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Power Comparisons of EDF Goodness-of-Fit Tests

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Abstract. In this article, the power of common goodness-of-fit (GoF) statistics is based on the empirical distribution function (EDF) where the critical values must be determined by simulation. The statistical power of Kolmogorov–Smirnov D_n, Cramér-von Mises W^2, Watson U^2, Liao and Shimokawa L_n, and Anderson–Darling A^2 statistics were investigated by the sample size, the significance level, and the alternative distributions, for the generalized Rayleigh model (GR). The exponential, the Weibull, the inverse Weibull, the exponentiated Weibull, and the exponentiated exponential distributions were considered among the most frequent alternative distributions.

Keywords: generalized Rayleigh distribution, Kolmogorov–Smirnov test, the Cramér-von Mises test (C-VM), Anderson–Darling test (A-D), Watson test (W), Liao and Shimokawa test (LS).

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Introduction

Statistical analysis means investigating trends, patterns, and relationships using quantitative data. It is an important research tool used by scientists, governments, businesses, and other organizations. Many statistical analysis tools rely on assumptions of underlying distributions. The goodness-of fit problem is to validate such assumptions before applying those tools to data, therefore it arises in applications of many statistical approaches. Many goodness of-fit tests (GoF) have been developed, and most of them are based on the empirical distribution functions (EDF), the old one, being the Kolmogorov–Smirnov (K-S) statistic D_n (Kolmogorov 1933). Later, Cramér-Von Mises W^2 statistics have been shown to be more powerful than a K-S test statistic (D_n) against a large class of alternative hypotheses. The Anderson–Darling statistic A^2 (Anderson and Darling 1954) can be considered as a limiting distribution of W^2 and it gives more weight to the tails than the statistic (D_n) does (see Darling 1957). Watson (1961a, 1962b) proposed a new test statistic U^2 as a generalization of Cramér-Von Mises test statistic W^2. Another new test statistic L_n, is developed by Liao et Shimokawa (1999) and applied for testing the GoF.

Let $(X_1, \dots, X_n)$ be a random sample from the distribution $F(x) = P(X \leqslant x)$. The main problem is that of testing hypotheses about F of the form:

$$\begin{cases} H_0 : F(x) = F_0(x) \\ H_1 : F(x) \neq F_0(x) \end{cases},$$

where $F_0(x)$ is a known distribution function.
The EDF is defined as

$$F_n(x) = \frac{\texttt{number of observations} \leqslant x}{n} = \frac{1}{n}\sum_{i=1}^{n} I(X_i \leqslant x), \tag{1}$$

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where I is an indicator function. Almost surely, the EDF $F_n(x)$ converges uniformly to the distribution function $F(x)$ (more detail, see the Glivenko-Cantelli theorem).

Many authors have addressed the problem of testing the null hypothesis in (1) when X follows a specified model, The EDF statistics are not distributed, but in the case of unknown parameters, their distribution will depend not only on the sample size but also on the hypothetical distribution. Using numerical methods, they developed modified test statistics, replacing the unknown parameters with their estimates. We find, for example, both Hassan from generalized exponential distribution (2005) and Al-Zahrani from Top-Leone distribution (2012) are obtained critical values for GoF tests based on a random sample and on the EDF tests. According to the critical tables which have been obtained by certain authors such as for example (for the two- and three parameter Weibull distributions (Evans, Johnson, and Green 1989), for the generalized Frechet distribution (Abd-Elfattah, Fergany, and Omima 2010) for the double Exponential distribution (Lemeshko and Lemeshko 2011a), it is particular that the statistic A^2 of AD test is the most powerful EDF test.

The generalized Rayleigh (GR) distribution plays an important role in the analysis of reliability and survival data (see, Kundu and Raqab 2007, Rao and Gadde Srinivasa 2014). This distribution was introduced by Surles and Padgett (2001). Originally, Mudholkar and Srivastava (1993), Mudholkar and al. (1995) proposed several distributions called the Burr distributions, whose generalized Rayleigh (GR) distribution is a special case of those of Burr Type X. Depending on the values of the parameters, Kundu and Raqab (2005) used different estimation methods for simple data so that Al-Khedhairi et al. (2007) calculated the estimators on grouped data and censored data. Fathipour et al. (2013) and Rao (2014) interested in estimating the weakness of the components described by GR distributions. Note that modified chi-square goodness-of-fit tests for this distribution have been developed for complete data and for censored data (D. Tilbi and Seddik-Amour 2017).

In this article, we explore the GoF for the generalized Rayleigh model with unknown parameters. After replacing the unknown parameters by their maximum likelihood estimates, we use R software and Monte Carlo methods, to provide tables of GoF critical values of the modified statistics D_n, L_n, W_n^2, U_n^2 and A_n^2 based on the FDE for this model. Finally, the power of these statistics is studied using alternative distributions (Weibull and exponential).

1. Generalized Rayleigh model

The Rayleigh distribution is widely used to model events that occur in different fields such as medicine, social and natural sciences. For instance, it is used in the study of various types of radiation, such as sound and light measurements. It is also used as a model for wind speed and is often applied to wind-driven electrical generation. Recently, Surles and Padgett (2001) considered the two parameter Burr Type X distribution by introducing a shape parameter and correctly named it as the generalized Rayleigh (GR) distribution. This distribution was studied by Mohammad Z. Raqab and Mohamed T. Madi (2011). If the random variable X has a two parameter GR distribution, then it has the cumulative distribution function (cdf)

$$F(x;\alpha,\lambda) = (1 - e^{-(\lambda x)^2})^\alpha, \quad x > 0,\ \alpha > 0,\ \lambda > 0, \tag{2}$$

and probability density function (pdf)

$$f(x;\alpha,\lambda) = 2\alpha\lambda^2 x e^{-(\lambda x)^2}(1 - e^{-(\lambda x)^2})^{\alpha-1}, \quad x > 0,\ \alpha > 0,\ \lambda > 0, \tag{3}$$

where α and λ are shape and inverse scale parameters, respectively. We denote the GR distribution with shape parameter α and inverse scale parameter λ as GR(α, λ). Its hazard and reliability functions are

$$h(x;\alpha,\lambda) = \frac{2\alpha\lambda^2 x e^{-(\lambda x)^2}(1-e^{-(\lambda x)^2})^{\alpha-1}}{1-(1-e^{-(\lambda x)^2})^{\alpha}}. \tag{4}$$

$$S(x;\alpha,\lambda) = 1-(1-e^{-(\lambda x)^2})^{\alpha}. \tag{5}$$

1.1. Maximum likelihood estimates

Suppose that $X_1, X_2, \Delta\Delta\Delta, X_n$ is a random sample from GR(α, λ). Then the log-likelihood function of the observed sample is

$$L(x;\alpha,\lambda) = n\ln 2 + n\ln\alpha + 2n\ln\lambda + \sum_{i=1}^{n}\ln x_i - \lambda^2\sum_{i=1}^{n}x_i^2 + (\alpha-1)\sum_{i=1}^{n}\ln(1-e^{-(\lambda x)^2}). \tag{6}$$

The MLEs of α and λ say $\hat{\alpha}$ and $\hat{\lambda}$, respectively, can be obtained as the solutions of the following equations

$$\frac{\partial L}{\partial\alpha} = \frac{n}{\alpha} + \sum_{i=1}^{n}\ln(1-e^{-(\lambda x)^2}) = 0. \tag{7}$$

$$\frac{\partial L}{\partial\lambda} = \frac{2n}{\lambda} - 2\lambda\sum_{i=1}^{n}x_i^2 + 2\lambda(\alpha-1)\sum_{i=1}^{n}\frac{x^2 e^{-(\lambda x)^2}}{1-e^{-(\lambda x)^2}} = 0. \tag{8}$$

We obtain

$$\hat{\alpha} = -\frac{n}{\sum_{i=1}^{n}\ln(1-e^{-(\lambda x)^2})}, \tag{9}$$

and $\hat{\lambda}$ can be obtained as the solution of the nonlinear equation $g(\lambda)=0$, where

$$g(\lambda) = \frac{\partial L(x;\alpha,\lambda)}{\partial\lambda} = \frac{2n}{\lambda} - 2\lambda\sum_{i=1}^{n}x_i^2 - 2\lambda\left(\frac{n}{\sum_{i=1}^{n}\ln(1-e^{-(\lambda x)^2})}+1\right)\sum_{i=1}^{n}\frac{x^2 e^{-(\lambda x)^2}}{1-e^{-(\lambda x)^2}}.$$

Therefore, $\hat{\lambda}$ can be obtained as solution of the nonlinear equation of the form $H(\lambda)=\lambda$, where

$$H(\lambda) = 2n\left[2\lambda\sum_{i=1}^{n}x_i^2 - 2\lambda\left(\frac{n}{\sum_{i=1}^{n}\ln(1-e^{-(\lambda x)^2})}+1\right)\sum_{i=1}^{n}\frac{x^2 e^{-(\lambda x)^2}}{1-e^{-(\lambda x)^2}}\right]^{-1}. \tag{10}$$

Since, $\hat{\lambda}$ is a fixed point solution of the non-linear equation (10), therefore, it can be obtained using an iterative scheme as $H(\lambda_j)=\lambda_{j+1}$, where λ_j is the jth iterate of $\hat{\lambda}$. The iteration procedure should be stopped when $|\lambda_j-\lambda_{j+1}|$ is sufficiently small. Once we obtain $\hat{\lambda}$, then $\hat{\alpha}$ can be obtained from (9).

2. GoF statistics based on the EDF

A goodness of fit test based on the empirical function (EDF), when the parameters are estimated, is called a modified goodness of fit test. The most popular nonparametric goodness-of-fit tests, namely; the Kolmogorov–Smirnov D_n, Cramér-von-Mises W^2, Anderson–Darling A^2, Watson U^2, and Liao–Shimokawa L_n test statistics. The critical values of the modified statistics did not exist in the statistical literature prior to the last decades. Through simulations, some authors have provided critical table values for classical models and some of their generalizations (for more details, see Lemeshko and Lemeshko 2011b). In this paper, using the Monte Carlo method and the R software, we offer tables of critical values of D_n, W^2, A^2, U^2, and L_n statistics for the generalized Rayleigh model when the parameters are unknown.

2.1. K-S test statistics D_n

The most popular GoF test is the Kolmogorov–Smirnov K-S test. The test statistic D_n is defined as

$$D_n = \max[D^+; D^-],$$

where

$$D^+ = \max_{1 \leqslant i \leqslant n} \left[\frac{i}{n} - F(x_i) \right],$$

and

$$D^- = \max_{1 \leqslant i \leqslant n} \left[F(x_i) - \frac{i-1}{n} \right],$$

with x_i is the order statistic. For Rayleigh model GR(α, λ) the D_n statistic becomes

$$D^+ = \max_{1 \leqslant i \leqslant n} \left[\frac{i}{n} - (1 - e^{-(\hat{\lambda} x_i)^2})^{\hat{\alpha}} \right], \tag{11}$$

and

$$D^- = \max_{1 \leqslant i \leqslant n} \left[(1 - e^{-(\hat{\lambda} x_i)^2})^{\hat{\alpha}} - \frac{i-1}{n} \right], \tag{12}$$

where $\hat{\alpha}$ and $\hat{\lambda}$ are the maximum likelihood parameter estimators of the unknown parameters.

2.2. C-VM test statistics W^2

The Cramér-von Mises test is an alternative to the Kolmogorov–Smirnov test (1933). C-VM test statistic W^2 may be considered as the sum of the quadratic differences between the empirical distribution function (EDF) and the theoretical cumulative distribution function (CDF). It is defined as

$$W^2 = \frac{1}{12n} + \sum_{i=1}^{n} \left(F(x_i) - \frac{2i-1}{2n} \right)^2. \tag{13}$$

So, for the GR(α, λ) distribution, we obtain

$$W_n^2 = \frac{1}{12n} + \sum_{i=1}^{n} \left((1 - e^{-(\hat{\lambda} x_i)^2})^{\hat{\alpha}} - \frac{2i-1}{2n} \right)^2. \tag{14}$$

2.3. A-D test statistics A^2

The A-D test statistic A^2 was developed by Anderson and Darling (1954) as a limiting distribution of the test of C-VM as in n$\to \infty$.
The A^2 is given by

$$A^2 = -n - \frac{1}{n} \sum_{i=1}^{n} (2i-1) \Big(\ln(F(x_i)) + \ln(1 - F(x_i)) \Big). \tag{15}$$

We obtain the test statistic for GR(α, λ) as follows

$$A_n^2 = -n - \frac{1}{n} \sum_{i=1}^{n} (2i-1) \Big(\ln((1 - e^{-(\hat{\lambda} x_i)^2})^{\hat{\alpha}}) + \ln(1 - (1 - e^{-(\hat{\lambda} x_i)^2})^{\hat{\alpha}}) \Big). \tag{16}$$

2.4. W test statistics U^2

Watson test statistic U^2 was developed for distributions which are cyclic and in 1961 it is based on the empirical distribution function. U^2 is a generalization of the C-VM test statistic. It is defined by

$$U^2 = W_2 + \sum_{i=1}^{n}\left(\frac{F(x_i)}{n} - \frac{1}{2}\right)^2. \tag{17}$$

The explicit form of this statistic for the GR(α, λ) model is

$$U_n^2 = \frac{1}{12n} + \sum_{i=1}^{n}\left((1-e^{-(\hat{\lambda}x_i)^2})^{\hat{\alpha}} - \frac{2i-1}{2n}\right)^2 + \sum_{i=1}^{n}\left(\frac{(1-e^{-(\hat{\lambda}x_i)^2})^{\hat{\alpha}}}{n} - \frac{1}{2}\right)^2. \tag{18}$$

2.5. LS test statistics L_n

The Liao–Shimokawa statistic measures the average of all weighted distances over the entire range of the data. For more details, we refer to Liao and Shimokawa (1999). The test statistic is given by

$$L_n = \frac{1}{\sqrt{n}}\sum_{i=1}^{n}\frac{\max_i\left(\frac{i}{n} - F(x_i), F(x_i) - \frac{i-1}{n}\right)}{\sqrt{F(x_i)[1-F(x_i)]}}. \tag{19}$$

For the distribution of GR(α, λ), L_n becomes

$$L_n = \frac{1}{\sqrt{n}}\sum_{i=1}^{n}\frac{\max_i\left(\frac{i}{n} - (1-e^{-(\hat{\lambda}x_i)^2})^{\hat{\alpha}}, (1-e^{-(\hat{\lambda}x_i)^2})^{\hat{\alpha}} - \frac{i-1}{n}\right)}{\sqrt{(1-e^{-(\hat{\lambda}x_i)^2})^{\hat{\alpha}}[1-(1-e^{-(\hat{\lambda}x_i)^2})^{\hat{\alpha}}]}}. \tag{20}$$

3. Critical values

The purpose of this paper is to provide critical adjustment values of the modified statistics D_n, A_n^2, W_n^2, U_n^2 and L_n for the generalized Rayleigh distribution when the parameters are unknown and replaced by their maximum likelihood estimates of the non grouped data. For this, we use Monte Carlo simulation method and R software to generate $10,000$ samples of different sizes n.

Under the null hypothesis H_0 that a sample $X = X_1, X_2, \ldots, X_n$ belongs to generalized Rayleigh model, we calculated the values of the various fit testing statistics mentioned above. To this end, the following steps are used to calculate the critical values for each statistic of the fit tests at different levels of significance $\alpha = 0.01$, 0.05, 0.10, 0.15 and 0.25 and sample sizes $n = 5, 10, 15, 20, 30, 50$ and 100:

Step 1. Generate n random variables $U(0,1)$ independent $U_1, U_2, \ldots, U_n$.

Step 2. For given values of the parameters α and λ, we set $x_i = F^{-1}(U_i)$.
Then $(x_1, x_2, \ldots, x_n)$ is the required sample size n of the GR distribution.

Step 3. Use the generated sample to estimate the unknown parameters using the maximum likelihood estimators given by (9) and (10).

Step 4. The unknown parameter estimators were used to determine the hypothetical cumulative distribution function of the GR distribution.

Step 5. The statistical tests D_n, L_n, W_n^2, U_n^2 and A_n^2 mentioned above are calculated for each generation random sample of different sizes.

Step 6. This procedure was repeated $10,000$ times independently. Therefore, we got $10,000$ values for each proposed test statistic. These values have been classified at different levels of significance 0.01, 0.05, 0.10, 0.15 and 0.25 are shown in the Tab. 1.

Table 1. Critical values for K-S, C-VM, A-D, W and LS tests

Sample size n	test statistics	Significance level α 0.01	0.05	0.10	0.15	0.25
5	D_n	0.0000	0.0006	0.0020	0.0053	0.0300
	W_n^2	0.0006	0.0061	0.0185	0.0312	0.0593
	A_n^2	0.0187	0.0700	0.1400	0.1990	0.3590
	U_n^2	0.0005	0.0044	0.00102	0.0199	0.0412
	L_n	0.0138	0.0449	0.0655	0.1022	0.1114
10	D_n	0.0000	0.0004	0.0017	0.0050	0.0111
	W_n^2	0.0004	0.0054	0.0182	0.0309	0.0587
	A_n^2	0.0156	0.0706	0.1359	0.1986	0.3840
	U_n^2	0.0004	0.0031	0.0099	0.0185	0.0391
	L_n	0.0125	0.0395	0.0592	0.0965	0.1072
15	D_n	0.0000	0.0004	0.0016	0.0049	0.0101
	W_n^2	0.0003	0.0048	0.0163	0.0305	0.0575
	A_n^2	0.0152	0.0762	0.1293	0.1836	0.3570
	U_n^2	0.0004	0.0029	0.0079	0.0178	0.0352
	L_n	0.0120	0.0345	0.0522	0.0960	0.1066
20	D_n	0.0000	0.0004	0.0015	0.0047	0.0100
	W_n^2	0.0003	0.0043	0.0140	0.0304	0.0569
	A_n^2	0.0147	0.0657	0.1297	0.1788	0.3470
	U_n^2	0.0004	0.0026	0.0072	0.0174	0.0332
	L_n	0.0115	0.0338	0.0452	0.0865	0.0987
30	D_n	0.0000	0.0003	0.0011	0.0045	0.0100
	W_n^2	0.0003	0.0042	0.0133	0.0289	0.0565
	A_n^2	0.0145	0.0700	0.1150	0.1755	0.1986
	U_n^2	0.0003	0.0020	0.0063	0.0170	0.0325
	L_n	0.0111	0.0332	0.0434	0.0799	0.0977
50	D_n	0.0000	0.0003	0.0009	0.0034	0.0079
	W_n^2	0.0002	0.0039	0.0126	0.0286	0.0559
	A_n^2	0.0129	0.0561	0.1132	0.1707	0.2590
	U_n^2	0.0002	0.0014	0.0039	0.0143	0.0291
	L_n	0.0101	0.0245	0.0398	0.0592	0.0923
100	D_n	0.0000	0.0001	0.0006	0.0030	0.0067
	W_n^2	0.0001	0.0032	0.0120	0.0284	0.0530
	A_n^2	0.0125	0.0524	0.1087	0.1585	0 .2500
	U_n^2	0.0001	0.0012	0.0036	0.0137	0.0278
	L_n	0.0097	0.0231	0.0341	0.0564	0.0878

From the table, we noticed that:

•For each statistical test, the power increases monotonically as the sample size increases and the level of significance increases.

•The **Anderson-Darling** A_n^2 statistical test is the most powerful of the proposed fit tests.

•The statistical test of **Komogorov-Smirnov** D_n is the least powerful among the fit tests proposed.

4. Simulation study

In this section, we performed a power comparison between D_n, L_n, W_n^2, U_n^2 and A_n^2 statistics for the GR model with unknown parameters. For this, we simulated $10,000$ random samples of different sizes $n = 10, 20, 50$ and 100, for each test at the significance level $\alpha = 0.05$ and from each of the alternative distributions:

1. The Exponential distribution $Exp(\lambda)$, with probability density function

$$f_X(x, \lambda) = \lambda \exp(-\lambda x),$$

and its cumulative distribution function is

$$F_{Exp}(x, \lambda) = 1 - \exp(-\lambda x). \tag{21}$$

2. The Weibull distribution $Wei(\gamma, \alpha)$, with probability density function

$$f(x; \gamma, \alpha) = \frac{\gamma}{\alpha}\Big(\frac{x}{\alpha}\Big)^{\gamma-1} \exp(-(\frac{x}{\alpha})^{\gamma}),$$

and its cumulative distribution function is

$$F_{Wei}(x; \gamma, \alpha) = 1 - \exp\Big(-\Big(\frac{x}{\alpha}\Big)^{\gamma}\Big). \tag{22}$$

3. The Inverse Weibull distribution $InWei(\alpha, \gamma)$, with probability density function

$$f(x; \gamma, \alpha) = \gamma\alpha^{\gamma} x^{-(\gamma+1)} \exp\Big(-\Big(\tfrac{\alpha}{x}\Big)\Big)^{-\gamma},$$

and its cumulative distribution function is

$$F_{InWei}(x; \gamma, \alpha) = \exp\Big(-\alpha\Big(\frac{1}{x}\Big)^{-\gamma}\Big). \tag{23}$$

4. The Exponentiated Weibull distribution $ExpWei(\alpha, \gamma, \lambda)$, with probability density function

$$f(x; \gamma, \alpha, \lambda) = \alpha\gamma\lambda^{\gamma} x^{\gamma-1}(1 - \exp(-\lambda x^{\gamma}))^{\alpha},$$

and its cumulative distribution function is

$$F_{ExpWei}(x; \gamma, \alpha, \lambda) = (1 - \exp(-\lambda x^{\gamma}))^{\alpha}. \tag{24}$$

5. The Exponentiated Exponential distribution $EE(\alpha, \lambda)$, with probability density function

$$f_{EE}(x; \alpha, \lambda) = \alpha\lambda(1 - \exp(-\lambda x))^{\alpha-1} \exp(-\lambda x),$$

and its cumulative distribution function is

$$F_{EE}(x; \alpha, \lambda) = (1 - \exp(-\lambda x))^{\alpha}. \tag{25}$$

The power results of tests statistics D_n, L_n, W_n^2, U_n^2 and A_n^2, for each alternative distribution at significance level $\alpha = 0.05$ are presented in Tab. 2.

From the table, we notice that:

•According to the test power values for the different statistics, are indicating that the generalized Rayleigh model is distinct from competing distributions of all sizes of the sample.

•The power of the test statistic increases as the sample size increases.

The modified test statistics D_n, L_n, W_n^2, U_n^2 and A_n^2 provided in this work and their critical values can detect the difference betwen the GR model and different alternatives with high Power.

Table 2. Power of statistics tests for GR distribution where $Exp, Wei, InWei, ExpWei$ and EE are the alternative distributions

Alternatives	test statistics	Sample size n 10	20	50	100
Exponential $Exp(1)$	D_n	1.0000	1.0000	1.0000	1.0000
	W_n^2	0.1016	0.3653	0.9119	0.9997
	A_n^2	0.4158	0.7834	0.9976	1.0000
	U_n^2	0.1004	0.3202	0.9164	0.9786
	L_n	0.1059	0.3728	0.9993	1.0000
Weibull $Wei(1,2)$	D_n	1.0000	1.0000	1.0000	1.0000
	W_n^2	0.0995	0.3542	0.9080	0.9998
	A_n^2	0.0644	0.0603	0.0539	0.0495
	U_n^2	0.0244	0.0282	0.0393	0.0450
	L_n	0.0159	0.0228	0.0324	0.0445
Inverse Weibull $InWei(1,2)$	D_n	0.9249	0.9992	1.0000	1.0000
	W_n^2	0.1059	0.3101	0.9463	0.9459
	A_n^2	0.8286	0.9324	0.9981	1.0000
	U_n^2	0.1083	0.3089	0.9059	0.9228
	L_n	0.1055	0.3076	0.9034	0.9210
Exponentiated Weibull $ExpWei(1,2,3)$	D_n	0.9999	0.9996	1.0000	0.9999
	W_n^2	0.1035	0.3588	0.9997	0.9995
	A_n^2	0.9999	0.9998	0.9992	0.8853
	U_n^2	0.1030	0.3438	0.9127	0.9960
	L_n	0.1011	0.3298	0.9037	0.9860
Exponentiated Exponential $EE(1,2)$	D_n	1.0000	1.0000	1.0000	1.0000
	W_n^2	0.1055	0.3676	0.9087	0.9994
	A_n^2	0.0592	0.0726	0.0639	0.0597
	U_n^2	0.0548	0.0526	0.0611	0.0684
	L_n	0.0539	0.0523	0.0601	0.0672

Conclusion

We have provided critical values for the statistics D_n, L_n, W_n^2, U_n^2 and A_n^2 for the generalized Rayleigh model when the parameters are unknown. The 1 and 2 tables given in this manuscript can be used to check whether the sample data fits this pattern which helps practitioners to choose the appropriate pattern for their analysis.

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Сравнение мощностей тестов согласия EDF

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Аннотация. В этой статье сила общей статистики согласия (GoF) основана на эмпирической функции распределения (EDF), где критические значения должны быть определены путем моделирования. Статистическая мощность Колмогорова–Смирнова D_n, Крамéр-фон Мизеса W^2, Ватсона U^2, Ляо и Симокавы L_n , и статистика Андерсона–Дарлинга A^2 исследовалась по размеру выборки, уровню значимости и альтернативным распределениям для обобщенной модели Рэлея (GR). Экспоненциальное, Вейбулла, обратное Вейбулла, экспоненциальное Вейбулла и экспоненциальное распределения были рассмотрены среди наиболее частых альтернативных распределений.

Ключевые слова: обобщенное распределение Рэлея, критерий Колмогорова–Смирнова, критерий Крамера-фон Мизеса (C-VM), критерий Андерсона–Дарлинга (AD), критерий Ватсона (W), критерий Ляо и Симокавы (LS).