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Filtration of Two Immiscible Liquids in a Viscoelastic Porous Medium

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Abstract. Governing equations for the motion of two immiscible fluids in a poroelastic skeleton are obtained within the framework of the theory of interacting continua. The stability of the steady-state solution of the system is investigated.

Keywords: poroelasticity, two-phase filtration, Darcy's law, stability, viscoelasticity.

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Introduction

The problem of global warming is one of the most important modern scientific problems. The emission of CO2 is one of the causes leading to global changes in the Earth's climate.

Geological storage of carbon dioxide in deep geological formations is considered a key transition method for reducing greenhouse gas emissions into the atmosphere and, therefore, their feedback on the climate. Such method has been used for several decades in applications related to enhanced oil recovery. A number of industrial, demonstration and pilot projects are underway, and the processes and techniques associated with geological carbon dioxide storage have been theoretically and experimentally studied. Deep saline formations are geological units that are estimated to have the highest storage potential due to their worldwide distribution. Methods for modelling and monitoring CO2 storage in such formations are rapidly developing in many parts of the world. The basic assumption underlying the modelling of such processes is that after CO2 injection, the void space within the formation is occupied by two fluids: natural brine and injected CO2 [1].

Two-phase models are also applied to describe CO2 sequestration in producing gas fields. In [2], CO2 sequestration scenarios through three injection wells in a producing gas field located in the river Po sedimentary basin (Italy) are modeled with the ultimate goal of understanding the geomechanical consequences of CO2 injection. The process is analyzed from a geomechanical point of view, with the following main issues being addressed: prediction of possible vertical uplift of the earth and the corresponding impact on the surface infrastructure; assessment of the stress state induced in the reservoir with possible formation of fractures and analysis of the risk of activation of existing faults.

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In the paper [3] a poromechanical model is developed to determine how chemical carbonation reactions can affect the mechanical behavior of well cement in the context of CO2 storage. A multiphase model is also considered, in which the pore fluid consists of dissolved components and a solvent (water).

Thus, the mathematical model of two-phase filtration in a poroelastic medium is quite relevant and describes well the processes of CO2 storage. A large number of works are devoted to mathematical modeling of the process of carbon dioxide burial under various conditions. Most of the known models do not take into account the variable porosity of the solid skeleton. Usually porosity is a given function or is assumed to be constant. However, taking into account variable porosity seems important, since it can lead to the detection of cracks and the release of CO2 to the surface during burial. The mathematical model we consider takes into account the compressibility of the solid skeleton and its poroelastic properties, i.e. variable porosity.

Work using variable porosity has been conducted since the 1920s. A relationship was discovered between the burial depth of sedimentary rocks and porosity. In particular, there is an exponential dependence of porosity on depth [4]. One of the first tools for constructing models of poroelastic media was the Terzaghi effective stress concept, which takes into account the mobility of the skeleton and its poroelastic properties [5]. Further, the theory of poroelasticity was developed in the works of Bio [6], where porosity was also a function of effective pressure. Porosity depended on pressure (but the deformation of the porous skeleton was not considered) in [7]. A model of two-phase filtration in a deformable porous medium was proposed in [8], in which the motion of a solid skeleton was described based on an analogue of Terzaghi's principle and a modified linear Hooke's law. The justification issues were not considered in this work. This was done in works [9, 10], where particular solutions were constructed in models of zero and first approximations. In the case of single-phase filtration in a deformable porous medium, the mathematical theory of the process was constructed in works [11–13].

1. Governing equations

We consider a system of differential equations describing the motion of two immiscible fluids in a deformable viscoelastic medium. The continuity equations for each phase, taking into account variable porosity in the absence of phase transitions, are as follows [15]:

$$\begin{gathered}\frac{\partial(\rho_1 s_1 \phi)}{\partial t} + \nabla\cdot(\rho_1\phi s_1\vec{v}_1) = 0, \quad \frac{\partial(\rho_2 s_2 \phi)}{\partial t} + \nabla\cdot(\rho_2\phi s_2\vec{v}_2) = 0, \quad s_2 + s_1 = 1, \\ \frac{\partial(1-\phi)\rho_3}{\partial t} + \nabla\cdot((1-\phi)\rho_3\vec{v}_3) = 0.\end{gathered} \tag{1}$$

Here $\rho_1, \rho_2, \rho_3, \vec{v}_1, \vec{v}_2, \vec{v}_3$ are true phase densities and velocities, respectively (1 is the wetting fluid, 2 is the non-wetting fluid, 3 is the solid deformable skeleton), s_1, s_2 are fluid saturations, ϕ is the porosity.

Instead of the equations of conservation of momentum in the theory of two-phase filtration, a generalized Darcy law for liquid phases is used, taking into account the motion of a solid skeleton [16, 17]:

$$s_1\phi(\vec{v}_1 - \vec{v}_3) = -K_0(\phi)\frac{k_{01}(s_1)}{\mu_1}(\nabla p_1 - \rho_1\vec{g}), \tag{2}$$

$$s_2\phi(\vec{v}_2 - \vec{v}_3) = -K_0(\phi)\frac{k_{02}(s_2)}{\mu_2}(\nabla p_2 - \rho_2\vec{g}), \tag{3}$$

where p_1, p_2 are fluid pressures, $k_{01}(s_1), k_{02}(s_2)$ are permeabilities, μ_1, μ_2 are dynamic viscosities, $\vec{g}$ is the acceleration vector of gravity. Taking into account capillary forces means that the phase pressures p_2 and p_1 differ by the magnitude of the capillary jump: $p_2 - p_1 = p_c(s_1)$, $p_c(s_1)$ is the capillary pressure (is a given function).

The system of equations (1)–(3) with respect to the sought functions of pressures, phase velocities and saturations of immiscible liquids moving in a non-deformable porous medium, in the isothermal case (the temperature in the flow is constant) is closed either by the assumption of incompressibility of liquids, i.e. the densities are assumed to be constant, or by an equation of state relating the densities and pressures of the phases.

The resulting mathematical model in the case of a stationary porous medium $\vec{v}_3 = 0$ is called the Musket–Leverett model (in the case of the absence of a capillary jump — the Buckley–Leverett model). The mathematical theory of the process for the Musket–Leverett model was justified in the monograph [18].

The fundamental point is to take into account the compressibility of the porous medium. Work using variable porosity began in the 1920s in connection with attempts to mathematically model filtration processes in sedimentary rocks [4]. At first, simple dependences of porosity on depth were used (see review in [19]]), obtained on the basis of experimental data. Then more complex dependences appeared for porosity through effective pressure [5], which, according to Terzaghi's concept, is defined as the difference between the total pressure and the fluid pressure. This position reflects the fact that the fluid bears part of the load. In this approach, the relationship between the deformation of the skeleton of the solid matrix and the processes of fluid flow is fundamental. Experimental data on unknown porosity are contained in the works of [20, 21].

The Maxwell-type relationship between porosity and effective pressure p_e is as follows [22–24]:

$$\nabla \cdot \vec{v}_3 = -(\alpha(\phi) p_e + \beta(\phi) \frac{dp_e}{dt}), \tag{4}$$

where $\alpha(\phi), \beta(\phi)$ are given functions that depend on porosity (parameters of the medium that are responsible for viscosity and elasticity, respectively), $\frac{d}{dt} = \frac{\partial}{\partial t} + (\vec{v}_3 \cdot \nabla)$ is the material derivative. The effective pressure p_e and the pressures in the liquid phases p_1, p_2 and the solid p_3 phases are related by the relations:

$$p_{tot} = \phi p_f + (1-\phi) p_3, \quad p_e = (1-\phi)(p_3 - p_f), \quad p_f = s_1 p_1 + s_2 p_2. \tag{5}$$

The balance equation of forces for the system as a whole has the form [22, 23, 25]:

$$\nabla p_{tot} = \rho_{tot} \vec{g} + \nabla \cdot \left((1-\phi)\eta \left(\frac{\partial \vec{v}_3}{\partial \vec{x}} + \left(\frac{\partial \vec{v}_3}{\partial \vec{x}} \right)^* \right) \right), \quad \rho_{tot} = \phi \rho_f + (1-\phi)\rho_3, \quad \rho_f = s_1 \rho_1 + s_2 \rho_2, \tag{6}$$

where p_{tot} is the total pressure, ρ_{tot} is the total density, η is the viscosity of the porous skeleton, $*$ is the symbol for the transposition operation. Here, the approach is used in which the deviator of the stress tensor in the liquid phase is neglected, because the viscosity of the liquid is much smaller than the shear viscosity of the skeleton.

Thus, the system of equations (1)–(6), describing the motion of two immiscible liquids in a deformable porous medium, takes the form [14]:

$$\begin{gathered} \frac{\partial(\rho_1 s_1 \phi)}{\partial t} + \nabla \cdot (\rho_1 \phi s_1 \vec{v}_1) = 0, \quad \frac{\partial(\rho_2 s_2 \phi)}{\partial t} + \nabla \cdot (\rho_2 \phi s_2 \vec{v}_2) = 0, \\ \frac{\partial(1-\phi)\rho_3}{\partial t} + \nabla \cdot ((1-\phi)\rho_3 \vec{v}_3) = 0, \end{gathered} \tag{7}$$

$$\begin{aligned} s_1\phi(\vec{v}_1-\vec{v}_3) &= -K_0(\phi)\frac{k_{01}(s_1)}{\mu_1}(\nabla p_1-\rho_1\vec{g}),\\ s_2\phi(\vec{v}_2-\vec{v}_3) &= -K_0(\phi)\frac{k_{02}(s_2)}{\mu_2}(\nabla p_2-\rho_2\vec{g}), \end{aligned} \tag{8}$$

$$\nabla\cdot\vec{v}_3 = -(\alpha(\phi)p_e+\beta(\phi)\frac{dp_e}{dt}),\quad \frac{d}{dt}=\frac{\partial}{\partial t}+(\vec{v}_3\cdot\nabla), \tag{9}$$

$$p_{tot}=\phi p_f+(1-\phi)p_3,\quad p_e=(1-\phi)(p_3-p_f),\quad p_2-p_1=p_c(s_1), \tag{10}$$

$$\nabla p_{tot}=\rho_{tot}\vec{g}+div\left((1-\phi)\eta\left(\frac{\partial\vec{v}_3}{\partial\vec{x}}+\left(\frac{\partial\vec{v}_3}{\partial\vec{x}}\right)^*\right)\right),\quad \rho_{tot}=\phi\rho_f+(1-\phi)\rho_3. \tag{11}$$

This model is quite complex for investigation, relatively new and has not been studied in sufficient detail. In the paper [26] a similar system of equations is investigated, for which some exact solutions are obtained in the thin layer approximation in the model case. In the paper [14] the solvability of the model problem in the Hele–Shaw cell approximation for the equations (7)–(11) is established. In the one-dimensional case for the system (7)–(11) at a constant temperature and single-phase filtration, the dependence of the liquid phase density on the pressure and in the absence of phase transitions, local solvability is established in [11]. With constant densities, global solvability is proved in [12]. In the papers [27, 28] the problems of two-phase filtration in a deformable medium with known porosity are considered. The purpose of this work is to study the stability of the stationary solution of the general system of equations (7)–(11).

2. A study of the stability of the problem of the motion of two immiscible fluids in a poroelastic medium

2.1. Steady-state solution of the system

To find an analytical solution to the system (7)–(11) we will use the following hypotheses:

•fluids and solid skeletons are incompressible, that is, $\rho_i^0 = const\,(i=1,2,3)$;

• gravity acceleration and capillary jump are equal to zero: $\overrightarrow{g}=0,\quad p_c=0$.

We consider a stationary solution in which the phase velocities are zero ($\vec{v}_1=\vec{v}_2=\vec{v}_3=0$), and the porosity and saturation are constant:

$$\phi=\phi^0,\quad s_1=s_1^0,\quad s_2=s_2^0,\quad (\phi^0,s_1^0,s_2^0)\in(0,1).$$

From the equation (11) it follows that $p_{tot}=h=const$.

From the absence of a capillary jump it follows that $p_1=p_2$.

Under these assumptions, equations (7)–(11) are satisfied automatically. From equation (9) it follows that $p_e=0$. From the equation for effective pressure: $p_e=p_{tot}-p_f$ we establish that $p_{tot}=p_f=p_1=p_2=p_3=h$.

Thus, the steady-state solution has the form:

$$s_1=s_1^0,\quad s_2=s_2^0,\quad \vec{v}_1=\vec{v}_2=\vec{v}_3=0,\quad \phi=\phi^0,\quad p_1=p_2=p_3=h.$$

2.2. Perturbed solution

The perturbed solution of the system (7)–(11) is sought in the vicinity of the stationary one and has the following form [29]:

$$\vec{v}_1 = \vec{\bar{v}}_1, \quad \vec{v}_2 = \vec{\bar{v}}_2, \quad \vec{v}_3 = \vec{\bar{v}}_3, \quad s_1 = s_1^0 + \bar{s}_1, \quad s_2 = s_2^0 + \bar{s}_2,$$
$$\phi = \phi^0 + \bar{\phi}, \quad s_1^0 + s_2^0 = 1, \quad \bar{s}_1 + \bar{s}_2 = 0,$$
$$p_1 = \bar{p}_1 + h, \quad p_2 = \bar{p}_2 + h, \quad p_3 = \bar{p}_3 + h, \quad \bar{p}_2 = \bar{p}_1.$$

where the functions $\overline{v_3}, \overline{v_1}, \overline{v_2}, \overline{p_1}, \overline{p_2}, \overline{p_3}, \overline{s_1}, \overline{s_2}, \overline{\phi}$ are small and have continuous derivatives. The functional parameters $K_0(\phi), k_{01}(s_1), k_{02}(s_2)$ can be represented as:

$$K_0(\phi) = K_0(\phi^0) + K_0'(\phi^0)\overline{\phi},$$
$$k_{01}(s_1) = k_{01}(s_1^0) + k_{01}'(s_1^0)\overline{s_1},$$
$$k_{02}(s_2) = k_{02}(s_2^0) + k_{02}'(s_2^0)\overline{s_2},$$
$$\alpha(\phi) = \alpha(\phi^0) + \alpha'(\phi^0)\bar{\phi}, \quad \beta(\phi) = \beta(\phi^0) + \beta'(\phi^0)\bar{\phi}.$$

Substituting the perturbed solution into the system (7)–(11) and discarding the nonlinear terms, we arrive at the following linear system (for convenience, we omit the dashes from above):

$$\frac{\partial(1-\phi)}{\partial t} + (1-\phi^0)\nabla\cdot\vec{v}_3 = 0, \tag{12}$$

$$\phi^0\frac{\partial(s_1)}{\partial t} + s_1^0\frac{\partial(\phi)}{\partial t} + \phi^0 s_1^0\nabla\cdot\vec{v}_1 = 0, \tag{13}$$

$$\phi^0\frac{\partial(s_2)}{\partial t} + s_2^0\frac{\partial(\phi)}{\partial t} + \phi^0 s_2^0\nabla\cdot\vec{v}_2 = 0, \tag{14}$$

$$s_1^0\phi^0(\vec{v}_1 - \vec{v}_3) = -K_0(\phi^0)\frac{k_{01}(s_1^0)}{\mu_1}\nabla p_1, \tag{15}$$

$$s_2^0\phi^0(\vec{v}_2 - \vec{v}_3) = -K_0(\phi^0)\frac{k_{02}(s_2^0)}{\mu_2}\nabla p_2, \tag{16}$$

$$\nabla\cdot\vec{v}_3 = (1-\phi^0)\left(\alpha(\phi^0)(p_3 - p_1) + \beta(\phi^0)\frac{\partial(p_3 - p_1)}{\partial t}\right), \tag{17}$$

$$(1-\phi^0)\nabla p_3 + \phi^0\nabla p_1 = \eta(1-\phi^0)(\Delta\vec{v}_3 + \nabla(\nabla\cdot\vec{v}_3)). \tag{18}$$

To find $\vec{v}_3$ we add the continuity equations (12)–(14). We get:

$$\nabla\cdot\vec{v}_3 = -\frac{\phi^0}{1-\phi^0}\left(s_1^0\nabla\cdot\vec{v}_1 + s_2^0\nabla\cdot\vec{v}_2\right). \tag{19}$$

After adding the equations (15), (16), and apply the *div* operator to both parts of the resulting equality we get:

$$\nabla\cdot(\phi^0 s_1^0\vec{v}_1 + s_2^0\phi^0\vec{v}_2) - \phi^0\nabla\cdot\vec{v}_3 = -K_0(\phi^0)\left(\frac{k_{01}(s_1^0)}{\mu_1} + \frac{k_{02}(s_2^0)}{\mu_2}\right).$$

Taking into account the relation (19), we obtain

$$\nabla\cdot\vec{v}_3 = K_0(\phi^0)\left(\frac{k_{01}(s_1^0)}{\mu_1} + \frac{k_{02}(s_2^0)}{\mu_2}\right)\Delta p_1. \tag{20}$$

Taking the *div* operator to both parts of the equation (18) and, taking into account the previous equality, we obtain

$$(1-\phi^0)\Delta p_3 = 2\eta\tilde{K}(1-\phi^0)\Delta^2 p_1 - \phi^0\Delta p_1. \tag{21}$$

Equation (17) taking into account (20) will take the form

$$\tilde{K}\Delta p_1 = (1-\phi^0)\Big(\alpha(\phi^0)(p_3-p_1)+\beta(\phi^0)\frac{\partial(p_3-p_1)}{\partial t}\Big),$$

where $\tilde{K} = K_0(\phi^0)\left(\frac{k_{01}(s_1^0)}{\mu_1}+\frac{k_{02}(s_2^0)}{\mu_2}\right)$. Taking the operator Δ to the previous equation, we get

$$\tilde{K}\Delta^2 p_1 = \alpha(\phi^0)((1-\phi^0)\Delta p_3)-\alpha(\phi^0)(1-\phi^0)\Delta p_1+\beta(\phi^0)\frac{\partial}{\partial t}((1-\phi^0)\Delta p_3)-\beta(\phi^0)(1-\phi^0)\frac{\partial}{\partial t}(\Delta p_1).$$

Taking into account equation (21), we have the equation for p_1

$$\frac{\partial}{\partial t}(\Delta p_1) - A\Delta^2 p_1 - B\frac{\partial}{\partial t}(\Delta^2 p_1) + C\Delta p_1 = 0, \tag{22}$$

where

$$A=\tilde{K}\frac{2\alpha(\phi^0)\eta(1-\phi^0)-1}{\beta(\phi^0)},\ B=2\eta\tilde{K}(1-\phi^0),\ C=\frac{\alpha(\phi^0)}{\beta(\phi^0)},\ \tilde{K}=K_0(\phi^0)\Big(\frac{k_{01}(s_1^0)}{\mu_1}+\frac{k_{02}(s_2^0)}{\mu_2}\Big).$$

Let us describe the scheme for finding all the desired functions. After finding p_1 from the equation (22) we find p_2, since $p_c = 0$ and, therefore, $p_1 = p_2$. We also obtain $div v_3$ from (20). We find p_3 from (21), and then we find v_3 from (18). We can find v_1 and v_2 from (15), (16), and ϕ from (12). From (13) we find s_1, and, therefore, s_2, since $s_1 + s_2 = 1$.

We now seek a plane wave solution of the form [30]

$$p_1 = \hat{p}_1 \exp(st)\exp(i\mathbf{k}\cdot\mathbf{x}),\quad s = \xi - i\bar{\eta},$$

where $\mathbf{k}$ is the wave vector of the plane wave, $\bar{\eta}$ is related to the velocity of propagation V by $V = \bar{\eta}/|\mathbf{k}|$, where $|\mathbf{k}|$ is the wave number.

Substituting this representation into (22), we obtain

$$\hat{p}_1 k^2(s + Ak^2 + Bsk^2 + C) = 0.$$

The solutions $\hat{p}_1 = 0$ represent transverse waves. We also have that

$$\xi = -\frac{k^2\tilde{K}(2\alpha(\phi^0)\eta(1-\phi^0)-1)+\alpha(\phi^0)}{\beta(\phi^0)(1+2\eta\tilde{K}(1-\phi^0)k^2)}. \tag{23}$$

From this equation we obtain the relationship between the degree of growth of the harmonic ξ perturbations and its wavelength (wave number $|\mathbf{k}| = 2\pi/\lambda$). For $\xi > 0$ the perturbations grow exponentially and, therefore, the initial solution is unstable, for $\xi < 0$ the perturbations decay and the solution is stable. It is easy to see that $\xi > 0$ for $|\mathbf{k}| \in (0, k_c)$, where

$$k_c = \left(\frac{\alpha(\phi^0)}{\tilde{K}(1-2\alpha(\phi^0)\eta(1-\phi^0))}\right)^{1/2},$$

if the condition $1 > 2\alpha\eta(1-\phi^0)$ is satisfied.

Note that in the absence of viscosity in the skeleton and the prevalence of its elastic properties, i.e., when $\alpha = 0$, we have unstable perturbations, since it is easy to see from the equality (23) that $\xi > 0$ for any initial data of the equations. In the presence of viscosity and when the condition is satisfied

$$1 < 2\eta\alpha(1 - \phi^0) \tag{24}$$

the process will be stable, since there are no real k_c. Therefore, viscosity can stabilize the process under consideration. In the absence of skeleton elasticity ($\beta = 0$) we have $k_c = \infty$ and the solution is always unstable. Therefore, elasticity also contributes to the stabilization of the process. In other words, the process will be stable if the (24) condition is met and the skeleton has viscoelastic properties.

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Фильтрация двух несмешивающихся жидкостей в вязкоупругой пористой среде

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Аннотация. В рамках теории взаимодействующих континуумов получены определяющие уравнения для движения двух несмешивающихся жидкостей в пороупругом скелете. Исследована устойчивость стационарного решения системы.

Ключевые слова: пороупругость, двухфазная фильтрация, закон Дарси, устойчивость, вязкоупругость.