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On the Effect of Delays in Self-oscillating System with External Influence

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Abstract. Self-oscillating system that interacts with energy source of limited power is considered in the presence of external force and joint action of delays in damping and elasticity. On the basis of the direct linearisation method, the solution of non-linear equations of the system is obtained. The equations of non-stationary motion, relations for calculating the amplitude and phase of stationary oscillations, the speed of the energy source and the load on it on the side of the oscillatory system are derived. Stability conditions of stationary oscillations were obtained with the use of the Routh–Hurwitz criteria. Calculations were carried out to study the influence of delays on dynamics of the system. The results show the combined effect of delays in elasticity and damping on dynamics of oscillations. Delays change the shape of the amplitude-frequency curve, shift it up/down and shift it in the frequency range. Delays also affect the stability of oscillations. If in the case of no delay there is no resonant curve then various intensity resonant curve may appear if delay is present. The intensity of resonant curve depends on the amount of delay. Considering the influence of delays on dynamics of oscillations, it was assumed that other parameters of the system are unchanged.

Keywords: self-oscillations, forced oscillations, energy source, delay, damping, elasticity, linearisation.

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Introduction

The theory of interaction between the oscillating system and the energy source is presented in many studies [1–8] et al. It is directly related to the solution of environmental problems that arose due to the large increase in energy consumption in modern conditions. Oscillatory processes of various types, including mixed oscillations, arise in many modern technical devices and technological processes under certain conditions. They can also be caused by delay caused by various factors [9]. Problems where properties of the energy source in systems with delay are not taken into account were considered in a number of works, for example, in [11–17].

The analysis of non-linear oscillatory systems is carried out using various methods of non-linear mechanics [17–23] which are very time consuming. The method of direct linearisation is not so time consuming, and it is easy of use [24–29] et al. Such features are very important when real technical devices are designed. A model of auto-oscillatory system with limited excitation in the presence of an external force and delays in elasticity and damping is considered below. The model is based on the direct linearisation method. The aim of the work is to study the joint effect of the delay of mixed forced and self-oscillation with limited power of the energy source on the system dynamics.

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1. Equations of motion

The model for the study of frictional self-oscillations in a system with a limited power source is based on the model given in [1–3, 29]. The dynamics of the system in the simplest case is described by non-linear differential equations

$$m\,\ddot{x} + k_0\,\dot{x} + c_0\,x = T(U), \tag{1}$$
$$I\ddot{\varphi} = M(\dot{\varphi}) - r_0\,T(U),$$

where $k_0 = const$, $c_0 = const$, $T(U)$ is the non-linear friction force causing self-oscillation, $U = r_0\dot{\varphi} - \dot{x}$, $r_0 = const$ is the distance to the point of application of force $T(U)$, $\dot{\varphi}$ is the speed of rotation of the engine, I is the total moment of inertia of the rotating parts, $M(\dot{\varphi})$ is the difference between the torque of the energy source and the torque of the forces resisting rotation.

In practical conditions $T(U)$ has various forms (see, for example, results of experiments in space [30]). Here the following form is assumed

$$T(U) = R(sgnU - \alpha_1 U + \alpha_3 U^3). \tag{2}$$

Here R is the normal force, $\alpha_1 = const$, $\alpha_3 = const$, $sgnU = 1$ in the case $U > 0$ and $sgnU = -1$ in the case $U < 0$.

Let us rewrite the first equation (1) in the form

$$m\,\ddot{x} + k_0\,\dot{x}_\eta + c_0\,x + c_1\,x_\tau + F(x) = T(U) - \lambda\,\sin\nu t, \tag{3}$$

where $F(x)$ is non-linear part of elastic force, $\lambda\,\sin\nu t$ is external force, $\dot{x}_\eta = \dot{x}(t-\eta)$, $x_\tau = x(t-\tau)$, $\eta = const$ and $\tau = const$ are delays, $c_1 = const$.

For greater generality characteristics of forces $T(U)$ and $F(x)$ are taken in the following form

$$T(U) = R\left[sgn\,U + f(\dot{x})\right], \quad f(\dot{x}) = \sum_i \alpha_i U^i = \sum_{n=0}^{5} \delta_n \dot{x}^n, \tag{4}$$

$$F(x) = \sum_s \gamma_s x^s, \quad s = 2, 3, 4, \dots$$
$$\delta_0 = \alpha_1 V + \alpha_2 V^2 + \alpha_3 V^3 + \alpha_4 V^4 + \alpha_5 V^5,$$
$$\delta_1 = -\left(\alpha_1 + 2\alpha_2 V + 3\alpha_3 V^2 + 4\alpha_4 V^3 + 5\alpha_5 V^4\right), \quad \delta_2 = \alpha_2 + 3\alpha_3 V + 6\alpha_4 V^2 + 10\alpha_5 V^3,$$
$$\delta_3 = -(\alpha_3 + 4\alpha_4 V + 10\alpha_5 V^2), \quad \delta_4 = \alpha_4 + 5\alpha_5 V, \quad \delta_5 = -\alpha_5,$$

where $\alpha_i = const$, $\gamma_s = const$.

Using the direct linearisation method [24], forces $f(\dot{x})$ and $F(x)$ are replaced with linear functions

$$f_*(\dot{x}) = B_f + k_f\,\dot{x}, \quad F_*(x) = B_F + k_F\,x, \tag{5}$$

where B_f, k_f, B_F, k_F are linearisation coefficients defined as

$$\begin{aligned} B_f &= \sum_n N_n \alpha_n \upsilon^n, \ n = 0, 2, 4 \ (n \text{ is even}), \\ k_f &= \sum_n \alpha_n \bar{N}_n \upsilon^{n-1}, \ n = 1, 3, 5 \ (n \text{ is odd}), \\ B_F &= \sum_s N_s \gamma_s a^s, \ s = 2, 4, 6, \dots \ (s \text{ is even}), \\ k_F &= \sum_s \bar{N}_s \gamma_s a^{s-1}, \ s = 3, 5, 7, \dots \ (s \text{ is odd}), \end{aligned} \tag{6}$$

$$N_n = (2r+1)/(2r+1+n), \quad \bar{N}_n = (2r+3)/(2r+2+n),$$
$$N_s = (2r+1)/(2r+1+s), \quad \bar{N}_s = (2r+3)/(2r+2+s),$$
$$a = \max|x|, \quad \upsilon = \max|\dot{x}|.$$

In expressions for N_n, $\bar{N}_n$, N_s, $\bar{N}_s$ symbol r represents the linearisation accuracy parameter which can be chosen in the interval (0,2) [24].

Taking into account (5), equations (1) take the form

$$m\ddot{x} + k_0\dot{x}_\eta + cx + c_1 x_\tau = B + R(sgnU + k_f\dot{x}) - \lambda \sin\nu t, \qquad (7)$$
$$I\ddot{\varphi} = M(\dot{\varphi}) - r_0 R(sgnU + B_f + k_f\dot{x}),$$

where $B = RB_f - B_F$, $c = c_0 + k_F$.

2. Solution of equations

To solve a non-linear equation with linearised functions the method of change of variables with averaging is used [24]. Then $x = a\cos\psi$, $\dot{x} = -\upsilon\sin\psi$, $\psi = pt + \xi$ are taken as a solution and the standard form of the equation for determining υ and ξ is derived. They can be used to study non-stationary and stationary processes. To solve the first equation (7) the standard form of equation is used. To solve the second equation the procedure described in [27] is used. In accordance with this procedure, $V = r_0\dot{\varphi}$ is replaced with $u = r_0\Omega$ in expressions for $\delta_0, \ldots, \delta_5$ in (4).

Taking into account $\upsilon = ap$ and $p = \nu$, the following equations for amplitude a, phase ξ and velocity u are obtained from (7) for $u \geqslant ap$

$$\frac{da}{dt} = -\frac{1}{2pm}\left[aA + \lambda\cos\xi\right],$$
$$\frac{d\xi}{dt} = \frac{1}{2pma}\left[aE + \lambda\sin\xi\right], \qquad (8)$$
$$\frac{du}{dt} = \frac{r_0}{I}\left[M\left(\frac{u}{r}\right) - r_0 T_0(1 + B_f)\right],$$

where $A = p(k_0\cos p\eta - T_0 k_f) - c_1\sin p\tau$, $E = m(\omega_0^2 - p^2) + k_F + c_1\cos p\tau$, $\psi_* = 2\pi - \arcsin(u/ap)$, $\omega_0^2 = c_0/m$.

Using technique described in [3], the following equations are obtained for $u < ap$

$$\frac{da}{dt} = -\frac{1}{2pm}\left[aA + \lambda\cos\xi - \frac{4T_0}{\pi ap}\sqrt{a^2p^2 - u^2}\right],$$
$$\frac{d\xi}{dt} = \frac{1}{2pma}\left[aE + \lambda\sin\xi\right],$$
$$\frac{du}{dt} = \frac{r_0}{I}\left[M\left(\frac{u}{r}\right) - r_0 T_0(1 + B_f) - \frac{r_0 T_0}{\pi}(3\pi - 2\psi_*)\right].$$

Equations for stationary oscillations when $\dot{a} = 0$, $\dot{\xi} = 0$, $\dot{u} = 0$ are obtained from (8). The amplitude and phase of these oscillations are determined by the relations

$$a^2(A^2 + E^2) = \lambda^2, \quad tg\xi = E/A. \qquad (9)$$

The approximate formula $ap \approx u$ can be used to calculate the amplitude for $u < ap$.

Equation for the stationary values of the velocity is obtained from the condition $\dot{u} = 0$

$$M(u/r_0) - S(u) = 0, \qquad (10)$$

where

a) $u \geqslant ap$, $S(u) = r_0 R(1 + B_F)$,

b) $u < ap$, $S(u) = r_0 R\left[(1 - B_F) + \pi^{-1}(3\pi - 2\psi_*)\right]$.

The term $S(u)$ can be simplified for $u < ap$ with the use of the approximate equality $ap \approx u$ for the amplitude.

3. Conditions of stability of stationary oscillations

Equations in variations are formulated for equations of non stationary motions, and the Routh–Hurwitz criteria is used. The conditions of stability are

$$D_1 > 0, \quad D_3 > 0, \quad D_1 D_2 - D_3 > 0, \tag{11}$$

where

$D_1 = -(b_{11} + b_{22} + b_{33})$, $D_2 = b_{11}b_{33} + b_{11}b_{22} + b_{22}b_{33} - b_{23}b_{32} - b_{12}b_{21} - b_{13}b_{31}$,
$D_3 = b_{11}b_{23}b_{32} + b_{12}b_{21}b_{33} - b_{11}b_{22}b_{33} - b_{12}b_{23}b_{31} - b_{13}b_{21}b_{32}$.

Taking into account that $u = r\,\Omega$ for $u \geqslant ap$, we have after averaging

$$b_{11} = \frac{r_0}{J}(Q - r_0 R\frac{\partial B_f}{\partial u}), \quad b_{12} = -\frac{r_0^2 R}{J}\frac{\partial B_f}{\partial a}, \quad b_{13} = 0, \quad b_{21} = \frac{aR}{2m}\frac{\partial k_f}{\partial u},$$

$$b_{22} = -\frac{1}{2m}(k_0 \cos p\eta - Rk_f - aR\frac{\partial k_f}{\partial a}), \quad b_{23} = \frac{\lambda \sin\xi}{2pm}, \quad b_{31} = 0,$$

$$b_{32} = \frac{1}{2pm}(\frac{\partial k_F}{\partial a} - \frac{\lambda}{a^2}\sin\xi), \quad b_{33} = \frac{\lambda\cos\xi}{2pma}, \quad Q = \frac{d}{du}M(\frac{u}{r_0}),$$

$$\frac{\partial B_f}{\partial u} = \frac{\partial \delta_0}{\partial u} + N_2\,(ap)^2\frac{\partial \delta_2}{\partial u} + N_4\,(ap)^4\frac{\partial \delta_4}{\partial u}, \quad \frac{\partial k_f}{\partial u} = \bar{N}_1\,\frac{\partial \delta_1}{\partial u} + \bar{N}_3\,(ap)^2\frac{\partial \delta_3}{\partial u} + \bar{N}_5\,(ap)^4\frac{\partial \delta_5}{\partial u},$$

$$\frac{\partial B_f}{\partial a} = 2ap^2(N_2\delta_2 + 2N_4\,\delta_4 a^2p^2), \quad \frac{\partial k_f}{\partial a} = 2ap^2(\bar{N}_3\delta_3 + 2\bar{N}_5\,\delta_5 a^2p^2),$$

$$\delta_0 = \alpha_1 u + \alpha_2 u^2 + \alpha_3 u^3 + \alpha_4 u^4 + \alpha_5 u^5, \quad \delta_1 = -(\alpha_1 + 2\alpha_2 u + 3\alpha_3 u^2 + 4\alpha_4 u^3 + 5\alpha_5 u^4),$$

$$\delta_2 = \alpha_2 + 3\alpha_3 u + 6\alpha_4 u^2 + 10\alpha_5 u^3, \quad \delta_3 = -(\alpha_3 + 4\alpha_4 u + 10\alpha_5 u^2), \quad \delta_4 = \alpha_4 + 5\alpha_5 u, \quad \delta_5 = -\alpha_5,$$

$$\frac{\partial \delta_0}{\partial u} = \alpha_1 + 2\alpha_2 u + 3\alpha_3 u^2 + 4\alpha_4 u^3 + 5\alpha_5 u^4, \quad \frac{\partial \delta_1}{\partial u} = -2\,(\alpha_2 + 3\alpha_3 u + 6\alpha_4 u^2 + 10\alpha_5 u^3),$$

$$\frac{\partial \delta_2}{\partial u} = 3\,(\alpha_3 + 4\alpha_4 u + 10\alpha_5 u^2), \quad \frac{\partial \delta_3}{\partial u} = -4(\alpha_4 + 5\alpha_5 u), \quad \frac{\partial \delta_4}{\partial u} = 5\alpha_5, \quad \frac{\partial \delta_5}{\partial u} = 0,$$

$$\frac{\partial k_F}{\partial a} = 2a(\bar{N}_3\gamma_3 + 2\bar{N}_5\,\gamma_5 a^2 + 3\bar{N}_7\,\gamma_7 a^4 + \cdots).$$

When $u < ap$ coefficients b_{13}, b_{23}, b_{31}, b_{33} remain as before but the following coefficients are changed

$$b_{11} = \frac{r_0}{I}\left[Q - r_0 R\frac{\partial B_f}{\partial u} - \frac{2r_0 R}{\pi\sqrt{a^2p^2 - u^2}}\right], \quad b_{12} = -\frac{r_0^2 R}{I}\left[\frac{\partial B_f}{\partial a} + \frac{2u}{\pi a\sqrt{a^2p^2 - u^2}}\right],$$

$$b_{21} = \frac{a}{2m}\left[R\frac{\partial k_f}{\partial u} + \frac{4uR}{\pi a^2p^2\sqrt{a^2p^2 - u^2}}\right],$$

$$b_{22} = -\frac{1}{2m}\left(k_0\cos p\eta - Rk_f - aR\frac{\partial k_f}{\partial a} + \frac{4Ru^2}{\pi a^2p^2\sqrt{a^2p^2 - u^2}}\right).$$

Let us note that when calculating $\partial B_f/\partial u$, $\partial B_f/\partial a$ only even powers of n (that is, δ_0, δ_2, δ_4) are taken into account, and when calculating $\partial k_f/\partial u$, $\partial k_f/\partial a$ odd powers of n (that is, δ_1, δ_3, δ_5) are taken into account. Similarly, odd powers of s and, respectively, γ_1, γ_3, $\gamma_5, \ldots$ are taken into account when calculating $\partial k_F/\partial a$.

4. Results of calculations

Calculations were carried out to obtain information on the effect of delays on the system at $\omega_0 = 1\text{s}^{-1}$, $m = 1\,\text{kgf}\cdot\text{s}^2\cdot\text{cm}^{-1}$, $k = 0.02\,\text{kgf}\cdot\text{s}\cdot\text{cm}^{-1}$, $c_1 = 0.05\,\text{kgf}\cdot\text{cm}^{-1}$, $\lambda = 0.02\,\text{kgf}$,

$r_0 = 1$ cm, $I = 1$ kgf·s·cm^2. Parameters of friction term (2) are $T_0 = 0.5$ kgf, $\alpha_1 = 0.84$ s·cm^{-1}, $\alpha_3 = 0.18$ s$^3 \cdot$ cm^{-3}. Parameters of delays are $p\eta = \pi/2,\ \pi,\ 3\pi/2$; $p\tau = \pi/2,\ \pi,\ 3\pi/2$.

The amplitude-frequency relationships $a(p)$ shown in Figs. 1–3 are obtained in the case of linear elastic force for $u = 1.2$, and a_a reflects the amplitude of self-oscillations. Relationships were obtained using the straight linearisation method with the accuracy parameter $r = 1.5$. Solid line in Figs. 1–3 corresponds to the absence of delays, and it is given for comparison with curves when delays are present. In particular, it also shows results based on the asymptotic averaging method that completely coincide with the results obtained with the direct linearisation method with the accuracy parameter $r = 1.5$. Some comparison of the numerical coefficients N and $\bar{N}$ indicated in (6) with the corresponding coefficients obtained on the basis of well-known and widely used methods of non-linear mechanics is given in [25]. Fig. 4 shows (without regard to stability and, therefore, feasibility) the relationship between amplitude and speed u for frequency $p = 1$ at $\eta = \pi/2$, $\tau = \pi/2$, $\tau = 3\pi/2$, where curve 1 corresponds to the absence of delays. On the section of curve 1 shown by the inclined straight line AB in Fig. 4 approximate equality $ap \approx u$ takes place.

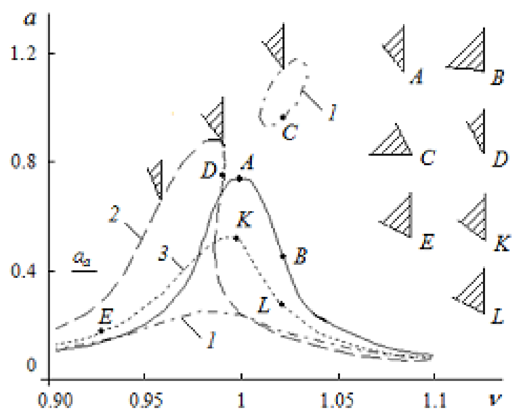

Fig. 1. Amplitude-frequency curves at $\eta = \pi/2$: curve 1 — $\tau = \pi/2$, curve 2 — $\tau = \pi$, curve 3 — $\tau = 3\pi/2$

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Oscillations are stable if the steepness $Q = dM(u/r_0)/du$ of the energy source characteristic is within the shaded sector. For all $p\eta = \pi/2,\ \pi,\ 3\pi/2$ the entire lower branch of curve 1 ($\tau = \pi/2$) and parts of curves 2 with small amplitudes are unstable. In contrast, there is stability for curve 3 ($\tau = 3\pi/2$) in the frequency range $p = 0.9 \div 1.1$ at $p\eta = \pi/2,\ 3\pi/2$ and in the frequency range $p = 0.9 \div 1.06$ at $p\eta = \pi$.

Conclusion

The obtained results show the combined effect of delays in elasticity and damping on the dynamics of oscillations. It was found that delays

- change the shape of the amplitude-frequency curve;
- shift the amplitude-frequency curve up/down and shift it in the frequency range (the resonance zone is displaced in frequency);
- have an effect on the stability of oscillations.

Summing up the results, one can conclude that if in the absence of delay there is no resonant curve (i.e. oscillations) then if delay is present resonant curve may appear with various intensities that dependin on the amount of delay.

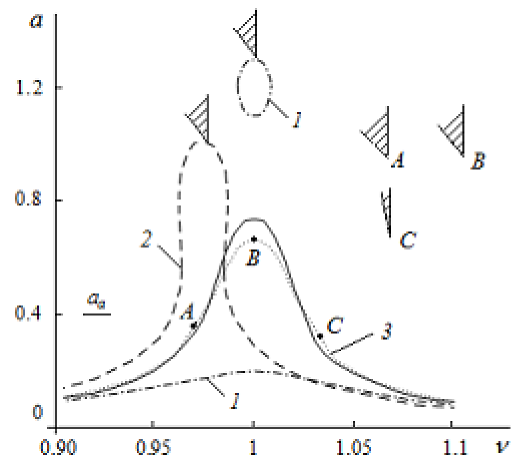

Fig. 2. Amplitude-frequency curves at $\eta = \pi$: curve 1 — $\tau = \pi/2$, curve 2 — $\tau = \pi$, curve 3 — $\tau = 3\pi/2$

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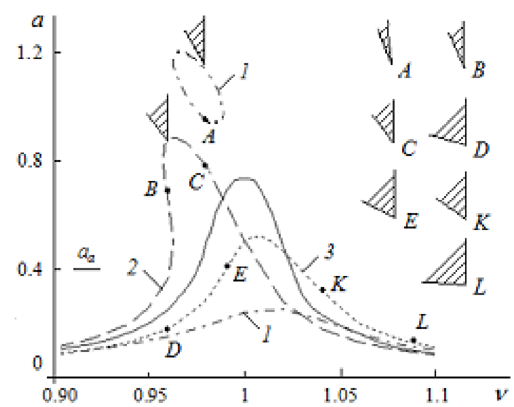

Fig. 3. Amplitude-frequency curves at $\eta = 3\pi/2$: curve 1 – $\tau = \pi/2$, curve 2 – $\tau = \pi$, curve 3 – $\tau = 3\pi/2$

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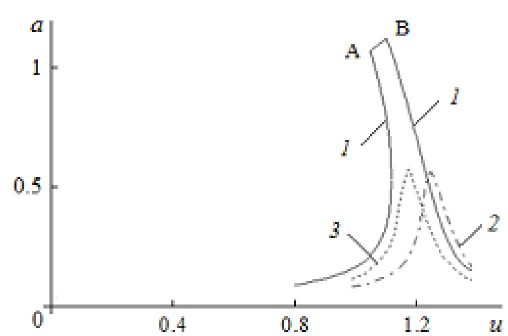

Fig. 4. Amplitude curves at $\eta = \pi/2$: curve 1 – $\tau = \pi/2$, curve 2 – $\tau = \pi$, curve 3 – $\tau = 3\pi/2$

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О вынужденных и автоколебаниях при запаздываниях

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Аннотация. Рассмотрена взаимодействующая с источником энергии ограниченной мощности автоколебательная система при наличии внешней силы и совместном действии запаздываний в демпфировании и упругости. На основе метода прямой линеаризации выполнено решение нелинейных уравнений системы. Выведены уравнения нестационарных движений и соотношения для вычисления амплитуды и фазы стационарных колебаний, скорости источника энергии и нагрузки на него со стороны колебательной системы. Получены условия устойчивости стационарных колебаний с использованием критериев Рауса–Гурвица. Проведены расчеты для получения информации о влиянии запаздываний на динамику системы. Результаты показывают совместное влияние запаздываний в упругости и демпфирования на динамику колебаний. Они изменяют форму амплитудно-частотной кривой, смещают ее вверх/вниз и сдвигают в частотной области (зона резонанса перемещается по частоте), оказывают действие на устойчивость колебаний. При остальных неизменных параметрах системы если в случае отсутствия запаздываний нет резонансной кривой, то при его наличии она может появиться разной интенсивностью в зависимости от величины запаздывания.

Ключевые слова: автоколебания, вынужденные колебания, источник энергии, запаздывание, демпфирование, упругость, линеаризация.