EDN: BKNAZF
УДК 517.955

On the Ill-posed Cauchy Problem for the Polyharmonic Heat Equation

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Received 30.10.2022, received in revised form 02.12.2022, accepted 20.12.2022

Abstract. We consider the ill-posed Cauchy problem for the polyharmonic heat equation on recovering a function, satisfying the equation $(\partial_t+(-\Delta)^m)u = 0$ in a cylindrical domain in the half-space $\mathbb{R}^n\times[0,+\infty)$, where $n \geqslant 1$, $m \geqslant 1$ and Δ is the Laplace operator, via its values and the values of its normal derivatives up to order $(2m-1)$ on a given part of the lateral surface of the cylinder. We obtain a Uniqueness Theorem for the problem and a criterion of its solvability in terms of the real-analytic continuation of parabolic potentials, associated with the Cauchy data.

Keywords: the polyharmonic heat equation, ill-posed problems, integral representation method.

Citation: I.A. Kurilenko, A.A. Shlapunov On the Ill-posed Cauchy Problem for the Polyharmonic Heat Equation, J. Sib. Fed. Univ. Math. Phys., 2023, 16(2), 194–203. EDN: BKNAZF.

In this short note we continue to investigate the ill-posed Cauchy problem for parabolic operators in various function spaces, see [1, 2] for the second order operators in the Hölder spaces, or [3–5] for the second order operators in the anisotropic Sobolev spaces. Actually the general schemes related to investigation of the ill-posed Cauchy problem for elliptic operators (see [6–8] for the second order operators or [9, 10] for the Cauchy–Riemann system in one and many complex variables or [12, 13] for general elliptic operators with the unique continuation property) are still applicable in this new situation.

In the present paper we concentrated our efforts on the solvability criterion of the ill-posed Cauchy problem for a simple class of Petrovsky $2m$-parabolic partial differential operators

$$(\partial_t + (-\Delta)^m), \tag{1}$$

where $m \geqslant 1$ and Δ is the Laplace operator in $\mathbb{R}^n$, $n \geqslant 1$, that are often called polyharmonic heat operators, see [14, Ch.2, Sec. 1], [15]. Namely the problem consists of the recovering a function, satisfying the equation $(\partial_t + (-\Delta)^m)u = 0$ in a cylindrical domain in the half-space $\mathbb{R}^n \times [0, +\infty)$, via its values and the values of its normal derivatives up to order $(2m-1)$ on a given part of the lateral surface of the cylinder. The crucial difference between the heat equation (or the parabolic Lamé system) and the polyharmonic heat equation is the fact that the fundamental solution of the polyharmonic heat operator is given by a non-elementary function. The situation resembles somehow the matter with the fundamental solutions to the Helmholtz operator $\Delta + c_0^2$: for $n = 3$ it is given by $\frac{-e^{\pm\iota c_0|x|}}{4\pi|x|}$ (here ι is the imaginary unit) while for $n = 2$ it is represented by the Hankel functions of the second kind (actually, some versions of the Bessel functions), see, [16, Ch. III, Sec. 11]. Of course, it is not a surprise, because after an application of the Laplace transform L with respect to the variable t (if applicable) to (1), one arrives at the parameter depending elliptic equation

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$$(\iota\tau + (-\Delta)^m) L(u) = 0, \tag{2}$$

coinciding with the Helmholtz equation for $m = 1$ regarding the generalized function $L(u)$ as an unknown and τ as a real parameter. Actually, this seemingly simple approach, reducing the parabolic equations to elliptic ones, is known for decades, see [17]. It gives a lot of qualitative information on the connection between the corresponding solutions of the differential equations of different kinds. However one needs very delicate properties of the Laplace transform in order to obtain really useful formulas solving the parabolic problems with the use of elliptic theory, see for instance, [3] for the heat equation and the related remark on properties of the Laplace transform [18]. Thus, we will act in the framework of mentioned above scheme invented by L.Aizenberg and developed in [12].

1. Preliminaries

Let Ω be a bounded domain in n-dimensional linear space $\mathbb{R}^n$ with the coordinates $x = (x_1, \dots, x_n)$. As usual we denote by $\overline{\Omega}$ the closure of Ω, and we denote by $\partial\Omega$ its boundary. In the sequel we assume that $\partial\Omega$ is piece-wise smooth. We denote by Ω_T the bounded open cylinder $\Omega \times (0, T)$ in $\mathbb{R}^{n+1}$ with a positive altitude T. Let also $\Gamma \subset \partial\Omega$ be a non empty connected relatively open subset of $\partial\Omega$. Then $\Gamma_T = \Gamma \times (0,\ T)$ and $\overline{\Gamma_T} = \overline{\Gamma} \times [0,\ T]$.

We consider the functions over subsets in $\mathbb{R}^n$ and $\mathbb{R}^{n+1}$. As usual, for $s \in \mathbb{Z}_+$ we denote by $C^s(\Omega)$ the space of all s times continuously differentiable functions in Ω. Next, for a (relatively open) set $S \subset \partial\Omega$ denote by $C^s(\Omega \cup S)$ the set of such functions from the space $C^s(\Omega)$ that all their derivatives up to order s can be extended continuously onto $\Omega \cup S$. The standard topology of these metrizable spaces induces the uniform convergence on compact subsets in $\Omega \cup S$ together with all partial derivatives up to order s. We will also use the standard Banach Hölder spaces $C^s(\overline{\Omega})$ and $C^{s,\lambda}(\overline{\Omega})$ (cf. [19], [20, Ch.1, Sec. 1], [21]), and the related metrizable spaces $C^{s,\lambda}(\Omega \cup S)$.

Let also $L^p(\Omega)$, $p \geqslant 1$, be the Lebesgue spaces, $H^s(\Omega)$, $s \geqslant 0$, stand for the Sobolev spaces if $s \in \mathbb{N}$ and for the Sobolev-Slobodetskii spaces if $s > 0$, $s \notin \mathbb{N}$.

To investigate the polyharmonic heat equation we need also the anisotropic ($2m$-parabolic) spaces, see [20, Ch. 1], [21, Ch. 8] for $m = 1$ and [14] for $m \geqslant 1$. With this aim, let $C^{2ms,s}(\Omega_T)$, $m \in \mathbb{N}$, stand for the set of all the continuous functions u in Ω_T, having in Ω_T the continuous partial derivatives $\partial_t^j \partial_x^\alpha u$ with all the multi-indexes $(\alpha, j) \in \mathbb{Z}_+^n \times \mathbb{Z}_+$ satisfying $|\alpha| + 2mj \leqslant 2ms$ where, as usual, $|\alpha| = \sum\limits_{j=1}^{n} \alpha_j$. Similarly, we denote by $C^{2ms+k,s}(\Omega_T)$ the set of continuous functions in Ω_T, such that all partial derivatives $\partial^\beta u$ belong to $C^{2ms,s}(\Omega_T)$ if $\beta \in \mathbb{Z}_+^n$ satisfies $|\beta| \leqslant k$, $k \in \mathbb{Z}_+$. Of course, it is natural to agree that $C^{2ms+0,s}(\Omega_T) = C^{2ms,s}(\Omega_T)$, $C^{0,0}(\Omega_T) = C(\Omega_T)$ and $C^0(\Omega) = C(\Omega)$. We also denote by $C^{2ms+k,s}((\Omega \cup S)_T)$ the set of such functions u from the space $C^{2ms+k,s}(\Omega_T)$ that their partial derivatives $\partial_t^j \partial_x^{\alpha+\beta} u$, $2mj + |\alpha| \leqslant 2ms$, $|\beta| \leqslant k$, can be extended continuously onto $(\Omega \cup S)_T$. The standard topology of these metrizable spaces induces the uniform convergence on compact subsets of $(\Omega \cup S)_T$ together with all partial derivatives used in its definition (the cases $S = \emptyset$ and $S = \partial D$ are included).

We use also the anisotropic Hölder spaces (cf., [20, Ch. 1], [21, Ch. 8]) for $m = 1$ and [14] for $m \geqslant 1$. Let $C^{2ms+k,s,\lambda,\lambda/2}((\Omega \cup S)_T)$ stand for the set of anisotropic Hölder continuous functions with a power λ over each compact subset of $(\Omega \cup S)_T$ together with all partial derivatives $\partial_x^{\alpha+\beta} \partial_t^j u$ where $|\beta| \leqslant k$, $|\alpha| + 2mj \leqslant 2ms$. Clearly, $C^{2ms+k,s,\lambda,\lambda/2}(\overline{\Omega_T})$ is a Banach space with the natural norm, see, for instance, [21, Ch. 8] for $m = 1$ and [14] for $m \geqslant 1$. In general, the space $C^{2ms+k,s,\lambda,\lambda/2}((\Omega \cup S)_T)$ can be treated again as a metrizable space, generated by a system of seminorms associated with a suitable exhaustion $\{\Omega_i\}_{i \in \mathbb{N}}$ of the set $\Omega \cup S$.

In order to invoke the Hilbert space approach, we need anisotropic ($2m$-parabolic) Sobolev spaces $H^{2ms,s}(\Omega_T)$, $s \in \mathbb{Z}_+$, see, [20, 22] for $m = 1$ or [14] for $m \geqslant 1$, i.e. the set of all the measurable functions u in Ω_T such that all the generalized partial derivatives $\partial_t^j \partial_x^\alpha u$ with all the multi-indexes $(\alpha, j) \in \mathbb{Z}_+^n \times \mathbb{Z}_+$ satisfying $|\alpha| + 2mj \leqslant 2ms$, belong to the Lebesgue class $L^2(\Omega_T)$. This is the Hilbert space with the natural inner product $(u, v)_{H^{2ms,s}(\Omega_T)}$. We also may define $H^{2ms,s}(\Omega_T)$ as the completion of the space $C^{2ms,s}(\overline{\Omega_T})$ with respect to the norm $\|\cdot\|_{H^{2ms,s}(\Omega_T)}$ generated by the inner product $(u, v)_{H^{2ms,s}(\Omega_T)}$. For $s = 0$ we have $H^{0,0}(\Omega_T) = L^2(\Omega_T)$.

We also will use the so-called Bochner spaces of functions depending on (x, t) from the strip $\mathbb{R}^n \times [T_1, T_2]$. Namely, for a Banach space $\mathcal{B}$ (for example, on a subdomain of $\mathbb{R}^n$) and $p \geqslant 1$, we denote by $L^p([T_1, T_2], \mathcal{B})$ the Banach space of all the measurable mappings $u : [T_1, T_2] \to \mathcal{B}$ with the finite norm $\|u\|_{L^p([T_1,T_2],\mathcal{B})} := \|\|u(\cdot, t)\|_{\mathcal{B}}\|_{L^p([T_1,T_2])}$, see, for instance, [23, ch. Sec. 1.2]. The space $C([T_1, T_2], \mathcal{B})$ is introduced with the use of the same scheme; this is the Banach space of all the continuous mappings $u : [T_1, T_2] \to \mathcal{B}$ with the finite norm $\|u\|_{C([T_1,T_2],\mathcal{B})} := \sup_{t \in [T_1,T_2]} \|u(\cdot, t)\|_{\mathcal{B}}$.

Let now $\Delta = \sum_{j=1}^{n} \partial^2_{x_j,x_j}$ be the Laplace operator in $\mathbb{R}^n$ and let $\mathcal{L}_m = \partial_t + (-\Delta)^m$ stand for the polyharmonic heat operator in $\mathbb{R}^{n+1}$. Of course, for $m = 1$ it coincides with the usual heat operator.

Now let $\partial_\nu = \sum_{j=1}^{n} \nu_j \partial_{x_j}$ denote the derivative at the direction of the exterior unit normal vector $\nu = (\nu_1, \dots, \nu_n)$ to the surface $\partial\Omega$. If $\partial\Omega \in C^{2m-1}$ then the higher order normal derivatives ∂_ν^j are defined near $\partial\Omega$. We fix also a Dirichlet system $\{B_j\}_{j=0}^{2m-1}$ of order $(2m - 1)$ consisting of boundary differential operators with smooth coefficients near $\partial\Omega$, i.e. $\mathrm{ord} B_j = j$ and for each $x \in \partial\Omega$ the characteristic polynomials $\sigma(B_j)(x, \zeta)$ related to the operators B_j do not vanish for $\zeta = \nu(x)$. The sets $(1, \partial_\nu, \partial_\nu^2, \dots \partial_\nu^{2m-1})$ and $(1, \partial_\nu, \Delta, \partial_\nu\Delta, \Delta^2, \dots \Delta^{m-1}, \partial_\nu\Delta^{m-1})$ are precisely the Dirichlet systems because $\sigma(\partial_\nu^j)(x, \nu(x)) = \sigma(\partial_\nu\Delta^j)(x, \nu(x)) = \sigma(\Delta^j)(x, \nu(x)) = 1$ for each $j \in \mathbb{N}$.

We consider the Cauchy problem for the polyharmonic heat equation in the cylinder Ω_T in the sense of the Cauchy–Kowalevski Theorem with respect to the space variables, cf. [24].

Problem 1. *Given $m \geqslant 1$, functions $u_j \in C^{2m-j+1,0}(\overline{\Gamma_T})$, $1 \leqslant j \leqslant 2m$, and $f \in C(\overline{\Omega}_T)$ find a function $u \in C^{2m,1}(\Omega_T) \cap C^{2m-1,0}((\Omega \cup \overline{\Gamma})_T)$ satisfying*

$$\mathcal{L}_m u = f \ \text{ in } \Omega_T, \tag{3}$$

$$B_j u(x, t) = u_{j+1}(x, t) \ \text{ on } \overline{\Gamma_T} \text{ for all } \ 0 \leqslant j \leqslant 2m - 1. \tag{4}$$

If the hypersurface Γ and the data of the problem are real analytic then the Cauchy–Kowalevski theorem implies that problem (3), (4) has one and only one solution in the class of (even formal) power series. However the theorem does not imply the existence of solutions to Problem 1 because it grants the solution in a small neighbourhood of the hypersurface Γ_T only (but not in a given domain Ω_T!). We emphasize that, unlike the classical case, we do not ask for the hypersurface Γ or/and the coefficients of the operators B_j or/and the data f or/and u_j to be real analytic.

Of course, the above trick with the Laplace transform suggests us that the problem is equivalent to an ill-posed problem for the strongly elliptic operator $(-\Delta)^m$ in Ω with the Cauchy data on Γ, i.e. Problem 1 is ill-posed itself, too.

2. Solvability conditions

We begin this section proving that Problem 1 can not have more than one solution in the spaces of differentiable (non-analytic) functions.

To investigate Problem 1, we use an integral representation constructed with the use the fundamental solution $\Phi_m(x, t)$ to polyharmonic heat operator $\mathcal{L}_m$. If $m = 1$ then

$$\Phi_1(x,t) = \begin{cases} \dfrac{e^{-\frac{|x|^2}{4\mu\, t}}}{\left(2\sqrt{\pi\mu\, t}\right)^n} & \text{if } t > 0, \\ 0 & \text{if } t \leqslant 0, \end{cases} \tag{5}$$

see, for instance, [19, 25]. Unfortunately, if $m > 1$ then the fundamental solution can not be represented as an elementary function, see, for instance, [14, Ch. 2, Sec. 1], [15],

$$\Phi_m(x,t) = \begin{cases} k_{n,m} t^{-n/2m} \displaystyle\int_0^{+\infty} \rho^{n-1} e^{-\rho^{2m}} \Big(\frac{|x|\rho}{t^{1/2m}}\Big)^{1-n/2} J_{n/2-1}\Big(\frac{|x|\rho}{t^{1/2m}}\Big) d\rho & \text{if } t > 0, \\ 0 & \text{if } t \leqslant 0, \end{cases} \tag{6}$$

where $k_{n,m}$ is a normalization constant and J_p is the Bessel function of the first kind and of order p (see, for example, [16, Ch. 5, Sec. 23]).

The fundamental solution allows to construct a useful integral Green formula for the operator $\mathcal{L}_m$. With this purpose, Denote by $\{C_0, \dots C_{2m-1}\}$ the Dirichlet system associated with the Dirichlet system $\{B_0, \dots B_{2m-1}\}$ via (first) Green formula for the operator Δ^m, i.e.

$$\int_{\partial\Omega} \Big(\sum_{j=0}^{2m-1} C_{2m-1-j} v B_j u\Big) ds = (\Delta^m u, v)_{L^2(\Omega)} - (u, \Delta^m v)_{L^2(\Omega)}$$

for all $u, v \in C^\infty(\overline{\Omega})$. For instance, if $\{B_0, \dots B_{2m-1}\} = (1, \partial_\nu, \Delta, \partial_\nu \Delta, \dots \Delta^{m-1}, \partial_\nu \Delta^{m-1})$ then $\{C_0, \dots C_{2m-1}\} = (1, -\partial_\nu, \Delta, -\partial_\nu \Delta, , \dots \Delta^{m-1}, -\partial_\nu \Delta^{m-1})$.

Consider the cylinder type domain $\Omega_{T_1,T_2} = \Omega_{T_2} \setminus \overline{\Omega_{T_1}}$ with $0 \leqslant T_1 < T_2$ and a closed measurable set $S \subset \partial\Omega$. For functions $f \in L^2(\Omega_{T_1,T_2})$, $v_j \in L^2([0,T], H^{2m-j-1/2}(S_T))$, $h \in H^{1/2}(\Omega)$ we introduce the following potentials:

$$I_{\Omega,T_1}(h)(x,t) = \int_\Omega \Phi(x-y,t) h(y) dy, \quad G_{\Omega,T_1}(f)(x,t) = \int_{T_1}^{t} \int_\Omega \Phi(x-y,\ t-\tau) f(y,\tau) dy d\tau,$$

$$V^{(j)}_{S,T_1}(v_j)(x,t) = \int_{T_1}^{t} \int_S C_j \Phi_m(x-y, t-\tau) v_j(y,\tau) ds(y) d\tau,\ 0 \leqslant j \leqslant 2m-1$$

(see, for instance, [19, Ch. 1, Sec. 3 and Ch. 5, Sec. 2], [20, Ch. 4, Sec. 1], [26, Ch. 3, Sec. 10] for $m = 1$). The potential $I_{\Omega,T_1}(h)$ is an analogue of the *Poisson integral* and the function $G_{\Omega,T_1}(f)$ is an analogue of the volume heat potential related to $m = 1$. The functions $V^{(0)}_{S,T_1}(v)$ and $V^{(1)}_{S,T_1}(v)$ are often referred to as *single layer heat potential* and *double layer heat potential*, respectively, if $m = 1$. By the construction, all these potentials are (improper) integrals depending on the parameters (x,t).

Next, we need the so-called Green formula for the polyharmonic heat operator.

Lemma 1. *For all $0 \leqslant T_1 < T_2$ and all $u \in {}^{2m,1}(\overline{\Omega_{T_1,T_2}})$ the following formula holds:*

$$\left.\begin{array}{l} u(x,t) \ \textit{in}\ \Omega_{T_1,T_2} \\ 0 \ \textit{outside}\ \overline{\Omega_{T_1,T_2}} \end{array}\right\} = I_{\Omega,T_1}(u) + G_{\Omega,T_1}(\mathcal{L}_m u) + \sum_{j=0}^{2m-1} V^{(j)}_{\partial\Omega,T_1}(B_j u)\,. \tag{7}$$

Proof. See, for instance, [27, ch. 6, Sec. 12] for $m = 1$ and [28, theorem 2.4.8] for more general operators, admitting fundamental solutions/parametreces. $\square$

Formulas (5), (6) mean that the kernels $\Phi_m(x-y, t-\tau)$ are smooth outside the diagonal $\{(x,t) = (y,\tau)\}$ and real analytic with respect to the space variables. In particular, this means that the $2m$-parabolic operator $\mathcal{L}_m$ is hypoelliptic. Moreover, any $C^{2m,1}(\Omega_{T_1,T_2})$-solution v to the polyharmonic heat equation $\mathcal{L}_m v = 0$ in the cylinder domain Ω_{T_1,T_2} belongs to $C^\infty(\Omega_{T_1,T_2})$ and, actually $v(x,t)$ is real analytic with respect to the space variable $x \in \Omega$ for each $t \in (T_1, T_2)$ (for $m = 1$, see, for instance, [25, Ch. VI, Sec. 1, Theorem 1] and for $m > 1$ see [14, Ch. Sec. 2, Sec. 1, Theorem 2.1]). Then Green formula (7) and the information on the kernel Φ_m provide us with a Uniqueness Theorem for Problem 1.

Theorem 1 (A Uniqueness Theorem)**.** *If Γ has at least one interior point in the relative topology of $\partial\Omega$ then Problem 1 has no more than one solution.*

Proof. For $m = 1$ see [1, Theorem 1, Corollary 1]. For $m > 1$ the proof can be done in the same way with natural modifications. Indeed, under the hypothesis of the theorem there is an interior (in the relative topology of Γ!) point x_0 on Γ. Then there is such a number $r > 0$ that $B(x_0,\ r) \cap \partial\Omega \subset \Gamma$ where $B(x_0,\ r)$ is ball in $\mathbb{R}^n$ with center at x_0 and radius r. Fix an arbitrary point $(x', t') \in \Omega_T$. Clearly, there is a domain $\Omega' \ni x'$ satisfying $\Omega' \subset \Omega$ and $\Omega' \cap \partial\Omega \subset \Gamma \cap B(x_0,\ r)$. Then $(x', t') \in \Omega'_{T_1,T_2}$ with some $0 < T_1 < T_2 < T$.

But $u \in C^{2m,1}(\Omega'_{T_1,T_2}) \cap C^{2m-1,0}(\overline{\Omega'_{T_1,T_2}})$ (for $m = 1$ see, for instance, [19, Ch. 1, Sec. 3 and Ch. 5, Sec. 2] and for $m > 1$ it follows from [14, Ch. 2, Sec. 1, Theorem 2.2]) and $\mathcal{L}_m u = 0$ in Ω'_{T_1,T_2} under the hypothesis of the theorem. Hence formula (7) implies:

$$\left.\begin{array}{r} u(x,t),\ (x,t) \in \Omega'_{T_1,T_2} \\ 0,\ (x,t) \notin \overline{\Omega'_{T_1,T_2}} \end{array}\right\} = I_{\Omega',T_1}(u)(x,t) + \sum_{j=0}^{2m-1} V^{(j)}_{\partial\Omega'\setminus\Gamma,T_1}(B_j u)(x,t), \tag{8}$$

because $B_j u \equiv 0$ on Γ_T for all $0 \leqslant j \leqslant 2m - 1$.

Taking into account the character of the singularity of the kernel $\Phi_m(x-y, t-\tau)$ we conclude that the following properties are fulfilled for the integrals, depending on parameter, from the right hand side of identity (8):

$$I_{\Omega',T_1}(u) \in C^{2m,1}(\{x \in \mathbb{R}^n, T_1 < t < T_2\}),$$
$$V^{(j)}_{\partial\Omega'\setminus\Gamma,T_1<t<T_2}(B_j u) \in C^{2m,1}(\{x \in \mathbb{R}^n \setminus (\partial\Omega' \setminus \Gamma), T_1 < t < T_2\})$$

(see, for instance, [19, Ch. 1, Sec. 3 and Ch. 5, Sec. 2], [20, Ch. 4, Sec. 1] or [26, Ch. 3, Sec. 10] for m=1). Moreover, as Φ_m is a fundamental solution to the polyharmonic heat operator then

$$\mathcal{L}_m(x,t)\Phi_m(x-y, t-\tau) = 0 \text{ for } (x,t) \neq (y,\tau),$$

and therefore, using Leibniz rule for differentiation of integrals depending on parameter we obtain:

$$\mathcal{L}_m I_{\Omega',T_1}(u) = 0 \text{ in the domain } \{x \in \mathbb{R}^n, T_1 < t < T_2\},$$
$$\mathcal{L}_m V^{(j)}_{\partial\Omega'\setminus\Gamma,T_1}(B_j u) = 0 \text{ in } \Omega''_{T_1,T_2} = \{x \in \mathbb{R}^n \setminus (\partial\Omega' \setminus \Gamma), T_1 < t < T_2\} \text{ for all } 0 \leqslant j \leqslant 2m-1.$$

Hence the function

$$v(x,t) = I_{\Omega',T_1}(u)(x,t) + V^{(j)}_{\partial\Omega'\setminus\Gamma,T_1}(B_j u)(x,t),$$

satisfies the polyharmonic heat equation $(\mathcal{L}_m v)(x,t) = 0$ in Ω''_{T_1,T_2}. As we mentioned above, this implies that the function $v(x,t)$ is real analytic with respect to the space variable $x \in \mathbb{R}^n \setminus (\partial\Omega' \setminus \Gamma)$ for any $T_1 < t < T_2$. By the construction the function $v(x,t)$ is real analytic with respect to x in the ball $B(x_0, r)$ and it equals to zero for $x \in B(x_0, r) \setminus \overline{\Omega}$ for all $T_1 < t < T_2$. Therefore, the Uniqueness Theorem for real analytic functions yields $v(x,t) \equiv 0$ in Ω''_{T_1,T_2}, and in the cylinder Ω'_{T_1,T_2}, containing point (x', t'). Now it follows from (8) that $u(x', t') = v(x', t') = 0$ and then, since the point $(x', t') \in \Omega_T$ is arbitrary we conclude that $u \equiv 0$ in Ω_T. □

Now we are ready to formulate a solvabilty criterion for Problem 1. As before, we assume that Γ is a relatively open connected subset of $\partial\Omega$. Then we may find a set $\Omega^+ \subset \mathbb{R}^n$ in such a way that the set $D = \Omega \cup \Gamma \cup \Omega^+$ would be a bounded domain with piece-wise smooth boundary.

It is convenient to set $\Omega^- = \Omega$. For a function v on D_T we denote by v^+ its restriction to Ω^+_T and, similarly, we denote by v^- its restriction to Ω_T. It is natural to denote limit values of $v^\pm$ on Γ_T, when they are defined, by $v^\pm_{|\Gamma_T}$. Actually, for $m = 1$ similar solvability criterions for Problem 1 were obtained in [1] and [4].

Theorem 2 (Solvability criterion)**.** *Let $\lambda \in (0,1)$, $\partial\Omega$ belong to $C^{2m-1+\lambda}$ and let Γ be a relatively open connected subset of $\partial\Omega$. If $f \in C^{0,0,\lambda,\lambda/2}(\overline{\Omega_T})$, $u_j \in C^{2m-j,0,\lambda,\lambda/2}(\overline{\Gamma_T})$, $1 \leqslant j \leqslant 2m$, then Problem* (3), (4) *is solvable in the space $C^{2m,1,\lambda,\lambda/2}(\Omega_T) \cap C^{2m-1,0,\lambda,\lambda/2}(\Omega_T \cup \Gamma_T)$ if and only if there is a function $F \in C^\infty(D_T)$ satisfying the following two conditions: 1) $\mathcal{L}_m F = 0$ in D_T, 2) $F = G_{\Omega,0}(f) + \sum\limits_{j=0}^{2m-1} V^{(j)}_{\overline{\Gamma},0}(u_{j+1})$ in Ω^+_T.*

Proof. Necessity. Let a function $u(x,t) \in C^{2m,1,\lambda,\lambda/2}(\Omega_T) \cap C^{2m-1,0,\lambda,\lambda/2}(\Omega_T \cup \Gamma_T)$ satisfy (3), (4). Clearly, the function $u(x,t)$ belongs to the space $C^{2m,1,\lambda,\lambda/2}(\Omega'_T) \cap C^{2m-1,0,\lambda,\lambda/2}(\overline{\Omega'_T}$ for each cylindrical domain Ω'_T with such a base Ω' that $\Omega' \subset \Omega$ and $\overline{\Omega'} \cap \partial\Omega \subset \Gamma$. Besides, $\mathcal{L}u = f \in C^{0,0,\lambda,\lambda/2}(\overline{\Omega'_T})$. Without loss of the generality we may assume that the interior part Γ' of the set $\overline{\Omega'} \cap \partial\Omega$ is non-empty. Consider in the domain D_T the functions

$$\mathcal{F} = G_{\Omega,0}(f) + \sum_{j=0}^{2m-1} V^{(j)}_{\overline{\Gamma},0}(u_{j+1}) \text{ and } F = \mathcal{F} - \chi_{\Omega_T} u, \tag{9}$$

where χ_M is a characteristic function of the set $M \subset \mathbb{R}^{n+1}$. By the very construction condition 2) is fulfilled for it. Note that $\chi_{\Omega_T} u = \chi_{\Omega'_T} u$ in D'_T, where $D' = \Omega' \cup \Gamma' \cup \Omega^+$. Then Lemma 1 yields

$$F = G_{\Omega\setminus\overline{\Omega'},0}(f) + \sum_{j=0}^{2m-1} V^{(j)}_{\overline{\Gamma},0}(u_{j+1}) - I_{\Omega',0}(u) \text{ in } D'_T. \tag{10}$$

Arguing as in the proof of Theorem 1 we conclude that each of the integrals in the right hand side of (10) is smooth outside the corresponding integration set and each satisfies homogeneous polyharmonic heat equation there. In particular, we see that $F \in C^\infty(D'_T)$ and $\mathcal{L}F = 0$ in D'_T because of [25, Ch. VI, Sec. 1, Theorem 1]. Obviously, for any point $(x,t) \in D_T$ there is a domain D'_T containing $(x,\ t)$. That is why $\mathcal{L}_m F = 0$ in D_T, and hence F belongs to the space $C^\infty(D_T)$. Thus, this function satisfies condition 1), too.

Sufficiency. Let there be a function $F \in C^\infty(D_T)$, satisfying conditions 1) and 2) of the theorem. Consider on the set D_T the function

$$U = \mathcal{F} - F. \tag{11}$$

As $f \in C^{0,0,\lambda,\lambda/2}(\overline{\Omega_T})$ then the results of [19, Ch. 1, Sec. 3], [20, Ch. 4, Secs. 11–14] for $m = 1$ and [14, Ch. 2, Sec. 1, Theorem 2.2] for $m > 1$ imply

$$G_{\Omega,0}(f) \in C^{2m,1,\lambda,\lambda/2}(\overline{\Omega^\pm_T}) \cap C^{2m-1,0,\lambda,\lambda/2}(D_T) \tag{12}$$

and, moreover,

$$\mathcal{L}_m G^-_{\Omega,0}(f) = f \text{ in } \Omega_T, \quad \mathcal{L}_m G^+_{\Omega,0}(f) = 0 \text{ in } \Omega^+_T. \tag{13}$$

Since $u_j \in C^{2m-j,0,\lambda,\lambda/2}(\overline{\Gamma_T})$ then the results of [20, Ch. 4, Secs. 11-14], [19, Ch. 5, Sec. 2] for $m = 1$ and [14, Ch. 2, Sec. 1, Theorem 2.2] for $m > 1$ yield

$$V^{(j)}_{\overline{\Gamma},0}(u_j) \in C^\infty(\Omega^\pm_T) \cap C^{2m-1,0,\lambda,\lambda/2}((\Omega^\pm \cup \Gamma)_T), \quad \mathcal{L}^{(j)} V_{\overline{\Gamma},0}(u_j) = 0 \text{ in } \Omega_T \cup \Omega^+_T. \tag{14}$$

Since $F \in C^\infty(D_T) \subset C^{1,0,\lambda,\lambda/2}((\Omega^+ \cup \Gamma)_T)$ then formulas (11)–(14) imply that U belongs $C^{2m,1,\lambda,\lambda/2}(\Omega^\pm_T) \cap C^{2m-1,0,\lambda,\lambda/2}((\Omega^\pm \cup \Gamma)_T)$ and $\mathcal{L}U = \chi_{D_T} f$ in $\Omega_T \cup \Omega^+_T$. In particular, (3) is fulfilled for U^-. Let us show that the function U^- satisfies (4). Since $F \in C^\infty(D_T)$ we see that $\partial^\alpha F^- = \partial^\alpha F^+$ on Γ_T for $\alpha \in \mathbb{Z}_+$ with $|\alpha| \leqslant 2m-1$ and

$$\partial^\alpha F^+_{|\Gamma_T} = \Big(\partial^\alpha G_{\Omega,0}(f) + \sum_{j=0}^{2m-1} \partial^\alpha \big(V^{(j)}_{\overline{\Gamma},0}(u_{j+1}) \big) \Big)^+_{|\Gamma_T}.$$

Thus, it follows from formula (12) that for all $0 \leqslant i \leqslant 2m-1$ we have

$$B_i U^-_{|\Gamma_T} = \Big(B_i \Big(\sum_{j=0}^{2m-1} V^{(j)}_{\overline{\Gamma},0}(u_{j+1}) \Big) \Big)^-_{|\Gamma_T} - \Big(B_i \Big(\sum_{j=0}^{2m-1} V^{(j)}_{\overline{\Gamma},0}(u_{j+1}) \Big) \Big)^+_{|\Gamma_T}. \tag{15}$$

Hence, in order to finish the proof we need the following lemma.

Lemma 2. *Let $\Gamma \in C^{2m-1+\lambda}$ and $u_j \in C^{2m-j,0,\lambda,\lambda/2}(\overline{\Gamma_T})$, $1 \leqslant j \leqslant 2m$. Then*

$$\Big(B_i \Big(\sum_{j=0}^{2m-1} V^{(j)}_{\overline{\Gamma},0}(u_{j+1}) \Big) \Big)^-_{|\Gamma_T} - \Big(B_i \Big(\sum_{j=0}^{2m-1} V^{(j)}_{\overline{\Gamma},0}(u_{j+1}) \Big) \Big)^+_{|\Gamma_T} = u_{i+1}, \ 0 \leqslant i \leqslant 2m-1. \tag{16}$$

Proof. It is similar to the proof of the analogous lemmas for the heat Single and Double Layer Potentials (see, for instance, [1, Lemma 3], [26, Ch. 3, Sec. 10, Theorem 10.1] for $m = 1$ and a different function class or [12, Lemma 2.7] for elliptic potentials). □

Using Lemma 2 and formulas (12), (15), we conclude that $B_j U^-_{|\Gamma_T} = u_{j+1}$ for all $0 \leqslant j \leqslant 2m-1$, i.e. the second equation in (4) is fulfilled for U^-. Thus, function $u(x,t) = U^-(x,t)$ satisfies conditions (3), (4). The proof is complete. □

We note that Theorem 2 is also an analogue of Theorem by Aizenberg and Kytmanov [10] describing solvability conditions of the Cauchy problem for the Cauchy–Riemann system (cf. also [11] in the Cauchy Problem for Laplace Equation or [13] in the Cauchy problem for general elliptic systems).

We note also that formula (11), obtained in the proof of Theorem 2, gives the unique solution to Problem 1. Clearly, if we will be able to write the extension F of the sum of potentials $G_{\Omega,0}(f) + \sum_{j=0}^{2m-1} V^{(j)}_{\overline{\Gamma},0}(u_{j+1})$ from Ω^+_T onto D_T as a series with respect to special functions or a limit of parameter depending integrals then we will get Carleman's type formula for solutions to Problem 1 (cf. [10]). However, for the best way for this purpose is to use the Fourier series in the framework of the Hilbert space theory, see [5]. Unfortunately, this is not a short story because one needs approximation theorems in spaces of solutions to the homogeneous polyharmonic heat equation that we are not ready to prove right now. Thus we finish our paper with a statement extending Theorem 2 to the anisotropic Sobolev spaces, leaving the construction of the Carleman's type formulae for the next article.

First of all, we need the following lemma.

Lemma 3. *Let $\partial\Omega \in C^{2m+1}$ and let Γ be a relatively open connected subset of $\partial\Omega$ with boundary $\partial\Gamma \in C^{2m+\lambda}$. If $u_j \in C^{2m+1-j,0,\lambda,\lambda/2}(\overline{\Gamma_T})$, $1 \leqslant j \leqslant 2m$, then there exist functions $\tilde{u}_j \in C^{2m+1-j,0,\lambda,\lambda/2}(\partial\Omega_T)$ such that $\tilde{u}_j = u_j$ on $\overline{\Gamma_T}$, $1 \leqslant j \leqslant 2m$, and a function $\tilde{u} \in C^{2m,1,\lambda,\lambda/2}(\overline{\Omega_T})$ such that $B_j \tilde{u} = \tilde{u}_{j+1}$ on $(\partial\Omega)_T$ for all $0 \leqslant j \leqslant 2m-1$.*

Proof. We may adopt the standard arguments from [29, Lemma 6.37] related to isotropic spaces. Indeed, according to it, under our assumptions, for any $s \leqslant 2m$ and any $v \in C^{s,\lambda}(\overline{\Gamma})$ there is $\tilde{v}_j \in C^{s,\lambda}(\partial\Omega)$ such that $v = v_0$ on $\overline{\Gamma}$. The construction of the extension involves the rectifying diffeomorphism of $\partial\Gamma$ and a suitable partition of unity of a neighbourhood of $\partial\Gamma$, only. Thus, we conclude there are functions $\tilde{u}_j \in C^{2m-j+1,0,\lambda,\lambda/2}(\partial\Omega_T)$ such that $\tilde{u}_j = u_j$ on $\overline{\Gamma_T}$, $1 \leqslant j \leqslant 2m$.

Next, we use the existence of the Poisson kernel $P_{\Delta^{2m},\Omega}(x,y)$ for the Dirichlet problem related to the operator Δ^{2m}, see [30]. It is known that the problem is well-posed over the scale of Hölder spaces in Ω. Namely, if $\partial\Omega \in C^{s+1,\lambda}$, $s \geqslant 2m-1$, then for each $\oplus_{j=0}^{2m-1} v_j \in C^{s-j,\lambda}(\partial\Omega)$ the integral

$$v(x) = \mathcal{P}_{\Delta^{2m},\Omega}(\oplus_{j=0}^{2m-1} v_j)(x) = \int_{\partial\Omega} \left(\sum_{j=0}^{2m-1} (B_j(y) P_{\Delta^{2m},\Omega})(x,y) v_j(y) \right) ds(y)$$

belongs to $C^{s,\lambda}(\overline{\Omega})$ and satisfies $\Delta^{2m} v = 0$ in Ω and $B_j v = v_j$ on $\partial\Omega$ for all $0 \leqslant j \leqslant 2m-1$.

Now, we set

$$\tilde{u}_0(x) = \mathcal{P}_{\Delta^{2m},\Omega}(\oplus_{j=0}^{2m-1} \tilde{u}_{j+1})(\cdot, 0)(x) \in C^{2m-1,\lambda}(\overline{\Omega}) \cap C^{2m,\lambda}(\Omega).$$

Now, we may take as $\tilde{u}(x,t) \in C^{2m,1,\lambda,\lambda/2}(\Omega_T) \cap C^{2m-1,0,\lambda,\lambda/2}(\overline{\Omega_T})$ the unique solution to the parabolic initial boundary problem

$$\begin{cases} \partial_t \tilde{u}(x,t) + \Delta^{2m} \tilde{u}(x,t) = 0 & \text{in} \quad \Omega_T, \\ \oplus_{j=0}^{2m-1} B_j \tilde{u}(x,t) = \oplus_{j=0}^{2m-1} \tilde{u}_{j+1}(x,t) & \text{on} \quad (\partial\Omega)_T, \\ \tilde{u}(x,0) = \tilde{u}_0(x) & \text{on} \quad \overline{\Omega}, \end{cases}$$

see, for instance, [20, Ch. 5, Sec. 6] for $m=1$ or [14, Ch. 3, Sec. 1] for $m \geqslant 1$. But of course, there are other possibilities to choose a function $\tilde{u}$ with the desired properties. □

Under the assumptions of Lemma 3, we set

$$\tilde{\mathcal{F}} = G_{\Omega,0}(f) + \sum_{j=0}^{2m-1} V^{(j)}_{\partial\Omega,0}(\tilde{u}_{j+1}) + I_{\Omega,0}(\tilde{u}). \tag{17}$$

Corollary 1. *Let $\lambda \in (0,1)$, $\partial\Omega$ belong to $C^{2m+1+\lambda}$ and let Γ be a relatively open connected subset of $\partial\Omega$ with boundary $\partial\Gamma \in C^{2m+\lambda}$. If $f \in C^{0,0,\lambda,\lambda/2}(\overline{\Omega_T})$, $u_j \in C^{2m-j+1,0,\lambda,\lambda/2}(\overline{\Gamma_T})$, then Problem (3), (4) is solvable in the space $C^{2m,1,\lambda,\lambda/2}(\Omega_T) \cap C^{2m-1,0,\lambda,\lambda/2}(\Omega_T \cup \Gamma_T) \cap H^{2m,1}(\Omega_T)$ if and only if there is a function $\tilde{F} \in C^\infty(D_T) \cap H^{2m,1}(D_T)$ satisfying the following two conditions: 1') $\mathcal{L}\tilde{F} = 0$ in D_T, 2') $\tilde{F} = \tilde{\mathcal{F}}$ in Ω^+_T.*

Proof. First of all, we note that, by Green formula (7), we have $\tilde{\mathcal{F}} = G_{\Omega,0}(f - \mathcal{L}\tilde{u}) + \chi_{\Omega_T}\tilde{u}$ and then $\tilde{\mathcal{F}} \in C^{2m,1,\lambda,\lambda/2}(\overline{\Omega^\pm_T})$ because of (12). On the other hand,

$$\tilde{\mathcal{F}} - \mathcal{F} = \sum_{j=0}^{2m-1} V^{(j)}_{\partial\Omega\setminus\Gamma,0}(\tilde{u}_{j+1}) + I_{\Omega,0}(\tilde{u}). \tag{18}$$

This means that the function $\tilde{\mathcal{F}} - \mathcal{F}$ satisfies the $\mathcal{L}(\tilde{\mathcal{F}} - \mathcal{F}) = 0$ in D_T and hence the function $\mathcal{F}$ extends to D_T as a solution of the heat equation if and only if function $\tilde{\mathcal{F}}$ extends to D_T as a solution of the polyharmonic heat equation, too.

Let Problem (3), (4) be solvable in the space $C^{2m,1,\lambda,\lambda/2}(\Omega_T) \cap C^{2m-1,0,\lambda,\lambda/2}(\Omega_T \cup \Gamma_T) \cap H^{2m,1}(\Omega_T)$. Then formulas (9) and (18) imply

$$\tilde{F} = \tilde{\mathcal{F}} - \chi_{\Omega_T} u \in H^{2m,1}(\Omega^\pm_T) \text{ and } \mathcal{L}\tilde{F} = 0 \text{ in } D_T.$$

Now, as $\tilde{F} \in H^{2m,1}(\Omega^\pm_T) \cap C^\infty(D_T)$ (see [25, Ch. VI, Sec. 1, Theorem 1]) we conclude that $\tilde{F} \in H^{2m,1}(D_T)$, i.e. conditions 1'), 2') of the corollary are fulfilled.

If conditions 1'), 2') of the corollary hold true then conditions 1), 2) of Theorem 2 are fulfilled, too. Moreover, formulas (11) and (18) imply that in D_T we have

$$U = \mathcal{F} - F = \tilde{\mathcal{F}} - \tilde{F} \in H^{2m,1}(\Omega^\pm_T) \tag{19}$$

and the U^- is the solution to Problem 1 in the space $C^{2m,1,\lambda,\lambda/2}(\Omega_T) \cap C^{2m-1,0,\lambda,\lambda/2}(\Omega_T \cup \Gamma_T) \cap H^{2m,1}(\Omega^\pm_T)$ by Theorem 2. □

The second author was supported by the Russian Science Foundation, grant no. 20-11-20117.

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О некорректной задаче Коши для решений полигармонического уравнения теплопроводности

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Аннотация. Мы рассматриваем некорректную задачу Коши для полигармонического оператора теплопроводности о восстановлении функции, удовлетворяющей уравнению $(\partial_t+(-\Delta)^m)u=0$ в цилиндрической области в полупространстве $\mathbb{R}^n \times [0,+\infty)$, где $n \geqslant 1$, $m \geqslant 1$, а Δ – оператор Лапласа, по заданным ее значениям и значениям ее нормальных производных до порядка $(2m-1)$ включительно на части боковой поверхности цилиндра. Нами получены теорема единственности для этой задачи Коши в анизотропных пространствах Соболева, а также необходимые и достаточные условия ее разрешимости в терминах вещественно-аналитического продолжения параболических потенциалов, ассоциированных с данными Коши.

Ключевые слова: полигармоническое уравнение теплопроводности, некорректные задачи, метод интегральных представлений.