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Mathematical Modeling of H-processes

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The problem of the discrete continuous processes having "tubular" structure in space "input-output" variables's modeling is investigated. The fact that when the trained parametrical models of "tubular" processes's creating, it's important to use corresponding nonparametric indicators, is reflected. Some private examples of "tubular" processes's modeling are reviewed. This examples proves that "tubular" processes proceed in the space of fractional dimension.

Keywords: priori information, identification, nonparametric model, nonparametric algorithms, H-model, fractional dimension space.
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Introduction

Identification of stochastic objects is often reduced to the identification of static systems with delay. This is due to the fact that some output variables of the object are controlled for much longer time than the input variables. For example, several variables are measured electrically (in this case, the discrete control can be brief) but the other variables are controlled by chemical analysis or physical-mechanical tests (in this case the discrete control ΔT is long, i.e. $\Delta T \gg \Delta t$).

The most common scheme of a discrete-continuous process is shown in Fig. 1:

The notation of Fig. 1: A is the investigate object (the process); $\overrightarrow{x(t)}$, $\overrightarrow{q(t)}$ and $\overrightarrow{z(t)}$ are the output vectors of the process; $\overrightarrow{u(t)}$ is the vector of control actions; $\overrightarrow{\mu(t)}$ is the uncontrolled but measured input vector of i-th process; $\overrightarrow{\lambda(t)}$ is the input vector of unmanaged and measured process variable; $\overrightarrow{\xi(t)}$ is the casual influence; $\overrightarrow{\omega^i(t)} : i = \overline{1,k}$ are the process variables controlled in object; t is time; $H^\mu, H^u, H^x, H^z, H^q, H^\omega$ are the communication channels corresponding to various variables, including control devices and devices for measurement of observed variables; $\mu_t, u_t, x_t, \omega_t$ are measurements of $\overrightarrow{\mu(t)}, \overrightarrow{u(t)}, \overrightarrow{x(t)}, \overrightarrow{\omega(t)}$ at time moment t; $h^\mu(t), h^u(t), h^x(t), h^{\omega_1}(t), \dots, h^{\omega_k}(t)$ are casual hindrances of measurements of the corresponding process variables.

This scheme is well known and it frequently occurs in many fields of research [1].

The distinctive features of output variables $\overrightarrow{z(t)}, \overrightarrow{q(t)}$ and $\overrightarrow{x(t)}$ are presented in Fig. 1. Output variable $\overrightarrow{x(t)}$ and input variables are controlled at interval $\Delta t, \overrightarrow{q(t)}$ is controlled at significantly

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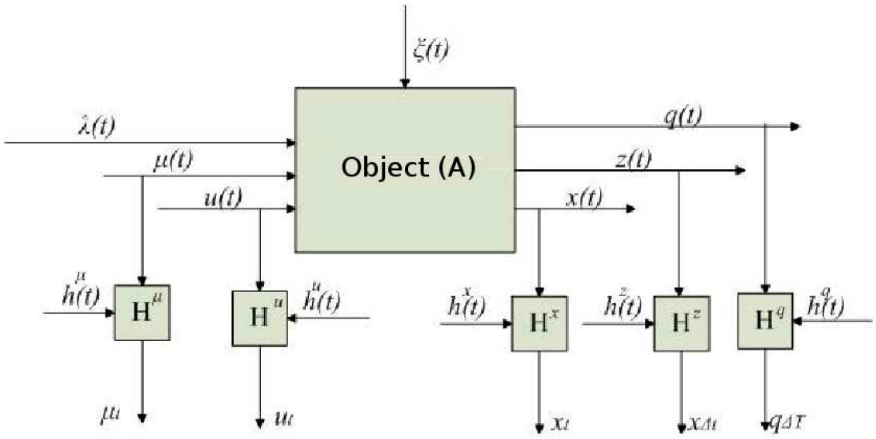

Fig. 1. The general scheme of the test process

biger interval ΔT. Variable $\overrightarrow{z(t)}$ is controlled at interval T, $T \ll \Delta T \ll \Delta t$. The control of variable $\overrightarrow{z(t)}$ is of importance in practical applications.

In this case output variables depend on input variables and $\omega(t)$:

$$x(t) = A\big(u(t), \mu(t), \omega(t), \lambda(t), \xi(t), t\big) \tag{1}$$

Consider various sampling of control of measurements $\overrightarrow{x(t)}, \overrightarrow{q(t)}$ and $\overrightarrow{z(t)}$. All variables used to predict $\overrightarrow{x(t)}, \overrightarrow{q(t)}, \overrightarrow{z(t)}$ can be used to predict $\overrightarrow{q(t)}$ and $\overrightarrow{z(t)}$:

$$\hat{x}(t) = \hat{A}\big(u(t), \mu(t), \omega(t), t\big) \tag{2}$$

$$\hat{q}(t) = \hat{A}\big(u(t), \mu(t), \omega(t), \hat{x}(t), t\big) \tag{3}$$

$$\hat{z}(t) = \hat{A}\big(u(t), \mu(t), \omega(t), \hat{x}(t), \hat{q}(t), t\big) \tag{4}$$

Variables $\hat{x}(t), \hat{q}(t), \hat{z}(t)$ in (2)–(4) are the estimates of variables $\overrightarrow{x(t)}, \overrightarrow{q(t)}$ and $\overrightarrow{z(t)}$. The values of ΔT and T are much bigger than the object time constant. Then processes considered are static processes with delay. Such processes play important role in problems of identification and stochastic systems control.

We reduce all input and output variables in a vector. The object can be represented as a static object with delay in the following form:

$$x(t) = f\big(u(t-\tau), \xi(t)\big), \tag{5}$$

$\overrightarrow{x(t)}$ is the output variable of the object, $u(t-\tau)$ is the aggregate input variable, τ is the time of delay, $\overrightarrow{\xi(t)}$ is the random perturbation, t is time.

1. Identification in "narrow" and "broad" sense

The "narrow" sense identification theory dominates now in the discrete modelling of continuous processes. In the first stage, on the basis of available a priory information the object structure (A^α) is defined, for example:

$$\hat{x}_\alpha(t) = A^\alpha(u(t), \alpha), \tag{6}$$

A^{α} is the parametric structure of the object, α is the vector of parameters.

In the second stage the assessment of α is carried out on the basis of available sample x_i, u_i, $i = \overline{1, s}$, s is the volume of selection. In this case the accuracy of identification significantly depends on operator (6).

The "broad" sense of identification does not assume any particular structure of the object. It is often much simpler to define a class of operators based on the data type. For example, it is linearity or nonlinearity, unambiguity or ambiguity of a process. In this case the problem of identification involves operator estimation. It is based on sample x_i, u_i, $i = \overline{1, s}$ [2, 3].

It should be noted that identification in "broad" sense requires high quality selection. We take the word quality to mean the accuracy of the data and uniformity of distribution of vector $\overrightarrow{u(t)}$ measurements. Quality of data is important because they are used for assessment if parametric operator is not available.

$$\hat{x_s}(t) = A_s(u(t), \overrightarrow{x_s}, \overrightarrow{u_s}), \tag{7}$$

$\overrightarrow{x_s} = (x_1, x_2, \ldots, x_s)$, $\overrightarrow{u_s} = (u_1, u_2, \ldots, u_s)$ are temporary vectors. Operator assessment A_s can be carried out by means of nonparametric statistics. The choice of a parametric structure is not considered here. Identification in "broad" sense is more adequate in real problems.

2. Identification of a static system

Let $\overrightarrow{u} = (u_1, \ldots, u_k) \in \omega(u) \subset R^k$, $x \in \omega(u) \subset R^k$. Generally speaking, every vector component $u_i \in [a_i, b_i], i = \overline{1, k}$, and $x \in [c; d]$. In practice, the values of coefficients a_i, b_i, c, d, $i = \overline{1, k}$ are always known. In technological processes the values of these coefficients are regulated by the production schedules. Further, we assume that all intervals are $[0; 1]$ [4]. Then $\omega(u)$ is the unit hypercube, $\omega(u) = [0; 1]$, i.e. $u \in [0; 1]$, $\omega_k(u, x) = [0; 1]$, $(u, x) \in [0; 1]$. The adaptive model in this case is 5

$$\hat{x}_s(u) = \hat{f}(u, \alpha_s). \tag{8}$$

Now we should define the parametric structure of the model. If at the first stage the considerable error is introduced then we would not obtain satisfactory model. This problem was explicitly discussed [2, 3]. Model of class (8) is hyper-surface in the space of "input-output" variables of the object, i.e. $(u, x) \in \omega(u, x) \subset R^{k+1}$.

If the process has "tubular" structure [2] then model (8) should be corrected:

$$\hat{x_s}(u) = I_s(u)\hat{f}(u, \alpha_s) \tag{9}$$

or

$$\hat{x_s}(u) = I_s(u) \sum_{j=1}^{N} \alpha_{sj} \varphi_j(u), \tag{10}$$

$\varphi_j(u)$ is the system of linear and independent functions, the indicator $I_s(u)$ is defined as

$$I_s(u) = \begin{cases} 1, \text{if } u \in \omega_s^H(u); \\ 0, \text{if } u \notin \omega_s^H(u). \end{cases} \tag{11}$$

Let us note that domain $\omega^H(u)$ is not known. Sample x_i, u_i, $i = \overline{1, s}$ is only known. If the indicator is equal to zero the estimate $\hat{x_s}(u)$ cannot be calculated, because the process does not exist at such values of vector$u \in \omega(u)$. However, the model without indicator produces estimate even in this case. If indicator $I_s(u)$ is equal to one at any value $u \in \omega(u)$ model (9) coincides with (8). One can use the following approximation for the indicator assessment $I_s(u)$:

$$I_s(u) = \operatorname{sgn} \sum_{i=1}^{s} \Phi\left(c_s^{-1}(x_s(u) - x_i)\right) \prod_{j=1}^{k} \Phi\bigl(c_s^{-1}(u^j - u_i^j)\bigr), \tag{12}$$

$$x_s(u) = \sum_{i=1}^{s} x_i \prod_{j=1}^{k} \Phi\Big(c_s^{-1}(u^j - u_i^j)\Big) \Big/ \sum_{i=1}^{s} \prod_{j=1}^{k} \Phi\Big(c_s^{-1}(u^j - u_i^j)\Big), \tag{13}$$

where parameter c_x and bell-shaped function $\Phi(*)$ should satisfy some conditions [2].

Bell-shaped function $\Phi(*)$ is a function that satisfies the following conditions

$$\frac{1}{c_s} \int_{-\infty}^{+\infty} \Phi\left(\frac{t - t_i}{c_s}\right) dt = 1, \tag{14}$$

$$\lim_{c_s \to 0} \frac{1}{c_s} \int_{-\infty}^{+\infty} \varphi(t) \Phi\left(\frac{t - t_i}{c_s}\right) dt = \varphi(t_i), \tag{15}$$

where c_s is the characteristic of the bell-shaped function called core width.

Thus, given value $u = u' \in \omega(u)$ we construct the first assessment $x_s(u = u')$, using (13). Then indicator $I_s(u)$ is calculated. Model (9) or (10) is used in the following stage if the indicator is equal to one. If indicator equals zero it means that $u' \in \omega(u)$ but $u' \notin \omega^H(u)$ and components of vector $u = u' = (u'_1, \ldots, u'_k)$ are not true. To put it in other words, the "tubular" process does not correspond to vector $u = u'$. This is because components of vector $u = u' = (u'_1, \ldots, u'_k)$ are not true or they are measured with the considerable error. This holds only for representative sample x_i, u_i, $i = \overline{1, s}$. One should note that traditional model (8) gives incorrect estimate $\hat{x}(u = u')$.

The object identification in the parametric statement should be also carried out with regard to the "tubular" structure of the object. Let us consider the following class of models of "tubular" process

$$\hat{x}(u) = I(u) \sum_{j=1}^{N} \alpha_j \varphi(u), \tag{16}$$

$\varphi(u)$, $j = \overline{1, N}$ is the system of linear and independent functions. Let us introduce the optimality criterion

$$R(\alpha) = \left(x(u) - I(u) \sum_{j=1}^{N} \alpha_j \varphi_j(u) \right)^2. \tag{17}$$

Our purpose is to find such $\alpha^* = (\alpha_1^*, \ldots, \alpha_N^*)$, that

$$R(\alpha^*) = min R(\alpha). \tag{18}$$

The solution of (18) is the system of recurrence relations

$$\alpha_s^l = \alpha_s^{l-1} + \gamma_s^{l-1} \left(x_s - I_s(u_s) \sum_{j=1}^{N} \alpha_{s-1}^j \varphi_j(u_s) \right) \varphi_j(u_s) I_s(u_s), \ l = 1, \ldots, N. \tag{19}$$

One can use the following approximation for assessment $I(u_s)$

$$I_s(u_s) = sgn \sum_{i=1}^{s} \prod_{j=1}^{k} \Phi\Big(\frac{u_s - u_i^j}{c_s}\Big). \tag{20}$$

It is clear that α_s tens to α_s^* when $s \to \infty$.

3. On one feature of the model operation of "tubular" processes

Let the object be described by the equation

$$x(u) = f(u_1, u_2, u_3) \tag{21}$$

where three-dimensional vector $\overrightarrow{u} = (u_1, u_2, u_3) \in R^3$ is the input variable and $x \in R^1$ is the output variable. The traditional way of modelling such process has been described (6). We define $\hat{x}(u) = \hat{f}(u_1, u_2, u_3, \alpha)$ and assessment parameters α, using observations, $(u_i, x_i,\, i = \overline{1, s})$, s is the sample size. Let us analyze this example from a different point of view. Let us assume that input variables $\overrightarrow{u} = (u_1, u_2, u_3)$ are independent. In this case we can use the traditional algorithm described above. Now we assume that objective components of the vector of input variables are not independent, for example,

$$u_2 = \varphi_1(u_1),\ u_3 = \varphi_2(u_2) = \varphi_2(\varphi_1(u_1)). \tag{22}$$

Naturally, we do not know about the existence of dependences (22). Such process was called the H-process. Otherwise it would be possible to substitute (22) into (21) and to obtain the dependence of x on one variable u_1:

$$x(u) = f\big(u_1, \varphi_1(u_1), \varphi_2(\varphi_1(u_1))\big). \tag{23}$$

When u_2 depends on u_1 we have

$$x(u) = f\big(u_1, \varphi_1(u_1), u_3\big), \tag{24}$$

i.e. x depends on u_1, u_3. Let us emphasize again that input variables are not independent. We do not know about the existence of interrelation between input variables. Now we analyse the most interesting case directly related to the H-process[[4]. Let us assume that u_3 and u_2 are related to each other stochastically [2]. First, if components of vector $\overrightarrow{u}$ are independent the process is described by the function of the three variables. If only two components of vector $\overrightarrow{u}$ are independent the process is described by the function of two variables. If two variables are related to each other stochastically the process is described by the function of more than two variables but less than three variables. It is possible to assume that we have fractional number of variables. Therefore, we deal with a space of fractional dimension. For example, B. Mondelbrot [5] noticed: "The vascular system of the person – pulsing alive – has dimension 2.7". For the first time fractional dimension of space was introduced in works of Hausdorff and Bezikovich.

Let us consider process (21).

In the case of a stochastic relation between variables $u_2(u_1), u_3(u_1)$ on the available training selections it is possible to calculate a squared error δ of the estimate $\hat{u_{2s}}(u_1), \hat{u_{3s}}(u_1)$

$$\delta_{21} = \sum_{i=1}^{s} \big(u_2 - \hat{u_{2s}}(u_1)^2\big)/\sigma^2_{u_2}, \quad \delta_{31} = \sum_{i=1}^{s} \big(u_3 - \hat{u_{3s}}(u_1)^2\big)/\sigma^2_{u_3}, \tag{25}$$

Here $\hat{u_{2s}}(u_1), \hat{u_{3s}}(u_1)$ there are nonparametric estimates [4], δ_{ij} is the squared error of the estimate u_i based on u_j.

The value of stochastic communication λ between any two variables can be calculated as

$$\lambda = 1 - \delta. \tag{26}$$

In the case strong functional communication $\lambda = 1$, lack of communication corresponds to $\lambda = 0$. In the case of stochastic relation between input variables $0 < \lambda < 1$. In general case, one can interpret such process as function of many variables. For example, this function can be expressed as follows [4]:

$$x = \begin{cases} f(t, u_1, \mathbf{u_2}, \mathbf{u_3}, u_4, u_5) - T_1 \\ f(t, u_1, u_2, \mathbf{u_3}, u_4, u_5) - T_2 \\ f(t, u_1, u_2, u_3, u_4, u_5) - T_3 \\ f(t, u_1, u_2, u_3, u_4, u_5, u_6) - T_4 \\ f(t, u_1, u_2, u_3, u_4, u_5, u_6) - T_5 \\ f(t, u_1, u_2, u_3, u_4, u_5, u_6) - T_6 \\ f(t, u_1, \mathbf{u_2}, u_3, u_4, \mathbf{u_5}, u_6) - T_7 \\ f(t, \mathbf{u_1}, \mathbf{u_2}, u_3, u_4, \mathbf{u_5}, \mathbf{u_6}, u_7) - T_8 \end{cases} . \quad (27)$$

Variables which have strong impact on x (functional relation) are designated in dark colour ($\mathbf{u_1}$). Less dark colour (u_1) means that this variable has weaker influence on x than ($\mathbf{u_1}$) (perhaps strong stochastic dependence). Variables marked as u_1 and u_1 have weaker influence on x than (u_1). Parameters $T_i, i = \overline{1,8}$ are time intervals. Role of each variable may change in real process. Given above relations show that some variables can lose their significance, some variables can restore their significance and some variables can appear for the first time such as u_6, u_7.

To treat function of many variables as a point in many-dimensional space we introduce space of fractional dimension F^λ. The dimension of F^λ can be calculated as

$$\dim F^\lambda = (n+1) - \sum_{i=1}^{n-1} \lambda_{i,i+1}\,, \quad (28)$$

n is the dimension of a vector u, $\lambda_{i,i+1}$ is the intensity of stochastic relation between u_i and u_{i+1}. There are other ways to calculate space dimension, for example,

$$\dim F^\lambda = (n+1) - \sum_{i=1}^{n-1} \lambda_{1,i}\,, \quad (29)$$

$\lambda_{1,i}$ is the relation between u_i and u_1.

When we deal with the Taylor series expansion of a function one should recall V. I. Arnold's phrase from the book "Catastrophe Theory" [1]: "Calculation in these applied studies[§].

4. Computing experiments

Let us consider process that is described by function $x = f(u_1, u_2)$ with the noise $\xi(t)$: $f(u_1, u_2) = u_1^2 + 2u_2 + \xi(t)$.

Let us assume that training selection s is equal to 500 and input variables are independent.

Fig. 3 illustrates that the space dimension F^λ decreases with decreasing sample size and the space dimension tends to 3 with increasing sample size.

Let us consider a process that has "tubular" structure, that is, H-process. We assume that relation between two variables is $u_2 = 3u_1$. Function $f(u_1, u_2)$ is given above. Fig. 2 shows that when input variables are independent the dimension of process is close to 3.

In the case of H-process the space dimension is close to 2 (Fig. 4).

We have $x = f(u_1, u_2)$ but this process has "tubular" structure, so $u_2 = g(u_1)$. Then $x = f(u_1, u_2) = f(u_1, g(u_1))$. As a result we have the process that is described by one variable.

[§]We are talking about the elasticity theory.

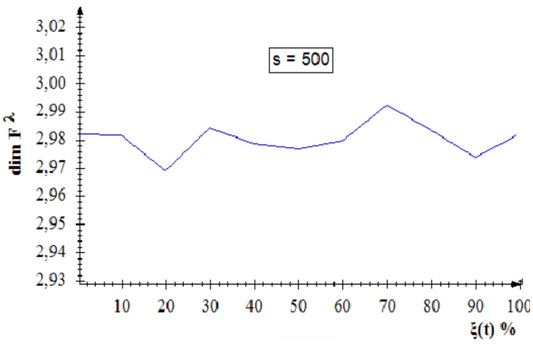

Fig. 2. dependence of dimension of space F^{λ} and noise level

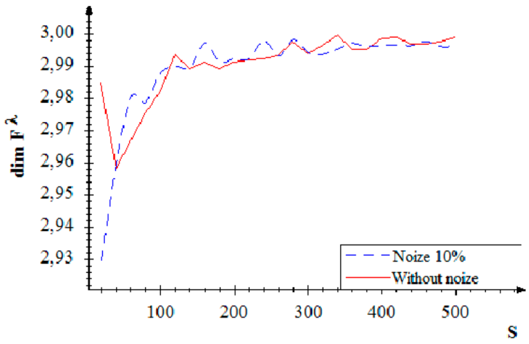

Fig. 3. dependence of dimension of space F^{λ} and sample size

When the noise level is increased the relation between u_1 and u_2 becomes weaker and dimension of the process grows.

Computational experiments show (Fig. 5) that space dimensions $dimF^{\lambda}$ are different. This is because u_2 stochastically depends on u_1 in the second experiment. Let us consider experiment with 10 input variables and one output variable.

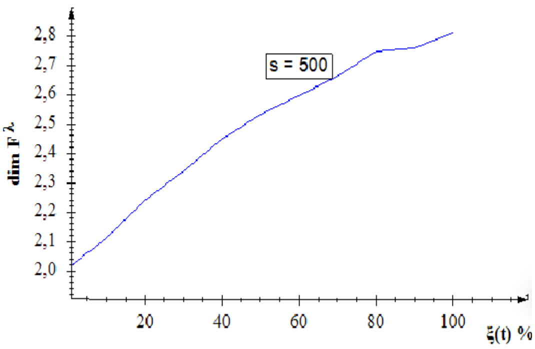

Fig. 4. dependence of dimension of space F^{λ} and noise level

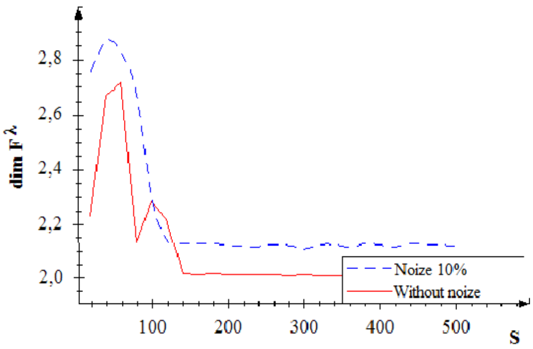

Fig. 5. dependence of dimension of space F^{λ} and sample size

Fig. 6 shows that the space dimension is close to 11. Now we consider two cases: in the former case we have 10% noise and no noise in the second case. As one would expect, the space dimension F^{λ} tends to 11 (Fig. 7). Let us calculate space dimension F^{λ} versus the level of noise if all input variables are stochastically related (Fig. 8).

Space dimension F^{λ} equals two when there are functional relations between input variables. Space dimension $dimF^{\lambda}$ is increased when functional relations become weaker. Space dimension $dimF^{\lambda}$ is increased with increasing the sample size (Fig. 9). This is due to more precise estimation of parameter δ.

Conclusion

The analysis of processes of "tubular" structure is presented. Such structure takes always place if components of the vector of input variables are stochastically related. In this case tradi-

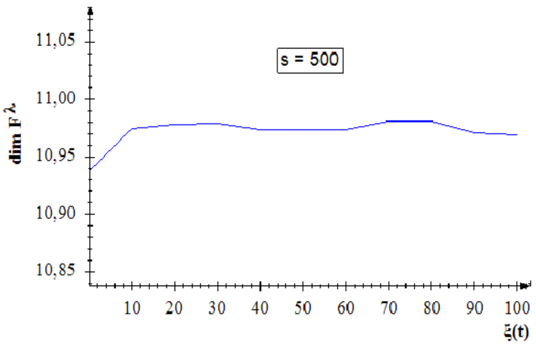

Fig. 6. dependence of dimension of space F^λ and noise level

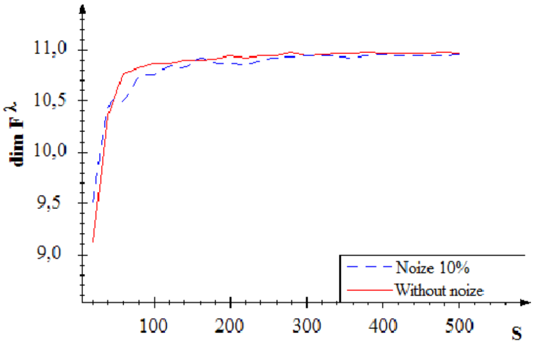

Fig. 7. dependence of dimension of space F^λ and sample size

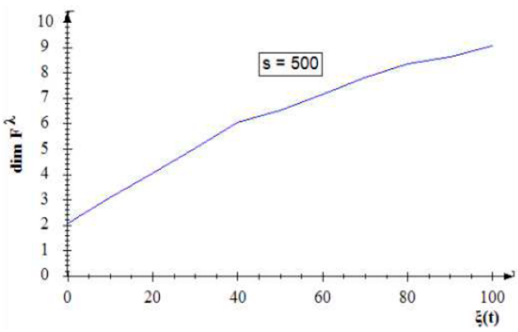

Fig. 8. dependence of dimension of space F^λ and noise level

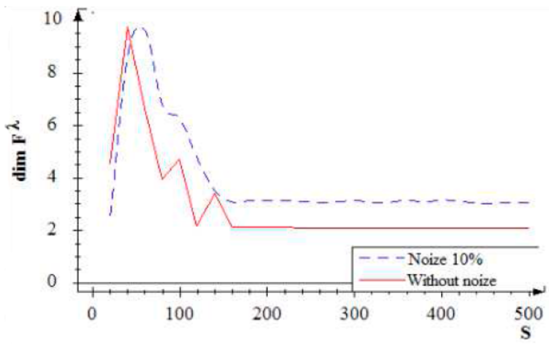

Fig. 9. dependence of dimension of space F^λ and sample size

tionally used models of static systems with delay are not applicable or they can give inaccurate results. It is found that one need to consider space of fractional dimension. It is also found that relation between input and output variables may appear and disappear. This is connected not only with the space of fractional dimension but also with the space of variable dimension.

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References

[1] Y.L.Fateev, D.D.Dmitriev, V.N.Tyapkin, E.N.Garin, V.V.Shaidurov, The phase ambiguity resolution in the angle-measuring navigation equipment, ICAAM, Shymkent, Kazakhstan, 2014, 12–15

[2] A.V.Medvedev, Analysis of the data in the problem identification, Computer analysis of simulation data, vol. 2, Minsk, BGU, 1995, 201–206 (in Russian).

[3] A.V.Medvedev, H-model for non-inertia systems with delay, *Vestnik SibGAU*, **45**(2012), no. 5, 84–89 (in Russian).

[4] A.V.Medvedev, Some notes on H - models for non-inertis systems with a delay, *Vestnik SibGAU*, **54**(2014), no. 5, 24–34 (in Russian).

[5] B.Mondelbrot, Fraktalnaya geometriya prirody [Fractal Geometry of Nature], Moscow–Izhevsk, Institute of Computer Science, NITS, Regular and Chaotic Dynamics, 2010.

[6] V.I.Arnol'd, Catastrophe theory, Springer-Verlag, Berlin, 1992.

Математическое моделирование Н-процессов

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Исследуется проблема моделирования дискретно-непрерывных процессов, имеющих «трубчатую» структуру в пространстве «входных–выходных» переменных. Отражено то, что при построении обучающихся параметрических моделей «трубчатых» процессов необходимо использование соответствующих непараметрических индикаторов. Рассмотрены некоторые частные примеры моделирования «трубчатых» процессов, из которых следует, что «трубчатые» процессы протекают в пространстве дробной размерности.

Ключевые слова: априорная информация, идентификация, непараметрическая модель, непараметрические алгоритмы, Н-модели, пространство дробной размерности.