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# Certain Integral Formulas Involving Products of Two Incomplete Beta Functions

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**Abstract.** The aim of this paper is to obtain some integral formulas involving products of two incomplete beta functions in terms of general triple hypergeometric series and Kampé de Fériet function. Some new particular integral formulas involving the incomplete beta function are also calculated as an application of our main results with the help of Whipple, Dixon and extension of Dixon summation theorems.

**Keywords:** incomplete beta function, Integral formulas, Kampé de Fériet function, General triple hypergeometric series.

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## 1. Introduction

The generalized hypergeometric function ${}_pF_q$ with $p$ numerator parameters and $q$ denominator parameters ($p$ and $q$ are positive integers or zero and $z$ is complex variable) is defined by (see [10, 11])

$$ {}_pF_q\left[\begin{matrix} a_1,\dots,a_p\,; \\ b_1,\dots,b_q\,; \end{matrix} z\right]=\sum_{n=0}^{\infty}\frac{(a_1)_n\dots(a_p)_n}{(b_1)_n\dots(b_q)_n}\frac{z^n}{n!}, \tag{1} $$

where $(\lambda)_n$ denotes the Pochhammer's symbol defined by

$$ (\lambda)_n=\begin{cases} 1 & , \quad (n=0) \\ \lambda(\lambda+1)(\lambda+2)\dots(\lambda+n-1) & , \quad (n\in\mathbb{N}) \end{cases} $$

$$ =\frac{\Gamma(\lambda+n)}{\Gamma(\lambda)}, \qquad (\lambda\in\mathbb{C}\setminus\mathbb{Z}_0^-) \tag{2} $$

and $\Gamma(\lambda)$ is the gamma function defined by

$$ \Gamma(\lambda)=\int_0^{\infty} t^{\lambda-1}e^{-t}dt, \qquad \Re(\lambda)>0. \tag{3} $$

The classical beta function $B(a,b)$ is defined by (see [11])

$$ B(a,b)=\begin{cases} \displaystyle\int_0^1 t^{a-1}(1-t)^{b-1}dt & , \quad \Re(a)>0,\ \Re(b)>0, \\ \displaystyle\frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)} & , \quad a,b\neq 0,-1,-2,\dots\,. \end{cases} \tag{4} $$

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The incomplete beta function is defined as follows [8]:

$$B_z(a,b)=\int_0^z t^{a-1}(1-t)^{b-1}dt, \quad 0\leqslant z\leqslant 1,\ a,b>0. \tag{5}$$

Further, the substitution $t=\sin^2(\theta)$ gives

$$B_z(a,b)=2\int_0^{\arcsin\sqrt{z}} \sin^{2a-1}(\theta)\cos^{2b-1}(\theta)d\theta. \tag{6}$$

The hypergeometric representation of incomplete beta function is given by [8]

$$B_z(a,b)=a^{-1}z^a{}_2F_1[a,1-b;a+1;z]. \tag{7}$$

Also, we recalling the following formulas for the incomplete beta function [7]:

$$B_z(a,b)=B(a,b)-B_{1-z}(b,a), \tag{8}$$

$$B_z(1,1)=z, \quad B_z(a,1)=\frac{z^a}{a}. \tag{9}$$

The Kampé de Fáriet function of two variables $F^{p:q;k}_{l:m;n}[x,y]$ is defined and represented as follows [10, 11]:

$$F^{p:q;k}_{l:m;n}\left[\begin{matrix}(a_p):(b_q);(c_k);\\ (\alpha_l):(\beta_m);(\gamma_n);\end{matrix}\ x, y\right]$$

$$=\sum_{r,s=0}^{\infty}\frac{\prod\limits_{j=1}^{p}(a_j)_{r+s}\prod\limits_{j=1}^{q}(b_j)_r\prod\limits_{j=1}^{k}(c_j)_s}{\prod\limits_{j=1}^{l}(\alpha_j)_{r+s}\prod\limits_{j=1}^{m}(\beta_j)_r\prod\limits_{j=1}^{n}(\gamma_j)_s}\frac{x^r}{r!}\frac{y^s}{s!}. \tag{10}$$

Furthermore, we recall that the general triple hypergeometric series $F^{(3)}[x,y,z]$ is defined by [10, 11]:

$$F^{(3)}[x,y,z]=F^{(3)}\left[\begin{matrix}(a)::(b);(b');(b''):(c);(c');(c'');\\ (e)::(g);(g');(g''):(h);(h');(h'');\end{matrix}\ x,y,z\right]$$

$$=\sum_{m,n,p=0}^{\infty}\Lambda(m,n,p)\frac{x^m}{m!}\frac{y^n}{n!}\frac{z^p}{p!}, \tag{11}$$

where, for convenience,

$$\Lambda(m,n,p)=\frac{\prod_{j=1}^{A}(a_j)_{m+n+p}\prod_{j=1}^{B}(b_j)_{m+n}\prod_{j=1}^{B'}(b'_j)_{n+p}\prod_{j=1}^{B''}(b''_j)_{p+m}}{\prod_{j=1}^{E}(e_j)_{m+n+p}\prod_{j=1}^{G}(g_j)_{m+n}\prod_{j=1}^{G'}(g'_j)_{n+p}\prod_{j=1}^{G''}(g''_j)_{p+m}}\times$$

$$\times\frac{\prod_{j=1}^{C}(c_j)_m\prod_{j=1}^{C'}(c'_j)_n\prod_{j=1}^{C''}(c''_j)_p}{\prod_{j=1}^{H}(h_j)_m\prod_{j=1}^{H'}(h'_j)_n\prod_{j=1}^{H''}(h''_j)_p} \tag{12}$$

and $(a)$ abbreviates the array of $A$ parameters $a_1,a_2,\ldots,a_A$, with similar interpretations for $(b),(b'),(b'')$ and so on.

Recently some works for the incomplete beta function with applications have been considered by several authors, see [1,3,4]. In this paper, we obtain some integral formulas involving products of two incomplete beta functions. Further, we apply these results with the help of Whipple, Dixon and extension of Dixon summation theorems to compute some new particular integral formulas involving incomplete beta function.

## 2. Integral formulas for the incomplete beta function

In this section, we establish four integral formulas involving products of two incomplete beta functions asserted by the following theorems:

**Theorem 2.1.** *The following integral formula holds true:*

$$\int_0^x z^{k-1}(1-z)^{p-1}B_z(a,b)B_z(c,d)dz=$$

$$=\frac{x^{a+c+k}}{ac(a+c+k)}F^{(3)}\left[\begin{matrix} a+c+k & :: -;-;-: a,1-b;c,1-d;1-p; \\ a+c+k+1 & :: -;-;-: \ a+1 \ ; \ c+1 \ ; \ - \ ; \end{matrix} x,x,x\right]. \tag{13}$$

*Proof.* Denoting the left hand side of (13) by $L$, replacing the two incomplete beta functions by their hypergeometric representations given in (7), expanding the two ${}_2F_1$ in a power series, changing the order of summation and integration and using (5), we get

$$L=\int_0^x z^{k-1}(1-z)^{p-1}B_z(a,b)B_z(c,d)dz=$$

$$=\frac{1}{ac}\sum_{m,n=0}^{\infty}\frac{(a)_m(1-b)_m(c)_n(1-d)_n}{(a+1)_m(c+1)_n m!n!}B_x(a+c+k+m+n,p). \tag{14}$$

Again, replacing the incomplete beta function in the right hand side of (14) by its hypergeometric representation given in (7) and expanding ${}_2F_1$ in a power series we have

$$L=\int_0^x z^{k-1}(1-z)^{p-1}B_z(a,b)B_z(c,d)dz=$$

$$=\frac{1}{ac}\sum_{m,n=0}^{\infty}\frac{(a)_m(1-b)_m(c)_n(1-d)_n x^m x^n}{(a+1)_m(c+1)_n m!n!}\times$$

$$\times\frac{x^{a+c+k}}{a+c+k+m+n}\sum_{s=0}^{\infty}\frac{(a+c+k+m+n)_s(1-p)_s x^s}{(a+c+k+1+m+n)_s s!}. \tag{15}$$

Finally, by using the following identities:

$$\frac{a}{a+m}=\frac{(a)_m}{(a+1)_m}, \tag{16}$$

$$(a)_{m+n}=(a)_m(a+m)_n, \tag{17}$$

we get the right hand side of (13).This completes the proof of Theorem 2.1. □

**Corollary 2.1.** *For $c=d=1$ in Theorem 2.1 yields the following result:*

$$\int_0^x z^{k-1}(1-z)^{p-1}B_z(a,b)dz=$$

$$=\frac{x^{a+k}}{a(a+k)}F^{1:2;1}_{1:1;0}\left[\begin{matrix} a+k & : a,1-b;1-p; \\ a+k+1 & : \ a+1 \ ; \ - \ ; \end{matrix} x\ ,\ x\right]. \tag{18}$$

**Theorem 2.2.** *The following integral formula holds true:*

$$\int_0^x z^{k-1}(1-z)^{p-1}B_z(a,b)B_{1-z}(c,d)dz=$$

$$=\frac{B(c,d)x^{a+k}}{a(a+k)}F\begin{array}{c}1:2;1\\ \\ 1:1;0\end{array}\left[\begin{array}{ll}a+k & :a,1-b;1-p;\\ & \qquad\qquad x\ ,\ x\\ a+k+1: & a+1\ ;\ -\ ;\end{array}\right]-$$

$$-\frac{x^{a+d+k}}{ad(a+d+k)}F^{(3)}\left[\begin{array}{ll}a+d+k & ::-;-;-:a,1-b;d,1-c;1-p;\\ & \qquad\qquad\qquad x,x,x\\ a+d+k+1 & ::-;-;-:\ a+1\ ;\ d+1\ ;\ -\ ;\end{array}\right]. \tag{19}$$

*Proof.* Denoting the left hand side of (19) by $L$ and then applying the result (8), we have

$$L=\int_0^x z^{k-1}(1-z)^{p-1}B_z(a,b)B_{1-z}(c,d)dz=$$

$$=\int_0^x z^{k-1}(1-z)^{p-1}B_z(a,b)(B(c,d)-B_z(d,c))dz=$$

$$=B(c,d)\int_0^x z^{k-1}(1-z)^{p-1}B_z(a,b)dz-\int_0^x z^{k-1}(1-z)^{p-1}B_z(a,b)B_z(d,c)dz$$

Now, using (13) and (18), we obtain the desired result. □

If we use the same technique as in the proof of the integral (13) asserted in the Theorem 2.1, we have the following theorem:

**Theorem 2.3.** *The following integral formula holds true:*

$$\int_0^1 z^{k-1}(1-z)^{p-1}B_z(a,b)B_{1-z}(c,d)dz=$$

$$=\frac{B(a+k,c+p)}{ac}F\begin{array}{c}0:3;3\\ \\ 1:1;1\end{array}\left[\begin{array}{ll}- & :a,1-b,a+k;c,1-d,c+p;\\ & \qquad\qquad 1\ ,\ 1\\ a+c+k+p: & a+1\ ;\ c+1\ ;\end{array}\right]. \tag{20}$$

**Corollary 2.2.** *For $c=d=1$ in Theorem 2.3 yields the following result:*

$$\int_0^1 z^{k-1}(1-z)^{p-1}B_z(a,b)dz=$$

$$=\frac{B(a+k,p)}{a}{}_3F_2\left[\begin{array}{l}a,1-b,a+k\ ;\\ \qquad\qquad 1\\ a+1,a+k+p\,;\end{array}\right]. \tag{21}$$

**Corollary 2.3.** *For $a=b=1$ in Theorem 2.3 yields the following result:*

$$\int_0^1 z^{k-1}(1-z)^{p-1}B_{1-z}(c,b)dz=$$

$$=\frac{B(k,c+p)}{c}{}_3F_2\left[\begin{array}{l}c,1-d,c+p\ ;\\ \qquad\qquad 1\\ c+1,c+k+p\,;\end{array}\right]. \tag{22}$$

**Theorem 2.4.** *The following integral formula holds true:*

$$\int_0^1 z^{k-1}(1-z)^{p-1}B_z(a,b)B_z(c,d)dz=$$
$$=\frac{B(c,d)B(a+k,p)}{a}{}_3F_2\left[\begin{matrix}a,1-b,a+k\ ;\\ a+1,a+k+p\ ;\end{matrix}\ 1\right]-$$
$$-\frac{B(a+k,d+p)}{ad}F\begin{matrix}0:3\,;3\\ 1:1\,;1\end{matrix}\left[\begin{matrix}-\ :a,1-b,a+k\,;d,1-c,d+p\,;\\ a+d+k+p:\quad a+1\quad;\quad d+1\quad;\end{matrix}\ 1\ ,\ 1\right]. \tag{23}$$

*Proof.* Denoting the left hand side of (23) by $L$ and then applying the result (8), we have

$$L=\int_0^1 z^{k-1}(1-z)^{p-1}B_z(a,b)B_z(c,d)dz=$$
$$=\int_0^1 z^{k-1}(1-z)^{p-1}B_z(a,b)(B(c,d)-B_{1-z}(d,c))dz=$$
$$=B(c,d)\int_0^1 z^{k-1}(1-z)^{p-1}B_z(a,b)dz-\int_0^1 z^{k-1}(1-z)^{p-1}B_z(a,b)B_{1-z}(d,c)dz$$

Now, using (20) and (21), we obtain the desired result. □

**Corollary 2.4.** *For $k=p=1$ in Theorem 2.4 yields the following result:*

$$\int_0^1 B_z(a,b)B_z(c,d)dz=$$
$$=B(c,d)B(a,b+1)-\frac{B(d,a+c+1)}{a(a+1)}{}_3F_2\left[\begin{matrix}a,1-b,a+c+1\ ;\\ a+2,a+c+d+1\ ;\end{matrix}\ 1\right]. \tag{24}$$

**Remark 2.1.** *Note that*

$$\int_0^1 B_z(a,b)B_{1-z}(c,d)dz=$$
$$=\frac{B(c,a+d+1)}{a(a+1)}{}_3F_2\left[\begin{matrix}a,1-b,a+d+1\ ;\\ a+2,a+c+d+1\ ;\end{matrix}\ 1\right]. \tag{25}$$

**Corollary 2.5.** *For $k=2$, $p=1$ in Theorem 2.4 yields the following result:*

$$\int_0^1 zB_z(a,b)B_z(c,d)dz=\frac{1}{2}B(c,d)(B(a,b)-B(a+2,b))-$$
$$-\frac{B(d,a+c+2)}{a(a+2)}{}_4F_3\left[\begin{matrix}a,1-b,a+2,a+c+2\ ;\\ a+1,a+3,a+c+d+2\ ;\end{matrix}\ 1\right]. \tag{26}$$

**Remark 2.2.** *Note that*

$$\int_0^1 zB_z(a,b)B_{1-z}(c,d)dz=$$

$$=\frac{B(c,a+d+2)}{a(a+2)}{}_4F_3\left[\begin{matrix}a,1-b,a+2,a+d+2\ ;\\ \\ a+1,a+3,a+c+d+2\ ;\end{matrix}\ 1\right]. \tag{27}$$

## 3. Some particular integrals with examples

In this section, we compute some particular integrals involving the incomplete beta function as an applications of our main results given in Section 2.

**I**. Taking $p=1$ in (18) and using the following result [9]:

$${}_3F_2\left[\begin{matrix}a,b,c\ ;\\ \\ b+1,c+1\ ;\end{matrix}\ x\right]=\frac{1}{c-b}\left[c\ {}_2F_1\left[\begin{matrix}a,b\ ;\\ \\ b+1\ ;\end{matrix}\ x\right]-b\ {}_2F_1\left[\begin{matrix}a,c\ ;\\ \\ c+1\ ;\end{matrix}\ x\right]\right], \tag{28}$$

thus, after considering the result (7) we obtain the following integral formula:

$$\int_0^x z^{k-1}B_z(a,b)dz=\frac{1}{k}[x^kB_x(a,b)-B_x(a+k,b)]. \tag{29}$$

**Example 3.1.** *For* $x=\frac{1}{2}$, $a=b=\frac{3}{2}$, $k=2$ *in (29), we get*

$$\int_0^{\frac{1}{2}} zB_z\left(\frac{3}{2},\frac{3}{2}\right)dz=\frac{1}{48}-\frac{\pi}{512}. \tag{30}$$

**Example 3.2.** *For* $x=\frac{1}{4}$, $a=b=\frac{3}{2}$, $k=3$ *in (29), we get*

$$\int_0^{\frac{1}{4}} z^2B_z\left(\frac{3}{2},\frac{3}{2}\right)dz=\frac{27\sqrt{3}}{5120}-\frac{13\pi}{4608}. \tag{31}$$

**Remark 3.1.** *For* $k=1$ *in (29), we get the well-known result [7]*

$$\int_0^x B_z(a,b)dz=xB_x(a,b)-B_x(a+1,b). \tag{32}$$

**Remark 3.2.** *For* $x=1$ *in (29), we get*

$$\int_0^1 z^{k-1}B_z(a,b)dz=\frac{1}{k}[B(a,b)-B(a+k,b)]. \tag{33}$$

*Further, using (8) in (33), we get*

$$\int_0^1 z^{k-1}B_{1-z}(a,b)dz=\frac{1}{k}[B(a,b+k)]. \tag{34}$$

**II**. Taking $a=b=\frac{1}{2}$ in (29) and using the result (6), we get

$$\int_0^{\sqrt{x}} t^{2k-1}\arcsin\ t\ dt=\frac{1}{2k}\left[x^k\arcsin\ \sqrt{x}-\int_0^{\arcsin\ \sqrt{x}}\sin^{2k}\ t\ dt\right]. \tag{35}$$

**Example 3.3.** *For* $x=\dfrac{1}{4}$, $k=2$ *in (35), we get*

$$\int_0^{\frac{1}{2}} t^3 \arcsin\ t\ dt = \frac{7\sqrt{3}}{256} - \frac{5\pi}{384}. \tag{36}$$

**Example 3.4.** *For* $x=\dfrac{1}{4}$, $k=3$ *in (35), we get*

$$\int_0^{\frac{1}{2}} t^5 \arcsin\ t\ dt = \frac{3\sqrt{3}}{192} - \frac{19\pi}{2304}. \tag{37}$$

**Remark 3.3.** *For* $x=1$ *in (35), we get the well-known result [5]*

$$\int_0^1 t^{2k-1} \arcsin\ t\ dt = \frac{\pi}{4k}\left[1 - \frac{(2k-1)!!}{2^k k!}\right]. \tag{38}$$

**III** Taking $a=b$, $c=d$ in (24) and using classical Whipple theorem for ${}_3F_2(1)$ [2], we get

$$\int_0^1 B_z(a,a) B_z(c,c) dz = B(c,c)B(a,a+1) - \frac{(a+c+\frac{1}{2})B(a+c+1,\frac{1}{2})}{2^{2(a+c)}ac}. \tag{39}$$

**Example 3.5.** *For* $a=\dfrac{3}{2}$, $c=\dfrac{1}{2}$ *in (39), we get*

$$\int_0^1 B_z\left(\frac{3}{2},\frac{3}{2}\right) B_z\left(\frac{1}{2},\frac{1}{2}\right) dz = \frac{\pi^2}{16} - \frac{2}{9}. \tag{40}$$

**Example 3.6.** *For* $a=\dfrac{5}{2}$, $c=\dfrac{1}{2}$ *in (39), we get*

$$\int_0^1 B_z\left(\frac{5}{2},\frac{5}{2}\right) B_z\left(\frac{1}{2},\frac{1}{2}\right) dz = \frac{3\pi^2}{256} - \frac{1}{25}. \tag{41}$$

**Remark 3.4.** *Note that*

$$\int_0^1 B_z(a,a) B_{z-1}(c,c) dz = \frac{(a+c+\frac{1}{2})B(a+c+1,\frac{1}{2})}{2^{2(a+c)}ac}. \tag{42}$$

**Example 3.7.** *For* $a=\dfrac{3}{2}$, $c=\dfrac{1}{2}$ *in (42), we get*

$$\int_0^1 B_z\left(\frac{3}{2},\frac{3}{2}\right) B_{1-z}\left(\frac{1}{2},\frac{1}{2}\right) dz = \frac{2}{9}. \tag{43}$$

**Example 3.8.** *For* $a=\dfrac{5}{2}$, $c=\dfrac{1}{2}$ *in (42), we get*

$$\int_0^1 B_z\left(\frac{5}{2},\frac{5}{2}\right) B_{1-z}\left(\frac{1}{2},\frac{1}{2}\right) dz = \frac{1}{25}. \tag{44}$$

**IV** Taking $a=c$, $b=d$ in (24) and using classical Dixon theorem for ${}_3F_2(1)$ [2], we get

$$\int_0^1 [B_z(a,b)]^2 dz = \frac{B(a,b)}{(a+b)}\left(b\ B(a,b) - \frac{B(a+\frac{1}{2},b+\frac{1}{2})}{B(a+b+\frac{1}{2},\frac{1}{2})}\right). \tag{45}$$

**Example 3.9.** *For* $a=\frac{3}{2}$, $b=\frac{1}{2}$ *in (45), we get*

$$\int_0^1 \left[B_z\left(\frac{3}{2},\frac{1}{2}\right)\right]^2 dz=\frac{\pi^2}{16}-\frac{1}{3}. \tag{46}$$

**Example 3.10.** *For* $a=\frac{5}{2}$, $b=\frac{1}{2}$ *in (45), we get*

$$\int_0^1 \left[B_z\left(\frac{5}{2},\frac{1}{2}\right)\right]^2 dz=\frac{3\pi^2}{128}-\frac{2}{15}. \tag{47}$$

**V** Taking $a=c$, $b=d$ in (26) and using the extension of Dixon theorem for ${}_4F_3(1)$ [6], we get

$$\int_0^1 z[B_z(a,b)]^2 dz=\frac{B(a,b)}{2(a+b)(a+b+1)}\left((b^2+2ab+b)B(a,b)-\frac{(2a+1)B(a+\frac{1}{2},b+\frac{1}{2})}{B(a+b+\frac{1}{2},\frac{1}{2})}\right). \tag{48}$$

**Example 3.11.** *For* $a=\frac{3}{2}$, $b=\frac{1}{2}$ *in (48), we get*

$$\int_0^1 z\left[B_z\left(\frac{3}{2},\frac{1}{2}\right)\right]^2 dz=\frac{3\pi^2}{64}-\frac{2}{9}. \tag{49}$$

**Example 3.12.** *For* $a=\frac{5}{2}$, $b=\frac{1}{2}$ *in (48), we get*

$$\int_0^1 z\left[B_z\left(\frac{5}{2},\frac{1}{2}\right)\right]^2 dz=\frac{39\pi^2}{2048}-\frac{1}{10}. \tag{50}$$

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## Некоторые интегральные формулы, включающие произведения двух неполных бета-функций

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**Аннотация.** Целью данной статьи является получение некоторых интегральных формул, включающих произведения двух неполных бета-функций в терминах общих тройных гипергеометрических рядов и функции Кампаé де Фéриета. Некоторые новые частные интегральные формулы, включающие неполную бета-функцию, также вычисляются как приложение наших основных результатов с помощью теорем Уиппла, Диксона и расширения теоремы Диксона о суммировании.

**Ключевые слова:** неполная бета-функция, интегральные формулы, функция Кампа де Фье, общий тройной гипергеометрический ряд.