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Symmetry Analysis of Radial Profiles of Magnetic Skyrmions

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Abstract. The search for analytical profiles of chiral magnetic structures such as 2D magnetic skyrmions (MS) is important for their theoretical study. Since the Euler–Lagrange (EL) equations for such excitations are not solved exactly, the MSs are described using analytical ansatzs. In this work, we validate one of the widely used ansatzs based on a symmetry analysis of the 1D analog of the EL equations, which characterizes the radial profile of the MS. As a development of this approach, a profiles of skyrmion bags are proposed.

Keywords: magnetic skyrmions, chiral interactions, symmetry analysis of differential equations, skyrmion bags.

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Introduction

Skyrmions are topologically nontrivial field configurations that are solutions of nonlinear differential equations of physics. At the first time, such solutions has been considered by T. Skyrme in nuclear physics for the baryon field [1, 2]. Later, the similar fields, $\vec{m}(\vec{r})$, were found in magnetic systems [3] and liquid crystals, where $\vec{m} \in \mathbb{S}^2$, and $\vec{r} \in \mathbb{R}^{2,3}$. Recently, magnetic skyrmions were found experimentally [4–6], arousing scientific interest in themselves as promising objects for logic and memory devices of a new generation [7, 8]. The main practical interest to MS is due to their topological stability or, in other words, the inability to convert a nontrivial distribution $\vec{m}(\vec{r})$ with a topological index Q into the another one (corresponding to homotopy class with different Q) without overcoming a high energy barrier. The vast majority of studies considered the MSs with $Q = -1$ (in what follows we fix the chirality of the MS). However, the more exotic magnetic vortices has recently been predicted numerically [9–11] and experimentally confirmed [12, 13]. The last ones are, for example, magnetic skyrmionium ($Q = 0$) or skyrmion bags of nontrivial morphology (arbitrary Q).

The finding of new 2D magnetic structures actualized the problem of their analytical description. So, when describing the ordinary MS with $Q = -1$, the three types of analitycal ansatz are widely used: i) linear, ii) exponential, and iii) 2π-domain wall ansatz. In this paper, we

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demonstrate that the type iii) ansatz correspond to the symmetry of EL equations describing the MS profile in the XoZ plane. We also generalize type iii) ansatz to the case of skyrmion bags.

1. Energy and profile of the magnetic skyrmion

Let us consider a continuum model of a 2D chiral magnet with competing interactions. We introduce the magnetization field $\vec{m}(\vec{r})$: $|\vec{m}\,| = 1$, $\vec{r} \in \mathbb{R}^2$. Then, the excitation energy of an arbitrary configuration, $\vec{m}(\vec{r})$, compared to the ferromagnetic case, $m_z(r) \equiv 1$, has the form:

$$E_{cont} = \int\int \Big[\mathcal{J} \cdot \left((\nabla m_x)^2 + (\nabla m_y)^2 + (\nabla m_z)^2 \right) + \\ + \mathcal{D} \cdot \left(m_z \frac{\partial m_x}{\partial x} - m_x \frac{\partial m_z}{\partial x} + m_z \frac{\partial m_y}{\partial y} - m_y \frac{\partial m_z}{\partial y} \right) + \mathcal{A} \cdot (1 - m_z^2) \Big] dx \wedge dy \quad (1)$$

The first two terms describe the Heisenberg and the Dzyaloshinskii–Moriya interactions of the strengths $\mathcal{J}$ and $\mathcal{D}$, respectively. These interactions are competing, making it possible to exist of non-trivial configurations, such as MS. The third term describes the "easy-axis" single-ion anisotropy with an amplitude of $\mathcal{A}$, which defines a chosen space direction along the z-axis.

The MSs are axially symmetric configurations, and are standardly described by the parametrization:

$$m_x = \sin\tilde{\Theta}(r)\,\cos\varphi, \quad m_y = \sin\tilde{\Theta}(r)\,\sin\varphi, \quad m_z = \cos\tilde{\Theta}(r). \quad (2)$$

Here φ is the azimuthal magnetization angle, which coincides with the polar angle of the vector $\vec{r}$ in XoY plane, $\tilde{\Theta}(r)$ is the «skyrmion angle», depending on the polar radius r (the origin of polar coordinates coincides with MS center). The $\tilde{\Theta}$ vs. r dependence can be found from the EL equation:

$$l_w^2 \Delta_r \tilde{\Theta} - \frac{(l_w^2 + r^2)}{2r^2} \sin 2\tilde{\Theta} + 2\mathcal{E} \frac{\sin^2\tilde{\Theta}}{r/l_w} + \frac{\mathcal{H}}{2\mathcal{K}} \sin\tilde{\Theta} = 0. \quad (3)$$

where $l_w = \sqrt{\dfrac{\mathcal{J}}{\mathcal{K}}}$, $\mathcal{E} = \dfrac{\mathcal{D}}{2\sqrt{\mathcal{J}\mathcal{K}}}$, $\Delta_r \tilde{\Theta}(r) = \dfrac{1}{r}\,(\partial_r \tilde{\Theta} + \partial_r^2 \tilde{\Theta}\, r)$ — the Laplace operator in spherical coordinates. The equation (3) can be obtained by substituting (2) into the functional (1) and varying the latter against the $\tilde{\Theta}(r)$ and its derivatives. By now, no exact solution of Eq. (3) has been obtained. In order to simulate the numerical solutions the 2π-domain wall ansatz is widely used

$$\tilde{\Theta}_{-1}(r, R, w) = 2\,\mathrm{arctg}\left(\frac{\cosh R/w}{\sinh r/w} \right) \quad (4)$$

with two parameters R and w, where w describes the domain wall width, whereas R is the distance from the skyrmion center to domain wall middle.

Due to complexity of Eq. (3), let us consider a simplified situation where

$$m_x = \sin\Theta(x), \quad m_y = 0, \quad m_z = \cos\Theta(x). \quad (5)$$

Such situation correspond to the flat domain wall, varying along the Ox axis. Since morphologically MS is an axially symmetric domain wall, one can assume that $\Theta(x)$ approximates the radial dependence $\tilde{\Theta}(r)$ of the skyrmion angle for large MS (see Fig. 1). This assumption does not take into account azimuthal distortions of the MS texture. However, for this picture, it is possible to obtain a simple EL equation

$$\frac{d^2\Theta}{dx^2} = \frac{\mathcal{A}}{2\mathcal{J}} \sin 2\Theta \tag{6}$$

for the angle $\Theta(x)$. The exact solution

$$\Theta(x) = 2 \arctan(e^{x/w}), \tag{7}$$

of Eq. (6) can be obtained if $\mathcal{J} > 0$, $\mathcal{A} > 0$. It describes the π-domain wall, where $\vec{m}$ wraps by an angle π in the XoZ plane. The solution (7) is a partial one of the Eq. (6). Another solutions can be obtained by symmetry analysis of the Eq. (6).

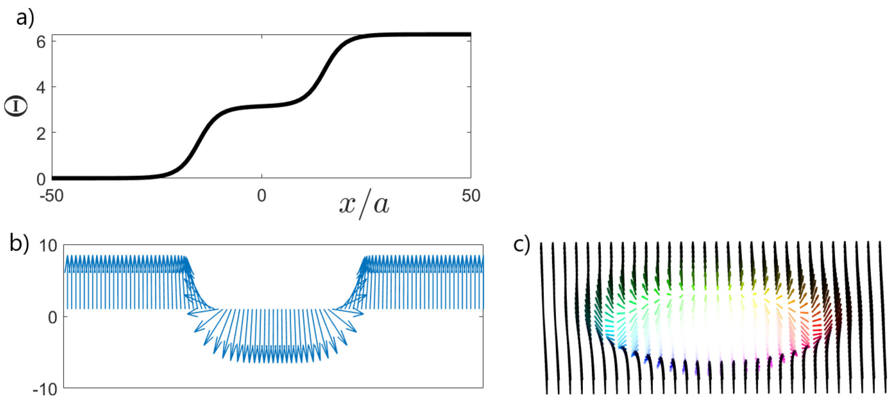

Fig. 1. a) visualization of a 2π-domain wall using a vector field (5) with an angle function $\Theta(x)$ defined by Eq. (22) with parameters $R = 15$, $w = 3$; b) the function $\Theta(x)$ profile; c) visualization of the vector field MS (see Eq. (2)) with the function $\Theta(r)$, see Eq. (4), with parameters $R = 15$, $w = 3$. The lengths are defined in the lattice parameter $a = 1$

2. Symmetry analysis of the approximate equation

Let us perform a symmetry analysis of the Eq. (6), and first consider its discrete symmetries. It can be seen that the equation retains under transformations: $x \to -x$ and $\Theta(x) \to -\Theta(x)$. Then, taking into account Eq. (7), one get a set of four exact solutions of Eq. (6):

$$\Theta(x) = \pm 2 \arctan(e^{\pm x/w}). \tag{8}$$

Now consider the continuous (Lie) symmetries of the equation. Then the replacing of variables

$$x' = F(t;\ x, \Theta, \dot{\Theta}), \quad \Theta' = \Phi(t;\ x, \Theta, \dot{\Theta}), \quad \dot{\Theta}' = \Psi(t;\ x, \Theta, \dot{\Theta}) \tag{9}$$

becomes continuously dependent on the parameter t, forming an infinite set. Hereafter $\dot{\Theta} = d\Theta/dx$, and Θ' denotes a new function retaining the form of Eq. (6) in new variables. In contrast to the discrete transformations (8), it can be shown that continuous ones should involve both the variable x as well as the functions Θ and $\dot{\Theta}$, see Eq. (9). The main goal of this changing is to find the mappings, that retain the form of Eq. (6). Then, knowing the discrete set of solutions (8), as well as the infinite set of mappings (9), one can obtain new solutions that depends on t.

It is convenient to implement this idea in the framework of the so called geometric approach to differential equations [16]. Let us consider a second-order equation of the general form

$$\mathcal{F}(x,\, \Theta,\, \dot{\Theta},\, \ddot{\Theta}) = \ddot{\Theta} - f(x, \Theta, \dot{\Theta}), \tag{10}$$

as a surface in the space of *independent* parameters $\mathcal{D} = \{x,\, \Theta,\, \dot{\Theta},\, \ddot{\Theta}\}$. In the geometric approach, the replacement of variables (9) corresponds to continuous paths in the space $\mathcal{D}$, depended on the starting points $\{x_0,\, \Theta_0,\, \dot{\Theta}_0,\, \ddot{\Theta}_0\}$ (in fact being a discrete set of solutions (8)) on the surface $\mathcal{F}$. This idea is visualized in Fig. 2 in the reduced space $\{x,\, \Theta,\, \dot{\Theta}\}$ for simplicity. By this

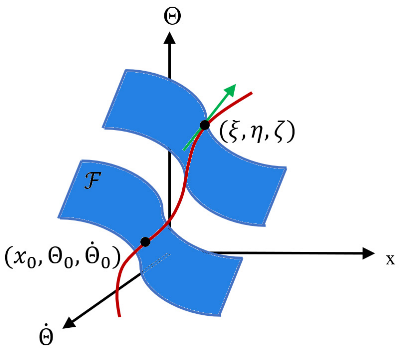

Fig. 2. Visualization of the idea of continuous paths in space depending on the initial points on the initial surface

approach, the invariance of the Eq. 10 under the mapping (9) means preserving the form of the surface $\mathcal{F}$ with a deformation where the points $\{x_0,\, \Theta_0,\, \dot{\Theta}_0,\, \ddot{\Theta}_0\}$ move along the paths defined by Eq. (9). At the same time, these paths determine at each point of the surface $\mathcal{F}$ the tangent vectors $\vec{l} = (\xi\ ,\eta,\ \zeta)$:

$$\xi = \left.\frac{\partial x'}{\partial t}\right|_{t=0} = \left.\frac{\partial F}{\partial t}\right|_{t=0}; \quad \eta = \left.\frac{\partial \Phi}{\partial t}\right|_{t=0}; \quad \zeta = \left.\frac{\partial \Psi}{\partial t}\right|_{t=0}.$$

Then, the problem of invariance of the surface $\mathcal{F}$ is reduced to finding its tangents with zero directional derivative:

$$\begin{aligned} \left.\frac{d}{dt}\mathcal{F}\right|_{t=0} &= \frac{\partial \mathcal{F}}{\partial x'}\frac{\partial x'}{\partial t} + \frac{\partial \mathcal{F}}{\partial \Theta'}\frac{\partial \Theta'}{\partial t} + \frac{\partial \mathcal{F}}{\partial \dot{\Theta}'}\frac{\partial \dot{\Theta}'}{\partial t} = \\ &= \left(\xi\frac{\partial \mathcal{F}}{\partial x'} + \eta\frac{\partial \mathcal{F}}{\partial \Theta'} + \zeta\frac{\partial \mathcal{F}}{\partial \dot{\Theta}'}\right) = (\xi,\, \eta,\, \zeta)\cdot \nabla\mathcal{F}\,|_{t=0} = 0. \end{aligned} \tag{11}$$

Hence, the generator of Lie symmetries of Eq. (10) is determined as

$$\begin{aligned} &\hat{O} = \xi\partial_x + \eta\partial_\Theta + \zeta\partial_{\dot{\Theta}}, \\ &\hat{O}\,\mathcal{F} = 0. \end{aligned} \tag{12}$$

This property, together with the Eq.(10), defines the necessary conditions for finding the functions ξ, η and ζ. However, these conditions are not complete yet. So, considering in the space $\mathcal{D}$ the variables x, Θ, $\dot{\Theta}$ and $\ddot{\Theta}$ as independents, we neglected in fact the bonds between them:

$$d\Theta - \dot{\Theta}\, dx = 0, \quad d\dot{\Theta} - \ddot{\Theta}\, dx = 0. \tag{13}$$

In order to account the last ones, let us continue the space $\mathcal{D} = \{x,\, \Theta,\, \dot{\Theta},\, \ddot{\Theta}\}$ to the expanded space $\tilde{\mathcal{D}} = \{x,\, \Theta,\, \dot{\Theta},\, \ddot{\Theta},\, d\Theta,\, d\dot{\Theta},\, dx\}$, continuing the symmetry generator as well:

$$\hat{O} \to \hat{\tilde{O}} = \xi \partial_x + \eta \partial_\Theta + \zeta \partial_{\dot{\Theta}} + \dots .$$

Then, in the expanded space, besides the invariance of the equation, it is necessary to consider the invariance of differential bonds in a similar way. The latter is the subject of the theory of continuations described in [16]. This approach allows us to consider the invariance of the *differential*, and not the *algebraic* equation. Then, following the Ref. [16], the sufficient conditions for searching ξ, η and ζ are reduced to the so called system of splittings:

$$\eta_{xx} = f_x\, \xi + f_\Theta \eta - f\, \eta_\Theta + 2f\, \xi_x; \tag{14}$$

$$2\eta_{x\Theta} - \xi_{xx} = 3f\, \xi_\Theta; \tag{15}$$

$$\eta_{\Theta\Theta} - 2\xi_{x\Theta} = 0; \tag{16}$$

$$-\xi_{\Theta\Theta} = 0. \tag{17}$$

Here, the lower indices indicate the derivatives of the corresponding variables. In our case, $f = a \sin 2\Theta$, see Eq. (6) and Eq. (10).

Starting solving system from the last equation (16), we find:

$$\xi = A\Theta + B; \quad A = A(x), \quad B = B(x) \tag{18}$$

Next, from Eq.(15) we find also solution for η:

$$\eta = A'\Theta^2 + C\Theta + D; \quad C = C(x), \quad D = D(x). \tag{19}$$

Going to Eq. (14), we obtain that $3A''\Theta + 2C' = B'' + 3aA \sin 2\Theta$. Differentiating this equation by Θ, we get:

$$\frac{A''(x)}{A(x)} = f_\Theta(\Theta) \Rightarrow \frac{A''}{A} = f_\Theta = K = \text{const}. \tag{20}$$

Finally, consider Eq. (14). So that the solution is not trivial, let us put $A = 0$. Then, we get: $C_{xx}\Theta + D_{xx} = 2a\, C\, \Theta\, \cos 2\Theta + 2D\, a\, \cos 2\Theta - a\, C\, \sin 2\Theta + 2B_x\, a\, \sin 2\Theta$. Expanding $\cos 2\Theta$ and $\sin 2\Theta$ into a Taylor series and considering coefficients at various degrees of Θ as independent ones, we find that D, C, $B_x = 0$. If we consider again the operator $\hat{O}$, we will see that $\hat{O} = \partial_x$. Thus, the group of Lie symmetries of Eq. (6) is poor and reduces to translations $x \to x + C$. As a result, the analysis of Eq.(6) allows to generate from the exact solution (7) a set of others:

$$\Theta(x) = \pm 2 \arctan \left(e^{(\pm x + C)/w} \right). \tag{21}$$

Such solutions describe a set of domain walls, with the middle at points $x = \pm C$, where the angle $\Theta(x)$ changes from 0 to $\pm\pi$ (or from $\pm\pi$ to 0) when x changes from $x = -\infty$ to $x = \infty$. It can be shown that if two domain walls $\Theta_1(x)$ and $\Theta_2(x)$ from the class of solutions (21) overlap slightly, then the sum of $\Theta = \Theta_1 + \Theta_2$ is also an approximate solution of Eq. (6). This allows to build combinations of solutions of the type $0\pi-$ or 2π-domain walls. The latter one is:

$$2 \arctan \left(e^{\frac{-x-R}{w}} \right) + 2\, \mathrm{arctg} \left(e^{\frac{x-R}{w}} \right) = 2 \arctan \left(\frac{\cosh \frac{x}{w}}{\sinh \frac{R}{w}} \right) \tag{22}$$

It is a 1D variant of the already mentioned ansatz for MS, see Eq. (22).

Fig. 1a shows the dependence of the function $\Theta(x)$ (22). Next, Fig. 1b shows a visualization of the vector field $\vec{m}(x)$, see Eq. (5), for such a domain wall. Finally, Fig. 1c shows a similar visualization of the MS profile, see Eq. (2), where the free parameters for the functions $\tilde{\Theta}(r)$ and for $\Theta(x)$ are chosen the same. It can be seen that the MS profile can be considered with good accuracy as a 1D 2π-"rotation body" of the domain wall. This makes it possible to generalize the MS profile to more exotic magnetic structures.

3. Some generalizations of the skyrmion profile

Summarizing above, it is possible to construct other combinations that will describe axially symmetric domain walls. So, by choosing a combination of four exact solutions of Eq. (6) with replacing $x \to r$, one can obtain an expression for the so-called skyrmionium angle

$$\tilde{\Theta}_0(r) = \pi - 2\arctan\left(\frac{\cosh\left(R_1/w_1\right)}{\sinh\left(r\ /\ w_1\right)}\right) - 2\arctan\left(\frac{\cosh\left(R_2/w_2\right)}{\sinh\left(r\ /\ w_2\right)}\right), \tag{23}$$

generalizing the two-parameter MS ansatz (4). Magnetic excitation, which profile is described by (2) and (23) is called magnetic skyrmionium. It has a topological charge $Q = 0$, which is depicted by the lower subscript on the radial function $\Theta_0(r)$. A comparison of the radial dependencies of the skyrmion and skyrmionium magnetizations, $m_z = \cos\tilde{\Theta}$, is shown in Fig. 3. It can be seen that the latter can be considered as an axially symmetric pair of successive domain walls. At the same time, in (23), the parameters w_1 and R_1 (w_2 and R_2) characterize the width and distance to the middle of the first (second) domain wall. Recently, magnetic skyrmioniums have been discovered experimentally [12].

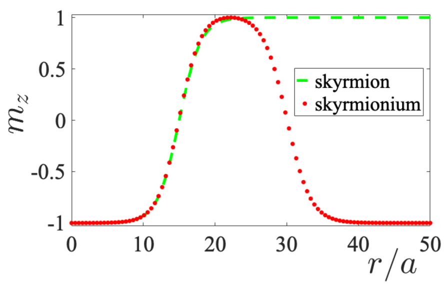

Fig. 3. Dependencies of the component $m_z = \cos\tilde{\Theta}$ for the skyrmion (4) and the skyrmionium (23) on the polar radius r. Parametrization for the skyrmionium corresponds to situations where the component $m_z = \cos\Theta_0 = -1$ at the points $r = 0$ and $r = \infty$. The parameters of ansatz (4) and (23) in units of a are: $R = R_1 = 15$, $R_2 = 30$, $w_1 = 3$, $w_2 = 3.3$

The knowledge of the skyrmionium profile allows us to construct profiles of C_g-symmetric exotic magnetic structures — the skyrmion bags. They have a topological charge $Q = g - 1$ and have recently been discovered both numerically [9–11] and experimentally [13]. Numerical experiments have shown [9] that to stabilize skyrmion bags with $Q = g - 1$, g magnetic skyrmioniums located near to each other should be chosen as the seed magnetic configuration. A similar idea can be developed in the analytical construction of skyrmion bags profiles. We will consider the skyrmionium (23) to be the generating function of the skyrmion bags. Consider the cut of the magnetic skyrmionium m_z-profile by the plane XoZ. This dependence has a two-peak structure, which can be obtained by mirroring the dependency in Fig. 3. Now, let us shift the origin from the point $(x, y) = (0, 0)$ to the peak point, $(x,\ y) = (-a,\ 0)$, and move on to the polar system centered at the latter. So, we first replace the variables in dependence $\vec{m}(r(\tilde{x}, \tilde{y}), \varphi(\tilde{x}, \tilde{y}))$ in the following way:

$$\begin{pmatrix} \tilde{x} \\ \tilde{y} \end{pmatrix} \to \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \tilde{x} \\ \tilde{y} \end{pmatrix} + \begin{pmatrix} a \\ 0 \end{pmatrix}, \tag{24}$$

followed by the second replacement $\tilde{x} \to r',\ \tilde{y} \to 0$. At this stage, we have introduced a new parametrization of the magnetic skyrmionium, whose field now depends on the variables $r',\ \theta$ of the new polar system: $\vec{m} = \vec{m}(r', \theta)$. Now, let us make the third substitution: $r' \to r^g$, $\theta \to g \cdot \varphi$, where $r \in [0, \infty)$, $\varphi \in [0, 2\pi)$. As a result, the g replicas of the skyrmionium profile are obtained, that are slightly deformed at small r and large g. Let us write the Eq. (24) in matrix form: $\tilde{\vec{r}} \to \vec{r} = \tilde{O}(\theta) \cdot \tilde{\vec{r}} + \tilde{\vec{a}}$ and apply the described transformations to the function $\tilde{\Theta}_0(\vec{r})$, see Eq. (23). Thus, one obtain the so-called angle of skyrmion bags $\tilde{\Theta}_g(r, \varphi)$:

$$\Theta_0(\tilde{\vec{r}}) \to \tilde{\Theta}_g(r,\varphi) = \pi \quad - \quad 2\arctan\left(\frac{\cosh\left(R_1/w_1\right)}{\sinh\left(\sqrt{a^2 + r^{2g} + 2a \cdot r \cdot \cos g\varphi}\ /\ w_1\right)}\right) - \\ - \quad 2\arctan\left(\frac{\cosh\left(R_2/w_2\right)}{\sinh\left(\sqrt{a^2 + r^{2g} + 2\,a \cdot r \cdot \cos g\varphi}\ /\ w_2\right)}\right)\Bigg]. \qquad (25)$$

The profiles of the components $m_z = \cos\tilde{\Theta}_g$ are shown in Fig. 4 for the cases: a) $g = 1$ – magnetic skyrmionium; b) $g = 2$ and c) $g = 3$. It can be shown that functions b) and c) approximate with a good accuracy the numerical profiles of skyrmion bags with topological charges $Q = 1$ and $Q = 2$, respectively.

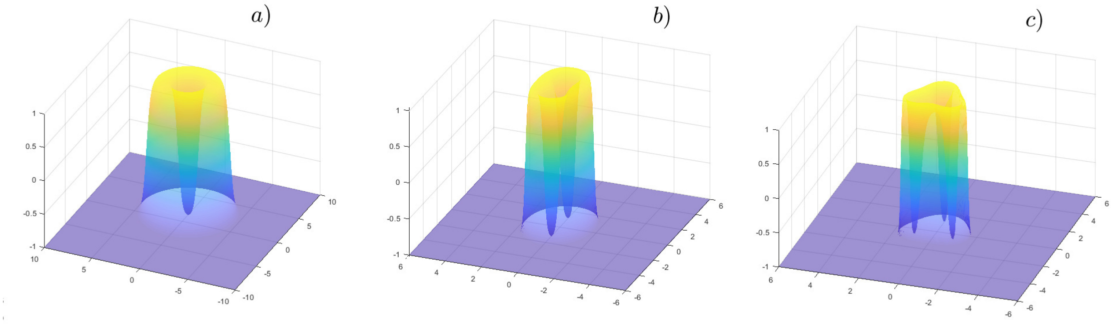

Fig. 4. Dependencies $m_z(r) = \cos\Theta_g$, where Θ_g is defined by (25). Parameters $\{R_1, R_2, w_1, w_2, g\}$ are: a) $\{7.66, 26.47, 2.80, 3.08,\ 1\}$, b) $\{7.86,\ 26.45,\ 2.83,\ 3.08,\ 2\}$, c) $\{7.56,\ 26.34,\ 2.90,\ 3.09,\ 3\}$

However, note that the Eq. (25) approximates the numerical profiles of the skyrmion bags with good accuracy only for small topological indices Q. For large Q, the peculiarities in the analytical profiles, that are absent in numerical one appear. A more correct construction of analytical profiles for skyrmion bags will be the subject for further studies.

Summary

The Euler–Lagrange equations are found for magnetic excitations in 2D chiral magnets with competing interactions: Heisenberg, Dzyaloshinskii–Moriya, and the "easy axis" single-ion anisotropy as well. These equations were found for two classes of magnetic excitations: i) axially symmetric magnetic vortices, such as magnetic skyrmions, and ii) flat domain walls. It is noted that in the case i) the EL equations cannot be solved analytically, and their numerical solutions can be approximated by an axially symmetric 2π-domain wall analytical ansatz. Whereas in case ii), the partial solutions as well as the Lie symmetries of corresponding EL equations can be obtained exactly, despite the approximate character of the description.

The symmetry analysis of the EL equations for the case ii) was carried out in the framework of the so-called "continuation theory". The main idea of latter is to find new exact solutions from the previous ones with the use of the symmetries of the EL equation. It was shown that the EL equations describing the flat domain walls (case ii)) have only the translation symmetry of the domain wall middle. Moreover, the sum of solutions for several domain walls displaced by a significant distance relative to each other is also an approximate solution of the equation.

The latter feature allowed us to justify the choice of a 2π-domain wall type ansatz for a magnetic skyrmion. Moreover, continuing this idea for more complex combinations of closed domain walls, including axially asymmetric ones, allowed us to propose analytical ansatzs for more exotic magnetic vortex structures — the skyrmion bags, recently discovered both numerically and experimentally. A detailed analysis and justification of the proposed parametrization will be the subject for further study.

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Анализ симметрии радиальных профилей магнитных скирмионов

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Аннотация. Поиск аналитических профилей киральных магнитных структур типа 2D магнитных скирмионов (МС) является важным при их теоретическом описании. Поскольку уравнения Эйлера–Лагранжа (ЭЛ) для таких возбуждений не решаются точно, описание МС проводят с помощью аналитических пробных функций — анзацев. В настоящей работе проводится обоснование одного из широко используемых анзацей на основе симметрийного анализа 1D версии уравнений ЭЛ, определяющего радиальный профиль МС. В развитии такого подхода предлагаются профили скирмионных мешков с топологическими зарядами $Q \geqslant 0$.

Ключевые слова: магнитные скирмионы, киральные взаимодействия, симметрийный анализ дифференциальных уравнений, скирмионные мешки.