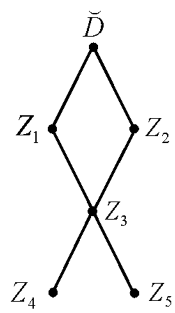

Figure 1

In the work, the class of semigroups, which consists of all semigroups $B_X(D)$ defined by some semilattice D belonging to the class $\Sigma_7(X, 6)$, is studied. Structures of idempotents, regular elements and maximal subgroups are described for the semigroups of of the considered class. Formulas for counting the number of idempotents and regular elements are found for the finite set X. It is proved that the set of all regular elements of any semigroup of the considered class form a subsemigroup of this semigroup.

References

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