EDN: JZZUNK
УДК 539.3

Solving Cauchy Problem for Elasticity Equations in a Plane Dynamic Case

Sergei I. Senashov*
Irina L. Savostyanova†
Reshetnev Siberian State University of Science and Technology
Krasnoyarsk, Russian Federation
Olga N. Cherepanova‡
Siberian Federal University
Krasnoyarsk, Russian Federation

Received 05.09.2024, received in revised form 24.10.2024, accepted 04.11.2024

Abstract. Equations of elasticity in a plane dynamic case are considered in this paper. The system of equations is replaced by system of first-order differential equations with the same solution. The solution-equivalent system is group fibration of the original system of equations. It is a combination of the resolving and automorphic systems. Special classes of conservation laws are found for the resolving system of equations. These laws allow one to find the solution of the original equations in the form of surface integrals over the boundary of an elastic body.

Keywords: equations of elasticity in a plane dynamic case, Cauchy problem, conservation laws, exact solutions.

Citation: S.I. Senashov, I.L. Savostyanova, O.N. Cherepanova, Solving Cauchy Problem for Elasticity Equations in a Plane Dynamic Case, J. Sib. Fed. Univ. Math. Phys., 2025, 18(1), 71–80. EDN: JZZUNK.

Introduction

Equations of linear elasticity theory were presented in the works of A.Cauchy, L. Navier, B. Saint–Venant and others as early as in the 19 century. Since then, attempts have been made to build solutions of the initial and boundary value problems. General solutions for equations of elasticity theory in a dynamic were built by G. Lame, P. F. Papkovich, H. Neuber, M. Yakovak, N. I. Ostrosablin and some others [1–3]. But according to the words of S. L. Sobolev " ...the knowledge of general solutions, with rare exception, gives nothing for solving important particular problems, ..., because we get, while solving these particular problems, a system of so complex functional relations for arbitrary functions that their finding is practically impossible [4]". To solve the elasticity theory problems a greate variety of contemporary mathematical methods are used. Thus, methods of group analysis of differential equations were used [5–8 and the references therein]. The theory of symmetries allowes one to build vast classes of invariant and partially-invariant solutions which describe stress-strain state of elastic medium.
Symmetries, by virtue of their locality, are not appropriate for solving initial and boundary value

*sen@mail.sibsau.ru
†ruppa@inbox.ru https://orcid.org/0000-0002-9675-7109
‡cheronik@mail.ru

problems. Here, conservation laws are more suitable for solving boundary value problems [9, 10 and the references therein]. In fact, conservation laws were used for solving linear equations by B. Riemann and V. Volterra [11]. It is known [12] that equations of elasticity theory can be presented with the use of group fibration in the form of a combination of two solution-equivalent systems of first-order differential equations: resolving system and automorphic system. This fact turned out to be very useful for constructing conservation laws and solving Cauchy problems with their use.
In this article the conservation laws are built for the resolving system of differential equations of elasticity theory which gave an opportunity to solve Cauchy problem for this system in the form of surface integrals over the boundary of an elastic body. Further, Cauchy problem for the automorphic system is solved. This allows one to build the solution of the initial problem for the equations of elasticity theory in a dynamic case.

1. Preliminaries

Let us consider the equations of elasticity in a plane case

$$\begin{aligned} w^1_{tt} &= (\lambda+2\mu)w^1_{xx} + \mu w^1_{yy} + (\lambda+\mu)w^2_{xy}, \\ w^2_{tt} &= (\lambda+2\mu)w^2_{yy} + \mu w^1_{xx} + (\lambda+\mu)w^1_{xy}, \end{aligned} \tag{1}$$

where λ, μ are Lame constants, w^1, w^2 are components of displacement vector, density is equal to one. On the plane $t = 0$, the Cauchy problem is set

$$\begin{aligned} w^1|_{t=9} &= f^1(x,y), w^2|_{t=9} = f^2(x,y). \\ w^1_t|_{t=9} &= g^1(x,y), w^2_t|_{t=9} = g^2(x,y). \end{aligned} \tag{2}$$

If functions f^i, g^i are continuous together with their derivatives on the plane $t = 0$ then all derivatives of functions w^1, w^2 in any direction are known on this plane. It is known that system of equations (1) is of hyperbolic type and it has characteristic surfaces defined as $\omega(t,x,y) = 0$ which satisfy the following equation [10]

$$[(\lambda+2\mu)(\omega_x^2+\omega_y^2) - \omega_t^2][\mu(\omega_x^2+\omega_y^2) - \omega_t^2] = 0. \tag{3}$$

It is known [4, 5] that system of equations (1) allows a group of point symmetries generated by operators

$$\begin{aligned} &X_1 = \partial_x, \ \ X_2 = \partial_y, \ \ X_0 = \partial_t, \\ &Z = y\partial_x - x\partial_y + w^2\partial_{w^1} - w^1\partial_{w^2}, \\ &P_0 = w^1\partial_{w^1} + w^2\partial_{w^2}, \ \ P_w = h^1\partial_{w^1} + h^2\partial_{w^2}, \ \ R = x\partial_x + y\partial_y + t\partial_t, \end{aligned} \tag{4}$$

where h^1, h^2 — arbitrary solution of equations (1). The presence of operator $P_w = h^1\partial_{w^1} + h^2\partial_{w^2}$ allows one to perform group fibration of system of equations (1) [4, 11], that is, to present it in the form of automorphic and resolving systems of equations. Let us consider operator $P_w = h_x\partial_{w^1} + h_y\partial_{w^2}$, where h is arbitrary harmonic function. Invariants of operator P_w are t, x, y.
Let us extend operator P_w on the first-order derivatives [4]

$$\underset{1}{p_w} = h\partial_{w^1} + h\partial_{w^2} + h_x(\partial_{w^1_x} - \partial_{w^2_y}) + h_y(\partial_{w^1_y} + \partial_{w^2_x}).$$

Differential invariants of the extended operator are

$$w_t^1,\ w_t^2,\ w_x^1 + w_y^2,\ w_x^2 - w_y^1.$$

Assigning differential invariants to be functions of invariants, one can obtain the automorphic system

$$w_t^1 = u(t, x, y),\ \ w_t^2 = v(t, x, y),\ \ \theta(t, x, y) = w_x^1 + w_y^2,\ \ \omega(t, x, y) = w_x^{2-} w_y^1. \tag{5}$$

Conditions of compatibility of equations (5) lead to the resolving system

$$u_t = (\lambda + 2\mu)\theta_x - \mu\omega_y,\ \ v_t = (\lambda + 2\mu)\theta_y + \mu\omega_x,\ \ \theta_t = u_x + v_y,\ \ \omega_t = v_x - u_y. \tag{6}$$

Solution of Lame system of equations (1) is equivalent to solution of systems (5), (6) [5, 6]. Using initial conditions for equations (1), it is not difficult to obtain initial conditions for the functions included in equations (5) and (6):

$$\theta|_{t=0} = \partial_x f^1 + \partial_y f^2,\ \ \omega|_{t=0} = \partial_x f^2 - \partial_y f^1,\ \ u|_{t=0} = g^1,\ \ v|_{t=0} = g^2. \tag{7}$$

2. Problem formulation

Let us find the conservation laws for the resolving system of equations. This allows one to solve Cauchy problem (7) for equations (6). Further on, using (5), one can solve Cauchy problem (2) for equations (1).

3. Conservation laws for resolving system

Let us consider system of equations (6) in the form

$$\begin{aligned} &F_1 = u_t - (\lambda + 2\mu)\theta_x + \mu\omega_y = 0,\ \ F_2 = v_t - (\lambda + 2\mu)\theta_y - \mu\omega_x = 0, \\ &F_3 = \theta_t - u_x - v_y = 0,\ \ F_4 = \omega_t - v_x + u_y = 0. \end{aligned} \tag{8}$$

Definition. Expression of the form

$$A_t + B_x + C_y = \sum_{i=1}^{4} \rho^i F_i \tag{9}$$

is called the conservation law for system of equations (8). Here ρ^i are some linear differential operators that are simultaneously not identically zero. Vector (A, B, C) is called conserved current for conservation law (9).
More general definitions of conservation laws can be found in [8, 9 and the references therein].
Let us assume that conserved current is written as

$$\begin{aligned} A &= \alpha^1 u + \beta^1 v + \gamma^1 \theta + \delta^1 \omega, \\ B &= \alpha^2 u + \beta^2 v + \gamma^2 \theta + \delta^2 \omega, \\ C &= \alpha^3 u + \beta^3 v + \gamma^3 \theta + \delta^3 \omega, \end{aligned} \tag{10}$$

where $\alpha^i, \beta^i, \gamma^i, \delta^i$ are smooth functions that depend only on t, x, y.

Note. System of equations (8) also has other conservation laws by virtue of linearity. However, for our purposes it is sufficient to have conservation laws with conserved current in form (10). Let us substitute (10) into (9). Then a first-degree polynomial with respect to derivatives $u_t, u_x, \dots, \omega_y$ and required functions u, v, θ, ω is obtained. Setting coefficients at these variables equal to zero,one can obtain

$$\begin{aligned}&\alpha^1 = \rho^1,\ \alpha^2 = -\rho^2,\ \alpha^3 = -\rho^4,\ \beta^1 = \rho^2,\ \beta^2 = -\rho^4,\ \beta^3 = -\rho^3,\\ &\gamma^1 = \rho^3,\ \gamma^2 = -(\lambda+2\mu)\rho^1,\ \gamma^3 = -(\lambda+2\mu)\rho^2,\ \delta^1 = \rho^4,\ \delta^2 = -\mu\rho^2,\ \delta^3 = -\mu\rho^1.\end{aligned} \tag{11}$$

$$\begin{aligned}&\alpha_t^1 - \gamma_x^1 + \delta_y^1 = 0, \beta_t^1 - \delta_x^1 - \gamma_y^1 = 0,\\ &\gamma_t^1 - (\lambda+2\mu)\alpha_x^1 - (\lambda+2\mu)\beta_y^1 = 0,\\ &\delta_t^1 - \mu\beta_x^1 + \mu\alpha_y^1 = 0.\end{aligned} \tag{12}$$

It follows from (10)—(12) that conserved current is written as

$$\begin{aligned}&A = \alpha^1 u + \beta^1 v + \gamma^1\theta + \delta^1\omega,\\ &B = -\gamma^1 u - \delta^1 v - (\lambda+2\mu)\alpha^1\theta - \mu\beta^1\omega,\\ &C = \delta^1 u - \gamma^1 v - (\lambda+2\mu)\beta^1\theta + \mu\alpha^1\omega.\end{aligned} \tag{13}$$

It follows from (12) that (γ^1, δ^1) is an arbitrary solution of equations of elasticity (1). Let us find the solution of equations (1) in the form of Lame

$$\gamma^1 = \Phi_x + \Psi_y,\ \delta^1 = \Phi_y - \Psi_x, \tag{14}$$

where Φ, Ψ are arbitrary solutions of equations

$$(\lambda+2\mu)(\Phi_{xx} + \Phi_{yy}) - \Phi_{tt} = 0, \tag{15}$$

$$\mu(\Psi_{xx} + \psi_{yy}) - \psi_{tt} = 0. \tag{16}$$

First, let us find the solution of equations (1) in the form

$$\gamma^1 = \Phi_x,\ \delta^1 = \Phi_y, \tag{17}$$

Then it follows from (12) that

$$\alpha_t^1 = 0, \quad \beta_t^1 = \Phi_{tt}/(\lambda+2\mu).$$

Further on, it is assumed that

$$\alpha^1 = 0, \quad \beta^1 = \Phi_t/(\lambda+2\mu). \tag{18}$$

Let us find the solution of equation (15) in the form of Kirchhoff

$$\Phi = \frac{1}{r}(G_1(t - t_0 + (\sqrt{\lambda+2\mu})^{-1}\, r) + G_2(t - t_0 - (\sqrt{\lambda+2\mu})^{-1}\, r),$$

where $r = \sqrt{(x-x_0)^2 + (y-y_0)^2}$, (t_0, x_0, y_0) is some point such that $t_0 \neq 0$. Let us assume that

$$G_1 = (t - t_0 + (\sqrt{\lambda+2\mu})^{-1}\, r)^{1+n},\ \ G_2 = -(t - t_0 - (\sqrt{\lambda+2\mu})^{-1}\, r)^{1+n}, \tag{19}$$

where $n \in R, \quad n > 1$.
Then

$$\begin{aligned}
\gamma^1 &= -\frac{x-x_0}{r^3}((t-t_0+(\sqrt{\lambda+2\mu})^{-1}r)^{1+n} - (t-t_0-(\sqrt{\lambda+2\mu})^{-1}r)^{1+n})- \\
&-\frac{(1+n)(x-x_0)(\sqrt{\lambda+2\mu})^{-1}}{r^2}((t-t_0+(\sqrt{\lambda+2\mu})^{-1}r)^n + (t-t_0-(\sqrt{\lambda+2\mu})^{-1}r)^n), \\
\delta^1 &= -\frac{y-y_0}{r^3}((t-t_0+(\sqrt{\lambda+2\mu})^{-1}r)^{1+n} - (t-t_0-(\sqrt{\lambda+2\mu})^{-1}r)^{1+n})- \\
&-\frac{(1+n)(y-y_0)(\sqrt{\lambda+2\mu})^{-1}}{r^2}((t-t_0+(\sqrt{\lambda+2\mu})^{-1}r)^n + (t-t_0-(\sqrt{\lambda+2\mu})^{-1}r)^n), \\
\beta^1 &= \frac{(1+n)}{r(\lambda+2\mu)}((t-t_0+(\sqrt{\lambda+2\mu})^{-1}r)^n + (t-t_0-(\sqrt{\lambda+2\mu})^{-1}r)^n), \quad \alpha^1 = 0.
\end{aligned} \tag{20}$$

Now let us find the solution of equations (1) in the form

$$\gamma^1 = \Psi_y, \quad \delta^1 = -\Psi_x. \tag{21}$$

Then from (12) it follows

$$\beta^1_t = 0, \quad \alpha^1_t = \Psi_{tt}/\mu.$$

Further on, it is assumed that

$$\beta^1 = 0, \quad \alpha^1 = \Psi_t/\mu. \tag{22}$$

Let us find the solution of equation (16) in the form of Kirchhoff

$$\Phi = \frac{1}{r}(G_3(t-t_0+(\sqrt{\mu})^{-1}\, r) + G_4(t-t_0-(\sqrt{\mu}\,)^{-1}r).$$

Let us assume that

$$G_3 = (t-t_0+(\sqrt{\mu})^{-1}\, r)^{1+m}, \ \ G_4 = -(t-t_0-(\sqrt{\mu}\,)^{-1}r)^{1+m}, \tag{23}$$

where $m \in R$.
Then

$$\begin{aligned}
\gamma^1 &= -\frac{x-x_0}{r^3}((t-t_0+(\sqrt{\mu})^{-1}r)^{1+m} - (t-t_0-(\sqrt{\mu})^{-1}r)^{1+m})- \\
&-\frac{(1+m)(x-x_0)(\sqrt{\mu})^{-1}}{r^2}((t-t_0+(\sqrt{\mu})^{-1}r)^m + (t-t_0-(\sqrt{\mu})^{-1}r)^m), \\
\delta^1 &= -\frac{y-y_0}{r^3}((t-t_0+(\sqrt{\mu})^{-1}r)^{1+m} - (t-t_0-(\sqrt{\mu})^{-1}r)^{1+m})- \\
&-\frac{(1+m)(y-y_0)\sqrt{\mu}}{r^2}((t-t_0+\sqrt{\mu}r)^m + (t-t_0-\sqrt{\mu}r)^m), \\
\alpha^1 &= \frac{(1+m)}{r\mu}((t-t_0+(\sqrt{\mu})^{-1}r)^m + (t-t_0-(\sqrt{\mu})^{-1}r)^m), \quad \beta^1 = 0.
\end{aligned} \tag{24}$$

4. Solving Cauchy problem for resolving system of equations

Characteristic cones with the origin at the point (t_0, x_0, y_0) are shown in Fig. 1. The lateral surface of the outer cone is given by the equation

$$S_1: \ (\lambda+2\mu)(t-t_o)^2 - (x-x_0)^2 - (y-y_0)^2 = 0, \tag{25}$$

and the lateral surface of the inner cone is given by the equation

$$S_2: \ \mu(t-t_o)^2 - (x-x_0)^2 - (y-y_0)^2 = 0. \tag{26}$$

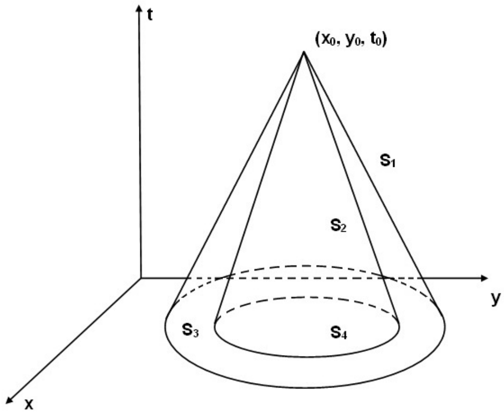

Fig. 1. Characteristic cones

Intersections of cones (25) and (26) with the plane $t = 0$ are circles S_3, S_4. Initial conditions on functions u, v, θ, ω are given inside these circles.
Let us consider domain V_1 bounded by surface S_1 and by plane $t = 0$. Then it follows from (9) that

$$\iiint\limits_{V_1} (A_t + B_x + C_y)dxdydt = 0. \tag{27}$$

Let us consider cylinder T_ε of radius $(x-x_0)^2 + (y-y_0)^2 = \varepsilon^2$ inside the outer cone as shown in Fig. 2.

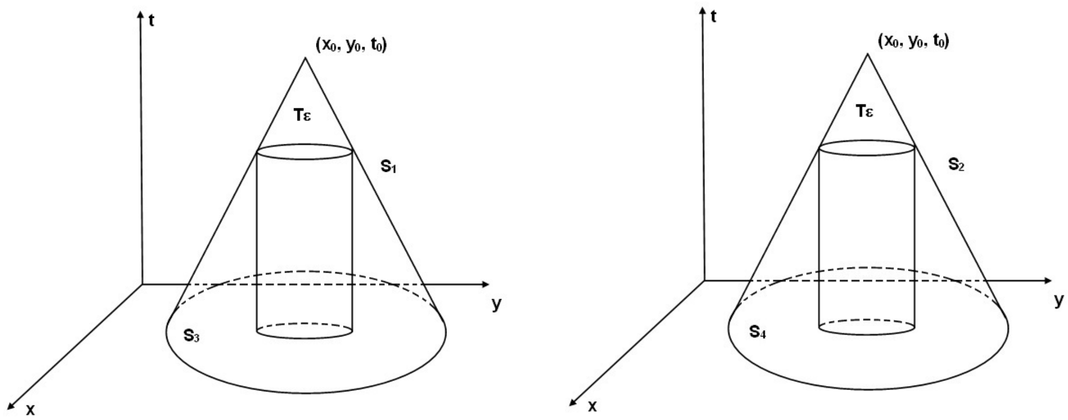

Fig. 2. Solving the Cauchy problem to find $\theta(x_0, y_0, t_0)$

Functions $\alpha^1, \beta^1, \gamma^1, \delta^1$ have no peculiarities inside the domain bounded by surface S_1, by cylindrical surface T_ε and by plane $t = 0$. Using the Gauss–Ostrogradskiy formula, one can obtain from (27) that

$$\begin{gathered}\iiint\limits_{V_1\setminus T_\varepsilon}(A_t+B_x+C_y)dxdydt=\iint\limits_{S_1}Adxdy+Bdydt+Cdtdx+\\+\iint\limits_{T_\varepsilon}Adxdy+Bdydt+Cdtdx+\iint\limits_{S_3}Adxdy+Bdydt+Cdtdx=0.\end{gathered}\tag{28}$$

By virtue of choosing function Φ the integral $\iint\limits_{S_1}Adxdy+Bdydt+Cdtdx=0$. It is not difficult to see that the integral $\iint\limits_{S_3}Adxdy+Bdydt+Cdtdx$ has no peculiarities. That is why, it is necessary to calculate only the integral

$$\iint\limits_{T_\varepsilon}Bdydt+Cdtdx\tag{29}$$

on the assumption that ε is small. Assume that $x-x_0=\varepsilon\cos\phi$, $y-y_0=\varepsilon\sin\phi$. Let us substitute these expressions into (29) and obtain

$$\begin{gathered}\iint\limits_{T_\varepsilon}Bdydt+Cdtdx=\\=\int\limits_0^{t_0}\varepsilon dt\int\limits_0^{2\pi}\big((-\gamma^1u-\delta^1v-(\lambda+2\mu)\alpha^1\theta-\mu\beta^1\omega)\cos\phi-(\delta^1u-\gamma^1v-(\lambda+2\mu)\beta^1\theta+\mu\alpha^1\omega)\sin\phi\big)d\phi.\end{gathered}$$

Since

$$\begin{aligned}\gamma^1&=-\frac{2\cos\phi}{\varepsilon\sqrt{\lambda+2\mu}}(2n+1)(t-t_0)^n+o(\varepsilon),\\ \delta^1&=-\frac{2\sin\phi}{\varepsilon\sqrt{\lambda+2\mu}}(2n+1)(t-t_0)^n+o(\varepsilon),\\ \alpha^1&=\frac{2}{\sqrt{\lambda+2\mu}}(n+1)(t-t_0)^n+o(\varepsilon),\beta^1=0,\end{aligned}$$

it follows that

$$\begin{aligned}&\iint\limits_{T_\varepsilon}Bdydt+Cdtdx=\\&=-(\lambda+2\mu)\int_0^{t_0}\left(\int_0^{2\pi}\theta(\alpha^1\cos\phi+\beta^1\sin\phi)d\phi-\mu\int_0^{2\pi}\omega(\beta^1\cos\phi-\alpha^1\sin\phi)d\phi\right)dt=\\&=2\pi\sqrt{\lambda+2\mu}(2n+1)\int_0^{t_0}(t-t_0)^n\theta(x_0,y_0,t)dt.\end{aligned}$$

The last expression is obtained with $\varepsilon\to0$.
Finally, it follows from (28) and (29) that

$$2\pi\sqrt{\lambda+2\mu}(2n+1)\int_0^{t_0}(t-t_0)^n\theta(x_0,y_0,t)dt=\iint\limits_{S_3}Adxdy.$$

Differentiating the last expression with respect to t_0, one can obtain that

$$\theta(x_0, y_0, t_0) = \frac{1}{2\pi(n+1)\sqrt{\lambda+2\mu}} \frac{\partial}{\partial t_0} \iint\limits_{S_3} A dx dy, \tag{30}$$

where $A = \alpha^1 u + \beta^1 v + \gamma^1 \theta + \delta^1 \omega$,

$$\gamma^1 = -\frac{x-x_0}{r^3}\Bigg(\Big(\frac{r}{\sqrt{\lambda+2\mu}} - t_0\Big)^{1+n} - \Big(-t_0 - \frac{r}{\sqrt{\lambda+2\mu}}\Big)^{1+n}\Bigg) - \\ -\frac{(x-x_0)(1+n)}{r^2\sqrt{\lambda+2\mu}}\Bigg(\Big(\frac{r}{\sqrt{\lambda+2\mu}} - t_0\Big)^{n} + \Big(-t_0 - \frac{r}{\sqrt{\lambda+2\mu}}\Big)^{n}\Bigg),$$

$$\delta^1 = -\frac{y-y_0}{r^3}\Bigg(\Big(\frac{r}{\sqrt{\lambda+2\mu}} - t_0\Big)^{1+n} - \Big(-t_0 - \frac{r}{\sqrt{\lambda+2\mu}}\Big)^{1+n}\Bigg) - \\ -\frac{(y-y_0)(1+n)}{r^2\sqrt{\lambda+2\mu}}\Bigg(\Big(\frac{r}{\sqrt{\lambda+2\mu}} - t_0\Big)^{n} + \Big(-t_0 - \frac{r}{\sqrt{\lambda+2\mu}}\Big)^{n}\Bigg),$$

$$\beta^1 = \frac{(1+n)}{r(\gamma+2\mu)}\Bigg(\Big(\frac{r}{\sqrt{\lambda+2\mu}} - t_0\Big)^{n} - \Big(-t_0 - \frac{r}{\sqrt{\lambda+2\mu}}\Big)^{n}\Bigg), \quad \alpha^1 = 0.$$

Now let us perform the same procedure for the inner cone but for solutions (20), (21) and obtain

$$\omega(x_0, y_0, t_0) = \frac{1}{2\pi(n+1)\sqrt{\mu}} \frac{\partial}{\partial t_0} \iint\limits_{S_3} A dx dy, \tag{31}$$

where $A = \alpha^1 u + \beta^1 v + \gamma^1 \theta + \delta^1 \omega$,

$$\alpha^1 = -\frac{y-y_0}{r^3}\Bigg(\Big(\frac{r}{\sqrt{\mu}} - t_0\Big)^{1+m} - \Big(-t_0 - \frac{r}{\sqrt{\mu}}\Big)^{1+m}\Bigg) - \\ -\frac{(y-y_0)(1+m)}{r^2\sqrt{\mu}}\Bigg(\Big(\frac{r}{\sqrt{\mu}} - t_0\Big)^{m} + \Big(-t_0 - \frac{r}{\sqrt{\mu}}\Big)^{m}),$$

$$\beta^1 = -\frac{x-x_0}{r^3}\Bigg(\Big(\frac{r}{\sqrt{\mu}} - t_0\Big)^{1+m} - \Big(-t_0 - \frac{r}{\sqrt{\mu}}\Big)^{1+m}\Bigg) - \\ -\frac{(y-y_0)(1+m)}{r^2\sqrt{\mu}}\Bigg(\Big(\frac{r}{\sqrt{\mu}} - t_0\Big)^{m} + \Big(-t_0 - \frac{r}{\sqrt{\mu}}\Big)^{m}\Bigg),$$

$$\gamma^1 = \frac{(1+m)}{r}\Bigg(\Big(\frac{r}{\sqrt{\mu}} - t_0\Big)^{m} - \Big(-t_0 - \frac{r}{\sqrt{\mu}}\Big)^{m}\Bigg), \quad \delta^1 = 0.$$

Now, taking into account (30)–(31) and initial conditions (2)

$$u_t = (\lambda+2\mu)\theta_x - \mu\omega_y, \;\; v_t = (\lambda+2\mu)\theta_y + \mu\omega_x$$

, one can obtain from (6) that

$$w_t^1 = u = \int_0^t ((\gamma+2\mu)\theta_x - \mu\omega_y)dt + g^1(x,y), \;\; w_t^2 = v = \int_0^t ((\gamma+2\mu)\theta_y + \mu\omega_x)dt + g^2(x,y).$$

Taking into account (5) and initial conditions (2), one can finally find that

$$\begin{aligned} w^1 &= \int_0^t u dt = \int_o^t \Bigg(\int_0^t ((\gamma+2\mu)\theta_x - \mu\omega_y)dt\Bigg)dt + g^1(x,y)t + f^1(x,y), \\ w^2 &= \int_0^t v dt = \int_o^t \Bigg(\int_0^t ((\lambda+2\mu)\theta_y + \mu\omega_x)dt\Bigg)dt + g^2(x,y)t + f^2(x,y). \end{aligned} \tag{32}$$

Relations (32) provide the solution of Cauchy problem for system of equations (1).

Note. The method of solving Cauchy problem stated in this paper can be used with some modifications to solve three-dimensional dynamic problems for equations of elasticity. This will be performed in the following works.

This paper was carried out by the team of the scientific laboratory "Smart Materials and Structures" within the state assignment of the Ministry of Science and Higher Education of the Russian Federation for the implementation of the project "Development of multifunctional smart materials and structures based on modified polymer composite materials capable to function in extreme conditions" (Project no. FEFE-2020-0015).

References

[1] V.Novatsky, Theory of elasticity, Novosibirsk, Nauka, 1983 (in Russian).

[2] N.I.Ostrosablin, Symmetry operators and general solutions of equations of linear theory of elasticity, *Applied Mechanics and Technical Physics*, **36**(1995), no. 5, 98–104.

[3] N.I Ostrosablin, General solutions and reduction of systems of equations of linear theory of elasticity to a diagonal form, *Applied Mechanics and Technical Physics*, **34**(1993), no. 5, 112–122.

[4] S.L.Sobolev, Equations of mathematical physics, Moscow: State Publishing House of Technical and Theoretical Literature, 1956 (in Russian).

[5] L.V.Ovsyannikov, Group analysis of differential equations, Moscow, Nauka, 1978 (in Russian).

[6] B.D.Annin, V.O.Bytev, S.I.Senashov, Group properties of elasticity and plasticity equations, Novosibirsk, Nauka, 1983 (in Russian).

[7] S.I.Senashov, I.L.Savostyanova, On elastic torsion around three axes, *Siberian Journal of Industrial Mathematics*, **24**(2021), no. 1, 120–125 (in Russian).
DOI: 10.33048/sibjim.2021.24.109

[8] B.D.Annin, V.D.Bondar, S.I.Senashov, Determination of elastic and plastic deformation regions in the problem of uniaxial tension of a plate weakened by holes, *Siberian Journal of Industrial Mathematics*, **23**(2020), no. 1, 11–16 (in Russian).

[9] O.V.Gomonova, S.I.Senashov, Group analysis and exact solutions of the equations of plane deformation of an incompressible nonlinear elastic body, *Applied Mechanics and Technical Physics*, **62**(2021), no. 1, 179–186 (in Russian).

[10] O.V.Gomonova, S.I.Senashov, O.N.Cherepanova, Group analysis of ideal plasticity equations, *Applied Mechanics and Technical Physics*, **62**(2021), no. 5, 208–216 (in Russian).

[11] V.I.Smirnov, Course of Higher Mathematics, Moscow, Nauka, 1981 (in Russian).

[12] V.Yu.Prudnikov, Yu.A.Chirkunov, Group bundle of Lame equations, *Solid State Mechanics*, **22**(2009), no. 3, 471–477 (in Russian).

Решение задачи Коши для уравнений упругости в плоском динамическом случае

Сергей И. Сенашов
Ирина Л. Савостьянова
Сибирский государственный университет науки и технологий имени академика М. Ф. Решетнева
Красноярск, Российская Федерация
Ольга Н. Черепанова
Сибирский федеральный университет
Красноярск, Российская Федерация

Аннотация. Рассмотрены уравнения упругости в плоском динамическом случае. Эта система заменена равносильной системой дифференциальных уравнений первого порядка. Равносильная система есть групповое расслоение исходной системы уравнений, она является объединением разрешающей и автоморфных систем. Для разрешающей системы уравнений найдены специальные классы законов сохранения, которые позволили найти решение исходных уравнений в виде поверхностных интегралов по границе упругого тела.

Ключевые слова: уравнения упругости в плоском динамическом случае, задача Коши, законы сохранения, точные решения