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Unification and Inference Rules in the Multi-modal Logic of Knowledge and Linear Time LTK

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We study unification of formulas in multi-modal LTK logic and give a syntactic description of all formulas which are non-unificable in this logic. Passive inference rules are considered, it is shown that in LTK logic there is a finite basis for passive rules.

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Introduction

The research of unification for various logic systems is one of the most rapidly developing areas of modern mathematical logic. Arisen in the field of Computer Science, primarily in the form of a question about the possibility to transform two different terms in syntactically equivalent by replacing the variables of certain other terms ([1,2]), from the time the task has changed course on the study of semantic equivalence ([3,4]).

For most of the non-classical logics (modal, pseudo-boolean, temporal, etc.), there are special dual equational theories of special algebraic systems, so their problems are reduced to the corresponding logical-unificational counterparts ([5–7]). Basic unificational problem can be viewed as a complex issue: whether the formula is to be transformed into a theorem after replacing the variables (keeping the same values of the coefficients parameters). This issue was investigated and partly resolved (including V. V. Rybakov [8–10]), for intuitionistic and modal logics S4 and Gr*z*.

Unification in intuitionistic logic and in propositional modal logic over the K4 investigated by S. Ghilardi [11–15] (with applications of projective algebra ideas and technology based on projective formulas). In these works the problem of constructing the finite complete sets of unifiers was solved for the considered logic, efficient algorithms were found. Such an approach proved to be a a useful and effective in dealing with the admissibility and the basis of admissible rules (Jerabek [16–18], Iemhoff, Metcalfe [19,20]). Indeed, the existence of computable finite sets of unifiers follows directly solution of the admissibility problem.

Temporal logic is also very dynamic area of mathematical logic and computer science (including Gabbay и Hodkinson [21–23]). In particular, LTL (linear temporal logic) has a significant application in the field of Computer Science (Manna, Pnueli [24,25], Vardi [26,27]). Solving the problem of admissibility of rules in the LTL was proposed by Rybakov [28], basis of admissible

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rules in LTL by Babenyshev and Rybakov in [29] (without the operator Until [30]). Solution of the unificational problem for LTL has also been found by Rybakov [31,32] and proposed for basic modal and intuitionistic logic in [33, 34]. Particularly, in [31] It proved that not all unified in LTL formula are projective, and in [32] proved the projectivity of any unified formula in LTL_u (it is a fragment of LTL, only with the operator Until). In the paper of Dzik and Wojtylak [35] they obtained the same result for $S4.3$.

Research conducted in the present paper, primarily based on the approach proposed in [36]. The key focus here is on the description of non-unifiable formulas in a wide class of modal logics. Especially, it proposed the criteria of non-unifiable (with the proofs) for modal extensions of S4 (Theorem 1.4 below) and $[K4+\Box\bot \equiv \bot]$ (Theorem 1.5). The aim of this article is to investigate the question of unification in linear temporal logic (LTK).

1. Fundamental definitions and notations

Before describing the main results, we introduce the most important definitions and notations. Proofs for the most of propositions, consequences and the theorems in this section are detailed in [36].

First, we define a unified formula in this logic. Let λ is a logic with the formula $\phi(p, q)$ which describes the equivalent formula. We say that α is equivalent to β in λ,and we write $\alpha \equiv_\lambda \beta$ if $\vdash_\lambda \phi(\alpha, \beta)$. For convenience, $\phi(\alpha, \beta)$ we denote $\alpha \equiv \beta$.

Definition 1.1. *Formula $\alpha(p_1, \dots, p_n)$ is unifiable in an algebraic logic λ iff there is a tuple of formulas $\delta_1, \dots, \delta_n$ such that $\vdash_\lambda \alpha(\delta_1, \dots, \delta_n)$.*

Definition 1.2. *Formulas $\alpha(p_1, \dots, p_n)$ and $\beta(p_1, \dots, p_n)$ are said to be unifiable in algebraic logic λ iff there is a tuple of formulas $\delta_1, \dots, \delta_n$ such that $\vdash_\lambda \alpha(\delta_1, \dots, \delta_n) \equiv \beta(\delta_1, \dots, \delta_n)$. In this case, the tuple $\delta_1, \dots, \delta_n$ is called an unifier for these two formulas.*

Corollary 1.3 (2.7 from [36]). *For all logics SIL, $S4_{ext}$, $K4 + \Box\bot \equiv \bot$ unifiers for formulas can be effectively found among sequences of formulas $\top$ end $\bot$.*

For example, by setting $\top$ everywhere in the performance of the variable p and $\bot$ otherwise.

Theorem 1.4 (2.10 from [36]). *For any modal logic λ extending S4 and any modal formula α, α is not unifiable in λ iff the formula $\Box\alpha \to \left[\bigvee_{p \in Var(\alpha)} \Diamond p \wedge \Diamond\neg p\right]$ if provable in λ.*

Theorem 1.5 (2.11 from [36]). *For any modal logic λ extending K4, where $\Box\bot \equiv \bot \in \lambda$ and any modal formula α, α is not unifiable at λ iff formula $\Box\alpha \wedge \alpha \to \left[\bigvee_{p \in Var(\alpha)} \Diamond p \wedge \Diamond\neg p\right]$ is provable at λ.*

Definition 1.6. *Rule $r := A/B$ is a consequence of the rules $r_1 := A_1/B_1, \dots, r_n := A_n/B_n$ in logic $L \Leftrightarrow \forall A \in Var(L) = \{A | A \vDash (\alpha = \top), \forall\alpha \in L\}$: if*

$$\forall i A \vDash (\alpha_i = \top) \Rightarrow (\beta_i = \top),$$

then

$$A \vDash (\alpha = \top) \Rightarrow (\beta = \top).$$

Let us recall the definition of algebra formulas, Lindenbaum algebra. Let For is the set of all formulas in the language of logic. We will use the following notation: $A \equiv B \Leftrightarrow (A \to B) \wedge (B \to A)$. We write $A \equiv_L B$, if $A \equiv B \in L$. Suppose that the logic L has theorem of replacing equivalent.

Namely, if $\oplus$ is any binary logic connective (for example $\rightarrow, \wedge, \vee$), and A_1, A_2, B_1 и B_2 are the formulas, then

$$(A_1 \equiv_L B_1, A_2 \equiv_L B_2) \Rightarrow A_1 \oplus A_2 \equiv_L B_1 \oplus B_2,$$

and if $\otimes$ is any unary logical connective, then

$$A_1 \equiv_L B_1 \Rightarrow \otimes A_1 \equiv_L \otimes B_1.$$

Lindenbaum algebra $A/_{\equiv}$ has the basic set $For_{\equiv}$, where

$$For_{\equiv} := \{[A]_{\equiv} | A \in For, [A]_{\equiv} := \{B | B \equiv_L A\}\}.$$

$$[A]_{\equiv} \oplus [B]_{\equiv} := [A \oplus B]_{\equiv}.$$

We define an algebra $A/_{\equiv}$ as follows: $A/_{\equiv} = \langle For_{\equiv}, \wedge, \vee, \rightarrow, \Box \rangle$, where

$$A \equiv_L B \Leftrightarrow (A \rightarrow B) \wedge (B \rightarrow A) \in L.$$

Theorem 1.7 (**Lindenbaum).** *For any modal formula α, modal logic L and variables $x_1, \dots, x_n \in Var(L)$: $\alpha(x_1, \dots, x_n) \in L \Leftrightarrow \alpha(x_1, \dots, x_n) \equiv \top$ is a truth on $A/_{\equiv}$.*

2. Semantics LTK

Alphabet of the language L^{LTK} includes a countable set of propositional variables $P := \{p_1, \dots, p_n, \dots\}$, braces $(,)$ default Boolean operations and a variety of single modal operators $\{\Box_{\leqslant}, \Box_e, \Box_1, \dots, \Box_n\}$. Every propositional variable $p \in P$ is well-formed formulae (wff), and if A is wff, then so are $\Box_{\leqslant} A, \Box_e A, \Box_i A (i \in I)$. We abbreviate $Fma(L^{LTK})$ as a set of all wff in language L^{LTK} (hereinafter referred to the *formula* will be understood as formula from the set $Fma(L^{LTK})$). Logic operations $\Diamond_{\leqslant}$, $\Diamond_e$, $\Diamond_i$ determined through $\Box_{\leqslant}$, $\Box_e$, $\Box_i$ as follows: $\Diamond_{\leqslant} = \neg \Box_{\leqslant} \neg$, $\Diamond_e = \neg \Box_e \neg$, $\Diamond_i = \neg \Box_i \neg$. The values of described modal operators are defined as follows: $\Box_{\leqslant} A$: A is a truth at the current time and in any future; $\Box_e A$: A is a truth at a given moment of time; $\Box_i A$ means that A is a truth in all informational points which available to the agent i. Semantics for the language L^{LTK} models linear and discrete stream of the computational process, in which each point in time is associated with a natural number n.

Semantically, our logic is defined on the Kripke frames.

Definition 2.1. *k-modal Kripke-frame is a tuple $F = \langle W_F, R_1, \dots, R_k \rangle$, where W_F is a non-empty set of worlds and each R_i is some binary relation on W_F.*

Definition 2.2. *Let $F = \langle W_F, R_1, \dots, R_k \rangle$ is Kripke-frame, and $\forall R_i$ R_i-cluster is a subset $C^{R_i} \in W_F$ such that $\forall v, z \in C^{R_i} : vR_i z \& zR_i v$ and $\forall z \in W_F, \forall v \in C^{R_i} : ((vR_i z \& zR_i v) \Rightarrow z \in C^{R_i})$. For any relation R_i, $C^{R_i}(v)$ is the R_i-cluster s.t. $v \in C^{R_i}$ or cluster, generated by the element v. R_i-cluster called: **degenerate**, if it consists of a single R_i-irreflexive point; **simple** if it consists of a single R_i-reflexive point; **proper**, if it contains at least two R_i-reflexive points.*

Definition 2.3. *LTK-frame is a k+2-modal Kripke-frame $F = \langle W_F, R_1, \dots, R_k, R_e, R_{\leqslant} \rangle$, where:*
a) W_F is the disjoint union of non-empty sets C^t, $t \in N$: $W_F := \bigcup_{t \in N} C^t$;
b) $R_1, ...R_k$ are some equivalence relations within each cluster C^t;
c) R_e is universal $S5$-relation of equivalence at any $C^t \in W_F$:

$$\forall w, z \in W_F (wR_e z \Leftrightarrow (w \in C^t) \& (z \in C^t));$$

d) $R_{\leqslant}$ is linear, reflexive, transitive binary temporal relation on W_F, specifying linear order of clusters (simple chain):

$$\forall v, z \in W_F(vR_{\leqslant}z \Leftrightarrow \exists i, j \in N((v \in C^i)\&(z \in C^j)\&(i \leqslant j)));$$

Also hold the following properties of matching these relations:
1) $wR_ez \Leftrightarrow (wR_{\leqslant}z)\&(zR_{\leqslant}w)$;
2) $wR_iz \Rightarrow wR_ez$.
We denote class of all such frames LTK.

Definition 2.4. *For two $R_{\leqslant}$-clusters C^m and C^j notation $C^m R_t C^j$ indicates that $\forall w \in C^m, \forall z \in C^j$ is performed $(wR_{\leqslant}z)$. Thus, C^m is $R_{\leqslant}$-precursor of cluster C^j, and C^j is $R_{\leqslant}$-follower of cluster C^m.*

Frames of this class model a situation in which each agent has all the information in the current temporary state C^t. Any temporary state C^t (i.e $R_{\leqslant}$-cluster) consists of a set of information points available at t. The relation $R_{\leqslant}$ is a connection into a linear stream of information points, wherein for two points w and z term $wR_{\leqslant}z$ means that either w and z are available at the time t, or z will be available at subsequent times in relation to w. Relation R_e connects all information points potentially available at the same moment of time, thus it represents knowledge that is potentially available at any given time. Each relation R_i, $i = 1, ..., n$, reflects the information available to a particular agent i.

Definition 2.5. *Model M_F on a LTK-frame F is a tuple $M_F = \langle F, V\rangle$, where V is a valuation of a set of propositional letters $p \in P$ on the frame, i.e $\forall p \in P$ $[V(p) \subseteq W_F]$. Given a model $M_F = \langle F, V\rangle$, where F is a LTK-frame W_F. Then $\forall w \in W_F$:*
a) $\langle F, w\rangle \Vdash_V p \Leftrightarrow w \in V(p)$;
b) $\langle F, w\rangle \Vdash_V \Box_{\leqslant}A \Leftrightarrow \forall z \in W_F(wR_{\leqslant}z \Rightarrow \langle F, z\rangle \Vdash_V A)$;
c) $\langle F, w\rangle \Vdash_V \Box_e A \Leftrightarrow \forall z \in W_F(wR_ez \Rightarrow \langle F, z\rangle \Vdash_V A)$;
d) $\forall i \in I, \langle F, w\rangle \Vdash_V \Box_i A \Leftrightarrow \forall z \in W_F(wR_iz \Rightarrow \langle F, z\rangle \Vdash_V A)$.

The relation $\Vdash_V$ here means truth relation on the element w of model M. Namely, $\langle F, w\rangle \Vdash_V A$ means that A true on the element w in model $\langle F, V\rangle$. If the formula A true on any element of the frame F with any valuation V, we called A true on the frame F and write $F \Vdash A$.

Definition 2.6. *The logic LTK is the set of all LTK-valid formulae on all frames: $LTK := \{A \in Fma(L^{LTK}) | \forall F \in LTK(F \Vdash A)\}$. If A belongs to LTK, then we say that A is a theorem of LTK.*

3. A criterion of non-unifiability

We immediately begin with the proof of the main statement of this section.

Theorem 3.1. *Any modal formula A is non-unifiable in LTK iff formula*

$$\Box_{\leqslant}A \to \left[\bigvee_{p \in Var(A)} \Diamond_{\leqslant}p \wedge \Diamond_{\leqslant}\neg p\right]$$

is a theorem in LTK.

Proof. 1. Prove the theorem by contradiction. Assume that

$$\Box_{\leqslant} A \to \left[\bigvee_{p\in Var(A)} \Diamond_{\leqslant} p \wedge \Diamond_{\leqslant} \neg p\right] \in LTK,$$

but at the same time, the formula A is unifiable in LTK.

Then by definition of unifier, there is a substitution (unifier) g s.t. $g(A) \in LTK$. By the fact that LTK is closed under substitution, we obtain $g\left(\Box_{\leqslant} A \to \left[\bigvee_{p\in Var(A)} \Diamond_{\leqslant} p \wedge \Diamond_{\leqslant} \neg p\right]\right) \in LTK$.

Let us consider LTK-frame F_1 with all single element clusters (i.e $\forall t : C^t = a$). Consider the valuation V for all variables q of formulas $g(p)$, where $p \in Var(A)$, on the $F_1 : V(q) = \oslash$. Then it is easy to check by the induction on the length of any formula B constructed on variables q that:

$$\forall b \in F_1, \forall c \in F_1 : b \Vdash_V B \Leftrightarrow c \Vdash_V B.$$

Consequently,

$$\forall b \in F_1 : b \nVdash_V \bigvee_{p\in Var(A)} \Diamond_{\leqslant} g(p) \wedge \Diamond_{\leqslant} \neg g(p).$$

At the same time,

$$\forall b \in F_1 : b \Vdash_V \Box_{\leqslant} g(A).$$

Thereby,

$$\forall b \in F_1 : b \nVdash_V g\left(\Box_{\leqslant} A \to \left[\bigvee_{p\in Var(A)} \Diamond_{\leqslant} p \wedge \Diamond_{\leqslant} \neg p\right]\right),$$

which contradicts the hypothesis:

$$g\left(\Box_{\leqslant} A \to \left[\bigvee_{p\in Var(A)} \Diamond_{\leqslant} p \wedge \Diamond_{\leqslant} \neg p\right]\right) \in LTK.$$

2. On the contrary, say that the formula A is non-unifiable in LTK, but at the same time $\Box_{\leqslant} A \to \left[\bigvee_{p\in Var(A)} \Diamond_{\leqslant} p \wedge \Diamond_{\leqslant} \neg p\right] \notin LTK$. Then, by finitary approximability of LTK, there is a certain root frame F that disproves this formula:

$$\exists a \in F : \langle F, a\rangle \nVdash_V \Box_{\leqslant} A \to \left[\bigvee_{p\in Var(A)} \Diamond_{\leqslant} p \wedge \Diamond_{\leqslant} \neg p\right].$$

That is $\langle F, a\rangle \Vdash_V \Box_{\leqslant} A$ и $\langle F, a\rangle \nVdash_V \left[\bigvee_{p\in Var(A)} \Diamond_{\leqslant} p \wedge \Diamond_{\leqslant} \neg p\right]$. Assume this element a as the root of the frame F_1 ($F_1 = a^{\leqslant}$). By $\langle F, a\rangle \nVdash_V \left[\bigvee_{p\in Var(A)} \Diamond_{\leqslant} p \wedge \Diamond_{\leqslant} \neg p\right], \forall p \in Var(A)$: either

$$(1)\ \forall b \in F_1 a R_{\leqslant} b : b \Vdash_V p,$$

or

$$(2)\ \forall b \in F_1 a R_{\leqslant} b : b \nVdash_V p.$$

Choose a substitution g for all of the variables p from the formula A as follows: $\forall p \in Var(A)$: $g(p) = \top$ if (1) and $g(p) = \bot$ in the case of (2). Then g is a unifier of the formula A. Indeed, if we take any frame F_2, any cluster $a_2 \in F_2$ and any valuation V_2:

$$a_2 \Vdash_{V_2} A \Leftrightarrow a \Vdash_V A.$$

Therefore, the formula A is unifiable in LTK. □

4. Passive inference rules

Definition 4.1. *Let $r := A_1, ..., A_n/\beta$ be an inference rule in the logic LTK. The rule r called passive for LTK if for any substitution g of formulas instead of variables in r never $g(A_1) \in LTK \& \dots \& g(A_n) \in LTK$. In other words r is a passive rule if formulas from its premise have no common unifiers.*

Proposition 4.2. *For multi-modal logic LTK the rules $r_n := \dfrac{\bigvee_{1\leqslant i\leqslant n} \Diamond_{\leqslant} p_i \wedge \Diamond_{\leqslant} \neg p_i}{\perp}$ form a basis for all passive rules for LTK.*

Proof. It is true that $\Box_{\leqslant} \bigvee_{1\leqslant i\leqslant n} \Diamond_{\leqslant} p_i \wedge \Diamond_{\leqslant} \neg p_i \to \left[\bigvee_{1\leqslant i\leqslant n} \Diamond_{\leqslant} p \wedge \Diamond_{\leqslant} \neg p\right] \in LTK$, and hence by Theorem 3.1 formula $A_n = \bigvee_{1\leqslant i\leqslant n} \Diamond_{\leqslant} p_i \wedge \Diamond_{\leqslant} \neg p_i$ does not unifiable in modal logic LTK, i.e any rule r_n is passive. Let us assume that a rule $t_1 := A_1, \dots, A_n/\beta$ is passive for LTK. Then the rule $t_2 := A_1 \wedge \dots \wedge A_n/\beta$ is also passive for LTK and formula $A_1 \wedge \dots \wedge A_n$ is not unifiable in LTK. Applying Theorem 4.1 we conclude

$$\Box_{\leqslant}(A_1 \wedge \dots \wedge A_n) \to \left[\bigvee_{p\in Var(A_1\wedge\dots\wedge A_n)} \Diamond_{\leqslant} p \wedge \Diamond_{\leqslant} \neg p\right] \in LTK.$$

Using the premise of rule t_2 we conclude

$$\bigvee_{p\in Var(A_1\wedge\dots\wedge A_n)} \Diamond_{\leqslant} p \wedge \Diamond_{\leqslant} \neg p$$

and then applying the rule r_n, where n is the number of variables in the conjunction of $A_1 \wedge \dots \wedge A_n$, we can derive the formula $\perp$. From $\perp \to \beta \in LTK$, in its turn holds β. Thus, all r_n really represent all passive rules in LTK. □

Now we consider the possibility of reduction infinite (due to an unlimited number of variables) basis of passive rules in LTK that was obtained in the Proposition 4.1 to a finite and more simple form.

Let us remind that the rule r is a consequence of the rules $r_i \in X$, $i \in I$ in a logic L, if for any algebra $A \in Var(L)$ and $\forall i \in I$: $A \vDash r_i \Rightarrow A \vDash r$. Accordingly, a rule r is not a consequence of the rules $r_i \in X$, $i \in I$ otherwise. A rule r true in the algebra A if for any substitution of elements from algebra instead of the variables of a rule r if all formulas from the premise of a rule r is true, then a conclusion formula of r is also true.

Theorem 4.3. *In multi-modal logic LTK the rule $r := \dfrac{\Diamond_{\leqslant} p \wedge \Diamond_{\leqslant} \neg p}{\perp}$ is a basis for all passive inference rules*

Proof. According to Proposition 4.2, it suffices to show that the rules r_n $(\forall n)$ are a consequence of r $(r \vdash r_n, \forall n)$.

Suppose that it is not true:

$$r_n := \frac{\bigvee_{1\leqslant i\leqslant n} \Diamond_{\leqslant} p_i \wedge \Diamond_{\leqslant} \neg p_i}{\perp}$$

is not a consequence of the rule r. Hence there is a finitely generated algebra A, in which the rule r is valid $(A \vDash r)$, but r_n is not $(A \nvDash r_n)$, thus $\forall i \in (1, \dots, n)$ there is $a_i \in A : \bigvee_{1\leqslant i\leqslant n} \Diamond_{\leqslant} a_i \wedge \Diamond_{\leqslant} \neg a_i = \top$. Get a subalgebra A_1 of algebra A generated by such elements a_i, $1 \leqslant i \leqslant n$,

$(A_1 = A_1(a_1, \dots, a_n) \subseteq A)$. A_1 is a $S4.3$-algebra on $\Box_{\leqslant}$. By Lemma 4.3.18 from [10] Kripke-frame A_1^+, associated to A_1 has a single element reflexive maximal cluster C. By the definition A_1^+, $\forall a \in A_1$, $a \subseteq A_1^+$. By hypothesis of proof, $\bigvee_{1\leqslant i\leqslant n} \Diamond_{\leqslant} a_i \wedge \Diamond_{\leqslant} \neg a_i \in A_1$, because A_1 is a subalgebra A, on the construction. Then $\bigvee_{1\leqslant i\leqslant n} \Diamond_{\leqslant} a_i \wedge \Diamond_{\leqslant} \neg a_i = \top = A_1^+$, but it is impossible on a single element reflexive maximal cluster ($C \nVdash \bigvee_{1\leqslant i\leqslant n} \Diamond_{\leqslant} a_i \wedge \Diamond_{\leqslant} \neg a_i$), and hence $\bigvee_{1\leqslant i\leqslant n} \Diamond_{\leqslant} a_i \wedge \Diamond_{\leqslant} \neg a_i \notin A_1^+$ that contradict with the proof conditions. $\Box$

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Унификация и правила вывода в многомодальной логике знания и линейного времени LTK

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В статье исследуется унификация формул в многомодальной логике LTK и предложено синтаксическое описание всех формул, которые не являются унифицируемыми в данной логике. Рассмотрен вопрос пассивных правил вывода, показано, что в логике LTK есть конечный базис для пассивных правил.

Ключевые слова: унификация, модальная темпоральная логика, пассивные правила вывода.