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Permeability Anisotropy of Fractured Reservoirs

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A mathematical model of permeability in fractured reservoir beds which have block structure has been presented. The functional dependence of permeability index for arbitrary direction on geometrical size of the block has been obtained.

Keywords: permeability anisotropy, permeability index, reservoir beds, oil and gas deposits.

Introduction

The permeability index K is one of the most important features of a reservoir bed, which reflects the ability of a bed to filter the fluids under the pressure gradient. Keeping this feature in mind is an essential condition for the correct operation of the oil and gas deposits.

For the usual, the so called porous, isotropic reservoirs with intergrain permeability, the coefficient K is determined by the Darcy filtration law [1]:

$$u = \frac{Q}{S} = K \frac{\Delta P}{\mu L} \quad (1)$$

in which u is filtration velocity, Q is fluid expenditure volume, S is the area of filtration, ΔP is the pressure differential over the distance L , μ is the coefficient of dynamic viscosity of the fluid. The permeability measure unit is $1 \mu\text{m}^2$. The permeable reservoirs are described as $K > 10^{-2} \mu\text{m}^2$, the impermeable ones are described as $K < 10^{-4} \mu\text{m}^2$.

The discovery of oil deposits in West Texas, in the Middle East and in the southern regions of the USSR gave rise to a heightened interest to fractured reservoirs in the 30-50s of the last century. It turned out that the permeability anisotropy, conditioned by fractureability of reservoirs, significantly influences upon the character of hydrodynamic processes, taking place in the working seam. There are numerous examples of the distinct difference in work modes of producing wells when they are located at an equal distance from the pumping well.

The majority of the research of the fractured reservoirs has been carried out overseas, though there are also significant works by the scientists of our country [1]. A sufficiently good survey of the existing information on the fractured reservoirs is presented in the monograph by T.D.Golf-Raht [2]. In the recent decades, when the main interest was focused on the deposits in Western Siberia, little attention was paid to the study of fractured reservoirs in Russian's oil-field geophysics. Also it should be noted that it was possible to describe more or less successfully the discovered and

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developed reserves (both in carbonate and in terrigenous stratum) using the framework of the reservoir pseudoisotropic permeability subordinate to the Darcy Law [2]. But recent research of perspective deposits in Eastern Siberia shows that the reservoir beds of these deposits are typical fractured beds with regular fracture formation. This calls for an appropriate modeling of the hydrodynamic processes in such beds considering their permeability anisotropy. Anisotropy permeability data can help essentially in matching production history, while interpreting the results of interference test and hydrodynamic research.

1. Statement of Problem

We shall treat the settled laminar flows in a fractured bed in which there are permeable fractures with flatly parallel walls and impermeable blocks. The flow laminarity is conditioned by the smallness of the Reynolds number $Re = \rho \cdot u \cdot 2g / \mu$ in narrow fractures with the typical opening value $g \sim 100 \mu\text{m}$ and the flow velocity $u \ll 1 \text{ m/s}$. The weight force compared to the pressure gradient ∇P is also neglected in equations of Navier-Stokes. Then the average velocity \vec{q}_k of Poiseuille's plane-parallel flow in fracture k with opening g_k can be determined as follows:

$$\vec{q}_k = -\frac{g_k^2}{12\mu} \cdot \nabla P. \quad (2)$$

The basic ideas of the fluid flow modeling through the fracture system in a bed belong to S.E.Romm [1]. According to [1] the fluid flow velocity in k direction is computed by means of the formula

$$\vec{u}_k = g_k f_k \vec{q}_k = -\frac{g_k^3}{12\mu} f_k (\nabla P \cdot \vec{k}) \cdot \vec{k}, \quad (3)$$

where f_k is the linear density of fractions (the number of fractions per linear unit in the direction perpendicular to the selected), \vec{k} is the unit vector in the k direction. The overall flow velocity \vec{u}_k is the sum of velocities in n fractures. Introducing the permeability tensor \mathbf{K} of the bed, the equation (3) can be written in the following form [1]

$$\vec{u} = -\mathbf{K} \frac{\nabla P}{\mu}, \quad (4)$$

where the tensor \mathbf{K} has the components

$$K_{ij} = \frac{1}{12} \sum_{k=1}^n g_k^3 f_k (\delta_{ij} - \alpha_{ik} \cdot \alpha_{jk}). \quad (5)$$

In the equation (5) δ_{ij} is the Kronecker symbol, α_{ik} are the cosines of the angles between the coordinate axes and the directions of the unit vectors perpendicular to vector \vec{k} . If the coordinate axes coincide with the main axes of the permeability tensor, then tensor \mathbf{K} assumes a diagonal shape.

Equations (4) and (5) are, as a matter of fact, a formal solution of the problem of permeability anisotropy of fractured reservoirs with impermeable matrix. But it is hard to use it in practice as it requires exhaustive information on the fractured reservoir (dip angles and strike azimuth, their linear density and what is more important their opening in different directions).

This type of information can be obtained with the help of fullbore formation microimager FMI. However, in the overwhelming majority of East Siberian cased boreholes the FMI was not applied. That is why one has to use other data, such as, the results of core macro fissuring analysis,

the well logging interpretation, seismic survey data, as well as the well testing permeability $K_{well\ test}$ determined by the results of hydrodynamic well tests. In [2] it is stated that $K_{well\ test}$ can be assumed to be the average for fractured rock. Estimation of fracture opening g_k according to core seems incorrect, as the g_k in the core raised to the day surface differs from their opening under the bed conditions [3].

The purpose of this work is to express the permeability anisotropy of fractured reservoirs by means of experimentally determined linear sizes of the bed by well testing results.

2. Permeability Coefficient Computing

Suppose that the reservoir is an aggregate of a large number of the same size and equally oriented in space rectangular blocks. We will orient the Cartesian system of coordinates along the sides of the blocks. Let x_0, y_0, z_0 be the block sizes along the corresponding axes, L be the typical linear size of the bed and $L^3 \gg x_0 \cdot y_0 \cdot z_0$. Then the number of fractions parallel to OX axis will be $N_x = \frac{L}{y_0} \cdot \frac{L}{z_0} = \frac{L^2}{y_0 z_0}$. Hence the linear fraction density along the OX axis is $f_x = \frac{N_x}{L} = \frac{L}{y_0 z_0}$. By analogy for the OY and OZ axes we obtain $f_y = \frac{N_y}{L} = \frac{L}{x_0 z_0}$, $f_z = \frac{N_z}{L} = \frac{L}{x_0 y_0}$. Let's assume that all the fracture openings are the same and equal to g , and the pressure gradient in all directions is constant and equal to $\Delta P/L$. Then according to (3)

$$u_x = -\frac{g^3}{12} \frac{L}{y_0 z_0} \frac{\Delta P}{\mu L}, \quad u_y = -\frac{g^3}{12} \frac{L}{x_0 z_0} \frac{\Delta P}{\mu L}, \quad u_z = -\frac{g^3}{12} \frac{L}{x_0 y_0} \frac{\Delta P}{\mu L}.$$

Anisotropy coefficients along the selected axes are:

$$K_x = \frac{g^3 L}{12 y_0 z_0}, \quad K_y = \frac{g^3 L}{12 x_0 z_0}, \quad K_z = \frac{g^3 L}{12 x_0 y_0}.$$

Let's introduce the bed anisotropy parameters:

$$a = \frac{u_x}{u_y} = \frac{K_x}{K_y} = \frac{x_0}{y_0}, \quad b = \frac{u_x}{u_z} = \frac{K_x}{K_z} = \frac{x_0}{z_0}. \quad (6)$$

Let at a certain moment at the origin of the selected system of coordinates a fluid pressure gradient be created in the bed. If the medium is isotropic in permeability, then the flow front equation at the moment of time τ may be written down as

$$\tau = \frac{\sqrt{x^2 + y^2 + z^2}}{u} = \text{const},$$

where u is the flow velocity. The flow front has the shape of a sphere. If the fluid spreads only along the fractures, then the flow front equation from the spot source at the moment τ will have the following form:

$$\tau = \frac{x}{u_x} + \frac{y}{u_y} + \frac{z}{u_z} = \text{const},$$

or taking into account (6)

$$x + a \cdot y + b \cdot z = \text{const}. \quad (7)$$

If the distance to the front exceeds the linear sizes of impermeable blocks, then in (7) we can proceed to increments:

$$dx + a \cdot dy + b \cdot dz = 0. \quad (8)$$

In the spherical coordinates (r, θ, φ) the equation (8) can be written as follows

$$\frac{dr}{r} = \frac{\sin \theta \cdot \sin \varphi - a \cdot \sin \theta \cdot \cos \varphi}{\sin \theta \cdot \cos \varphi + a \cdot \sin \theta \cdot \sin \varphi + b \cdot \cos \theta} d\varphi - \frac{\cos \theta \cdot \cos \varphi + a \cdot \cos \theta \cdot \sin \varphi - b \cdot \sin \theta}{\sin \theta \cdot \cos \varphi + a \cdot \sin \theta \cdot \sin \varphi + b \cdot \cos \theta} d\theta.$$

Hence

$$r(\theta, \varphi) = C \frac{1 + \operatorname{tg}^2 \frac{\varphi}{2}}{\chi(\theta, \varphi) \cdot \psi(\theta, \varphi)} = C \cdot w(\theta, \varphi), \quad (9)$$

where C is the integration constant,

$$\chi(\theta, \varphi) = \sin \theta \cdot \cos \varphi + a \cdot \sin \theta \cdot \sin \varphi + b \cdot \cos \theta, \quad (10)$$

$$\psi(\theta, \varphi) = \sin \theta \cdot \left(1 + 2 \cdot a \cdot \operatorname{tg} \frac{\varphi}{2} - \operatorname{tg}^2 \frac{\varphi}{2} \right) + b \cdot \cos \theta \cdot \left(1 + \operatorname{tg}^2 \frac{\varphi}{2} \right), \quad (11)$$

$$w(\theta, \varphi) = \frac{1 + \operatorname{tg}^2 \frac{\varphi}{2}}{\chi(\theta, \varphi) \cdot \psi(\theta, \varphi)}. \quad (12)$$

According to (9) the flow front geometry is determined by functional form $w(\theta, \varphi)$, which, in its turn, depends on anisotropy parameters of the bed a and b . The obtained solution (9) makes physical sense only for $\varphi \in [0, \pi/2]$, $\theta \in [0, \pi/2]$. The solution for other directions can be easily constructed proceeding from the flow symmetry relative to the XOZ , XOY and YOZ planes. The considered solution peculiarities are typical for the spherical coordinates system, in which the cross points of Cartesian system of coordinates with a sphere are singular points.

The absolute minimum $w(\theta, \varphi)$ corresponds to the polar angle $\theta_{min} = \arcsin \sqrt{\frac{1+a^2}{1+a^2+b^2}}$ and to the azimuth angle $\varphi_{min} = \arcsin \frac{a}{\sqrt{1+a^2}}$. In the direction of OX, OY, OZ axes the functions $w(\theta, \varphi)$ are: $w\left(\frac{\pi}{2}, 0\right)=1$, $w\left(\frac{\pi}{2}, \frac{\pi}{2}\right)=\frac{1}{a^2}$, $w(0, \varphi) = \frac{1}{b^2}$.

We will find the integration constant C in equation (9) proceeding from the fitting condition with the average permeability data $K_{well \ test}$ according to well test results. The bed is supposed to be isotropic in these measurements, that is why the seeming position of the flow front r_τ at the time τ in concordance with (1) is determined by the equation

$$r_\tau = |u| \cdot \tau = K_{well \ test} \cdot \frac{\Delta P}{\mu L} \cdot \tau = C_1 \cdot K_{well \ test}. \quad (13)$$

On the other hand, the value r_τ computed by means of the equation (9) corresponds to the average value of the radius vector, which determines the position of the real flow front, namely

$$r_\tau = \frac{1}{2\pi^2} \int_0^\pi \int_0^{2\pi} r(\theta, \varphi) d\theta d\varphi = C \cdot \bar{w}, \quad (14)$$

where

$$\bar{w} = \frac{1}{2\pi^2} \int_0^\pi \int_0^{2\pi} \frac{1 + \operatorname{tg}^2 \frac{\varphi}{2}}{\chi(\theta, \varphi) \cdot \psi(\theta, \varphi)} d\theta d\varphi. \quad (15)$$

For fractured ground at the same time τ

$$r(\theta, \varphi) = K(\theta, \varphi) \frac{\Delta P}{\mu L} \cdot \tau = C_1 \cdot K(\theta, \varphi) = C \cdot w(\theta, \varphi). \quad (16)$$

Thus, from (13), (14) and (16) we obtain the system of equations

$$\begin{cases} C_1 \cdot K_{well\ test} = C \cdot \bar{w}, \\ C_1 \cdot K(\theta, \varphi) = C \cdot w(\theta, \varphi). \end{cases}$$

After eliminating the constants C and C_1 we get

$$K(\theta, \varphi) = K_{well\ test} \cdot \frac{w(\theta, \varphi)}{\bar{w}}. \quad (17)$$

The equation (17) is the solution of the posed problem. The total permeability $K_{well\ test}$ is determined by the results of hydrodynamic survey of the wells, the $w(\theta, \varphi)$ and \bar{w} functions are computed by means of the equations (9) and (15) and depend only on the geometric sizes of the blocks. The fracture opening value is hard to determine experimentally, but we are not dealt with this value in the equation (17). It is also important to mention that the permeability in fractured beds is proportional to the cube of fracture opening.

3. Model Example of Permeability Anisotropy

To demonstrate how much $K_{well\ test}$ may differ from the real bed permeability in the given directions let's consider the following example. Let the block geometry be given by $a = x_0/y_0 = 2$, $b = x_0/z_0 = 5$. According to (15) $\bar{w} = 0,07$. Impermeable blocks are stretched out along the OX axis, the maximum linear fracture density is perpendicular to it, that is why $Max(K/K_{well\ test}) = 14,29$ is achieved along the OX axis ($\theta = \pi/2, \varphi = 0$). $Min(K/K_{well\ test}) = 0,47$ is for the direction $\theta_{min} = 24^\circ$. Therefore, $K_{max}/K_{min} = 30,4$. From this example it is evident that in certain directions the bed permeability may exceed the one measured by well test approximately in single order or more.

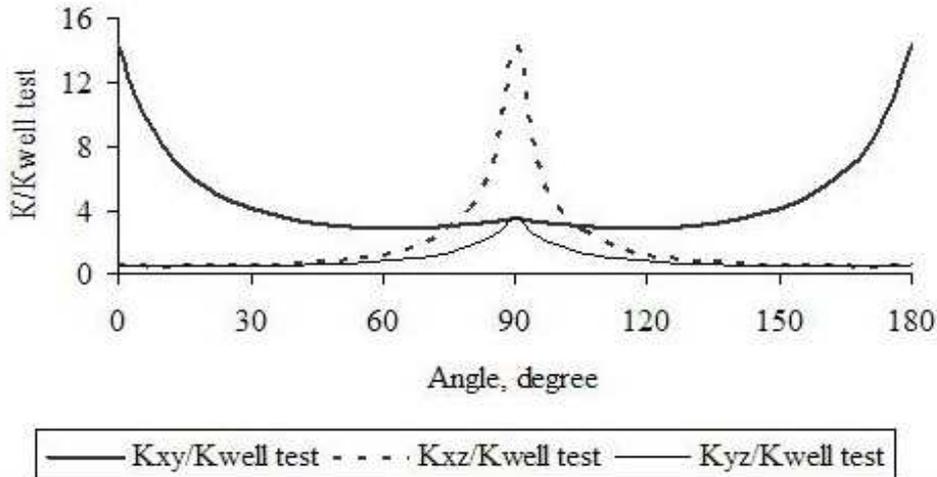


Fig. 1. Angle dependence of permeability in XOY, XOZ, YOZ planes ($a=2$, $b=5$)

The relative permeability changes in three main planes of the Cartesian system of coordinates for this case are presented in fig. 1.

The ratio K_{max}/K_{min} can be treated as the fractured bed's permeability anisotropy feature. The changes of this value depending on the value of parameter b are presented in fig. 2. The value of parameter a was fixed and equal to 1. In this case

$$\frac{K_{max}}{K_{min}} = \frac{1}{w(\theta_{min}, \varphi_{min})},$$

where $\theta_{min} = \arcsin \sqrt{\frac{2}{2+b^2}}$, $\varphi_{min} = \arcsin \frac{1}{\sqrt{2}} = 45^\circ$.

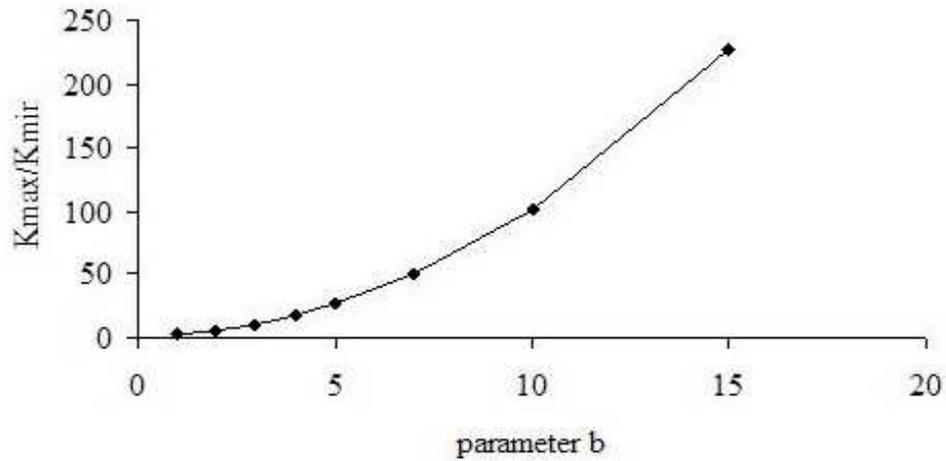


Fig. 2. Permeability anisotropy depending on b parameter ($a=1$)

According to computations for $a=1$, $b=1$ the ratio $K/K_{well\ test}$ changes from 0,66 to 1,98, for $a=1$, $b=10$ the ratio $K/K_{well\ test}$ changes from 0,19 to 19,61. In other words, the permeability of fractured beds in certain directions may be several times less, as well as dozens of times more than the permeability measured by the well test results.

4. Transformation of Coordinates

The geophysical survey is carried out and interpreted in the system of coordinates $OX'Y'Z'$ connected with the day surface: OZ' axis goes vertically down, OX' axis runs to the north, OY' axis to the east. To obtain type function $w(\theta, \varphi)$ in the system of coordinates $OX'Y'Z'$, it is necessary to turn the original system of coordinates $OXYZ$, connected with the system of orthogonal fractures around the general point of origin using the Euler angles ψ_0 , θ_0 and φ_0 [4]. If the OX axis is oriented along the line of knots, then these three angles in our case can be defined as follows: the angle of precession ψ_0 is the azimuth of the dip angle of the fractures oriented along the OZ axis; the nutation angle θ_0 is the supplementary angle to the dip angle of these fractures; the pure rotation angle φ_0 is the dip angle of the fractures oriented along the OX axis (the strike azimuth angle of these fractures is $\psi_0 = 180^\circ$). The rotation matrix of the $OXYZ$ system in the $OX'Y'Z'$ position around the general point of origin has the following components, expressed in terms of Euler angles [4]:

$$\begin{aligned}
c_{11} &= \cos \psi_0 \cos \varphi_0 - \sin \psi_0 \sin \varphi_0 \cos \theta_0; & c_{21} &= -\cos \psi_0 \sin \varphi_0 - \sin \psi_0 \cos \varphi_0 \cos \theta_0; \\
c_{12} &= \cos \psi_0 \sin \varphi_0 + \sin \psi_0 \cos \varphi_0 \cos \theta_0; & c_{22} &= -\sin \psi_0 \sin \varphi_0 + \cos \psi_0 \cos \varphi_0 \cos \theta_0; \\
c_{13} &= \sin \varphi_0 \sin \theta_0; & c_{23} &= \cos \varphi_0 \sin \theta_0; \\
c_{31} &= \sin \psi_0 \cos \theta_0; & c_{32} &= -\cos \psi_0 \sin \theta_0; & c_{33} &= \cos \theta_0.
\end{aligned} \tag{18}$$

Then the equation of transformation of coordinates (θ, φ) in the $OXYZ$ system into the (θ', φ') system in the matrix type will look as follows:

$$\begin{pmatrix} \sin \theta' \cos \varphi' \\ \sin \theta' \sin \varphi' \\ \cos \theta' \end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{pmatrix} \cdot \begin{pmatrix} \sin \theta \cos \varphi \\ \sin \theta \sin \varphi \\ \cos \theta \end{pmatrix}. \tag{19}$$

5. Conclusion

The main feature of the model of the fractured reservoir's permeability anisotropy presented above is that the fracture opening value that is hard to determine in experiments has not been dealt with. The permeability of the reservoir can be computed within the frames of this model in any selected direction. It is not difficult to compute the diagonal components of permeability tensor, if this is required by the need to use the obtained results in the standard simulation program of the hydrodynamic processes in the reservoirs. It is worthwhile to mention once again those simplifications which have been made regarding a real fractured reservoir. Firstly, the spatial orientation and linear sizes of impermeable blocks were taken to be constants, and the blocks were presented as rectangular parallelepipeds. Secondly, the fracture permeability in three orthogonal directions was taken to be a constant. Thirdly, we neglected the cavernous and intergrain permeability.

The permeability model presented above was used in computing for one of the Yurubchenskoe oil field deposit sites. It is expected that the peculiarities of geological, geophysical and fluidodynamic characteristics of this deposit allow us to apply simplified mathematical models in estimation of the physical properties of reservoirs. It has been illustrated by means of this example that in certain directions permeability can exceed more than one order.

The permeability computed by the results of well-logging, and the permeability anisotropy (maximal permeability ratio to the minimal one) may be more than one hundred. All these circumstances have to be taken into consideration while preparing this deposit for development.

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