EDN: TXBEZL
УДК 517.5

Applications of Two Summation Theorems of Gosper for the ${}_5F_4$ Hypergeometric Series

M. I. Qureshi*
Department of Applied Sciences and Humanities
Faculty of Engineering and Technology
Jamia Millia Islamia (A Central University)
New Delhi-110025, India

R. B. Paris†
Division of Computing and Mathematics
Abertay University
Dundee DD1 1HG, UK

M. Kashif Khan‡
Department of Mathematics and Statistics
Faculty of Science
Integral University
Lucknow-226026 (U.P.), India

Received 26.08.2022, received in revised form 16.09.2022, accepted 25.11.2022

Abstract. By means of two summation theorems of R. W. Gosper for a terminating ${}_5F_4$ hypergeometric series of arguments $1/4$ and 4, we derive two general double-series identities involving a bounded sequence of arbitrary complex numbers. These series are then applied to obtain two reduction formulas for the Srivastava–Daoust double hypergeometric function.

Keywords: generalised hypergeometric function, bounded sequence, Gosper's summation theorems, Srivastava–Daoust double hypergeometric function.

Citation: M.I. Qureshi, R.B. Paris, M. Kashif Khan, Applications of Two Summation Theorems of Gosper for the ${}_5F_4$ Hypergeometric Series, J. Sib. Fed. Univ. Math. Phys., 2023, 16(1), 142–148. EDN: TXBEZL

1. Introduction and preliminaries

We use the following standard notation:

$$\mathbf{N} := \{1, 2, 3, \cdots\}; \quad \mathbf{N}_0 := \mathbf{N} \cup \{0\}; \quad \mathbf{Z}_0^- := \{0, -1, -2, -3, \cdots\}$$

and the symbols $\mathbf{C}$, $\mathbf{R}$ for the sets of complex and real numbers, respectively. The Pochhammer symbol (or the *shifted* factorial) is given by $(\alpha)_n = \alpha(\alpha+1)\dots(\alpha+n-1) = \Gamma(\alpha+n)/\Gamma(\alpha)$, it being understood conventionally that $(0)_0 = 1$. In what follows we shall adopt the usual convention of writing the sequence $\alpha_1, \alpha_2, \dots, \alpha_A$ simply by (α_A).

*miqureshi_delhi@yahoo.co.in https://orcid.org/0000-0002-5093-1925
†r.paris@abertay.ac.uk https://orcid.org/0000-0001-7868-0278
‡md.kashifkhan85@gmail.com https://orcid.org/0000-0001-6013-8026

The generalized hypergeometric function ${}_AF_B(z)$ is defined by

$$ {}_AF_B\left[\begin{matrix}(\alpha_A)\\(\beta_B)\end{matrix};z\right] = {}_AF_B\left[\begin{matrix}\alpha_1,\alpha_2,\dots,\alpha_A\\ \beta_1,\beta_2,\dots,\beta_B\end{matrix};z\right] = \sum_{n=0}^{\infty}\frac{(\alpha_1)_n(\alpha_2)_n\dots(\alpha_A)_n}{(\beta_1)_n(\beta_2)_n\dots(\beta_B)_n}\,\frac{z^n}{n!}, \tag{1} $$

where A and $B \in \mathbf{N_0}$ and the variable $z \in \mathbf{C}$. The numerator parameters $\alpha_1, \alpha_2, \dots, \alpha_A$ and the denominator parameters $\beta_1, \beta_2, \dots, \beta_B$ can, in general, take on complex values, provided that

$$ \beta_j \neq 0, -1, -2, \dots, \quad (j = 1, 2, \dots, B). $$

Assuming that none of the numerator and denominator parameters is zero or a negative integer, the ${}_AF_B(z)$ function defined by equation (1) converges for $|z| < \infty$ $(A \leqslant B)$, $|z| < 1$ $(A = B+1)$ and $|z| = 1$ $(A = B+1$ and $\Re(s) > 0)$, where s is the parametric excess defined by $s := \sum\limits_{j=1}^{B}\beta_j - \sum\limits_{j=1}^{A}\alpha_j$.

The popularity and usefulness of the hypergeometric function of one variable have stimulated research of hypergeometric functions of several variables. Functions of the hypergeometric type constitute an important class of special functions. Many special functions can be expressed in terms of others. Therefore, a detailed study and systematization of the relationships between them is a fundamental problem. Serious investigations of functions of two variables which are natural generalisations of the Gaussian hypergeometric function were initiated by Appell and Kampé de Fériet [1]. The reducibility of the Kampé de Fériet function has been a subject to study for many authors (see papers by Buschman and Srivastava [3], Karlsson [12], Exton [10], Choi and Rathie [7], Kim et. al. [13] etc.)

In an earlier investigation, Srivastava and Daoust [18, p.199] defined a generalization of the Kampé de Fériet function [1, p.150] by means of the double hypergeometric series (for details, see [19]):

$$ F^{A:\,B;\,B'}_{C:\,D;\,D'}\left(\begin{matrix}[(\alpha_A):\vartheta,\varphi]:[(\beta_B):\psi];[(\beta'_{B'}):\psi'];\\ [(\gamma_C):\xi,\varepsilon]:[(\delta_D):\eta];[(\delta'_{D'}):\eta'];\end{matrix}x,y\right) = $$

$$ = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{\prod_{j=1}^{A}(\alpha_j)_{m\vartheta_j+n\varphi_j}\prod_{j=1}^{B}(\beta_j)_{m\psi_j}\prod_{j=1}^{B'}(\beta'_j)_{n\psi'_j}}{\prod_{j=1}^{C}(\gamma_j)_{m\xi_j+n\varepsilon_j}\prod_{j=1}^{D}(\delta_j)_{m\eta_j}\prod_{j=1}^{D'}(\delta'_j)_{n\eta'_j}}\,\frac{x^m}{m!}\,\frac{y^n}{n!}\ , \tag{2} $$

where the quantities

$$ \left\{\begin{matrix}\vartheta_1,\dots,\vartheta_A;\ \varphi_1,\dots,\varphi_A;\ \psi_1,\dots,\psi_B;\ \psi'_1,\dots,\psi'_{B'};\ \xi_1,\dots,\xi_C;\\ \varepsilon_1,\dots,\varepsilon_C;\ \eta_1,\dots,\eta_D;\ \eta'_1,\dots,\eta'_{D'}\end{matrix}\right. $$

are real and positive. Here, for the sake of convenience the quantity $[(\alpha_A):\vartheta,\varphi]$ represents the set of A parameters $[\alpha_1:\vartheta_1,\varphi_1], [\alpha_2:\vartheta_2,\varphi_2], \dots, [\alpha_A:\vartheta_A,\varphi_A]$. The values of the positive coefficients $\vartheta_1, \vartheta_2, \dots, \vartheta_A$ may be equal or different, with a similar interpretation for the coefficients $\varphi_1, \varphi_2, \dots, \varphi_A$ and others. The double power series in (2) converges for all complex values of x and y when $\Delta_1 > 0$, $\Delta_2 > 0$; for suitably constrained values of $|x|$ and $|y|$ when $\Delta_1 = \Delta_2 = 0$; and diverges (except in the trivial case $x = y = 0$) when $\Delta_1 < 0$, $\Delta_2 < 0$, where

$$ \Delta_1 = 1 + \sum_{j=1}^{C}\xi_j + \sum_{j=1}^{D}\eta_j - \sum_{j=1}^{A}\vartheta_j - \sum_{j=1}^{B}\psi_j, \quad \Delta_2 = 1 + \sum_{j=1}^{C}\varepsilon_j + \sum_{j=1}^{D'}\eta'_j - \sum_{j=1}^{A}\varphi_j - \sum_{j=1}^{B'}\psi'_j. $$

Influenced by the studies of Buschman and Srivastava [3], Chen et. al. [5], Choi et. al. [6–8], Kim et. al. [13], we derive two double series identities involving a bounded sequence of arbitrary complex numbers in Section 2 by making use of two known summation theorems for the terminating ${}_5F_4$ series of arguments $\frac{1}{4}$ and 4 due to R. W. Gosper. For non-negative integer n, these are given by [11, Eqs. (1.2), (1.3)] (see also [15, p. 568, Entries 7.6.4(1), 7.6.4(2)])

$$
{}_5F_4\left[\begin{matrix} -n, 2a, 2b, 1-2b, 1+\frac{2}{3}a \\ 1+a-b, a+b+\frac{1}{2}, \frac{2}{3}a, 1+2a+2n \end{matrix}; \frac{1}{4}\right] = \frac{(a+\frac{1}{2})_n(a+1)_n}{(a+b+\frac{1}{2})_n(1+a-b)_n} \tag{3}
$$

and

$$
{}_5F_4\left[\begin{matrix} -n, a, b, a-b+\frac{1}{2}, 1+\frac{2}{3}a \\ 1+a+\frac{1}{2}n, 2a-2b+1, 2b, \frac{2}{3}a \end{matrix}; 4\right] = \begin{cases} 0 & (n=2N+1) \\ \dfrac{(2N)!(a+1)_N 2^{-2N}}{N!(1+a-b)_N(b+\frac{1}{2})_N} & (n=2N), \end{cases} \tag{4}
$$

where $n, N \in \mathbf{N_0}$, respectively.

We now present some preliminary results necessary for our investigation. First, we state Cauchy's double series identity [16, p. 56]

$$
\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\Theta(m,n) = \sum_{m=0}^{\infty}\sum_{n=0}^{m}\Theta(m-n,n) \ , \tag{5}
$$

provided that the associated double series are absolutely convergent. We also have the following identities involving the Pochhammer symbol:

$$
(-n)_m = \frac{(-1)^m n!}{(n-m)!} \qquad (0 \leqslant m \leqslant n) \tag{6}
$$

and

$$
(a)_{2n} = 2^{2n}\Big(\frac{1}{2}a\Big)_n\Big(\frac{1}{2}a+\frac{1}{2}\Big)_n. \tag{7}
$$

It should be observed that throughout we tacitly exclude any values of the parameters and arguments in Sections 2 and 3 leading to results that do not make sense.

2. Two double-series identities

In this section, we derive two double-series identities involving a bounded sequence. The first identity takes the following form:

Theorem 2.1. *Let $\{\Psi(\mu)\}_{\mu=1}^{\infty}$ be a bounded sequence of complex (or real) numbers such that $\Psi(0) \neq 0$. Then, the following double-series identity holds true:*

$$
\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\Psi(n+m)\,\frac{(a+\frac{1}{2})_{n+m}(1+a)_{n+m}(2a)_m(2b)_m(1-2b)_m(1+\frac{2}{3}a)_m}{(1+2a)_{2n+3m}(\frac{2}{3}a)_m(1+a-b)_m(a+b+\frac{1}{2})_m}\,\frac{(-1)^m 4^n z^{n+m}}{m!n!} =
$$

$$
= \sum_{n=0}^{\infty}\frac{\Psi(n)\,(a+\frac{1}{2})_n(1+a)_n}{(a+b+\frac{1}{2})_n(1+a-b)_n}\,\frac{z^n}{n!} \tag{8}
$$

provided $1+2a$, $\frac{2}{3}a$, $2b$, $1-2b$, $1+a-b$, $a+b+\frac{1}{2} \in \mathbf{C}\backslash\mathbf{Z}_0^-$ and the infinite series on both sides of (8) are absolutely convergent.

Proof of Theorem 2.1. Let

$$L(z) := \sum_{n=0}^{\infty}\sum_{m=0}^{\infty} \Psi(n+m) \frac{(a+\frac{1}{2})_{n+m}(1+a)_{n+m}(2a)_m(2b)_m(1-2b)_m(1+\frac{2}{3}a)_m}{(1+2a)_{2n+3m}(\frac{2}{3}a)_m(1+a-b)_m(a+b+\frac{1}{2})_m} \frac{(-1)^m 4^n z^{n+m}}{m!n!}.$$

Replacing n by $n-m$ and using Cauchy's double series identity (5), we have

$$L(z) = \sum_{n=0}^{\infty}\sum_{m=0}^{n} \Psi(n) \frac{(a+\frac{1}{2})_n(a+1)_n(2a)_m(2b)_m(1-2b)_m(1+\frac{2}{3}a)_m}{(1+2a)_{2n+m}(\frac{2}{3}a)_m(1+a-b)_m(a-b+\frac{1}{2})_m} \frac{(-1)^m 4^{n-m} z^n}{m!(n-m)!} =$$

$$= \sum_{n=0}^{\infty} \frac{\Psi(n)z^n}{n!} \sum_{m=0}^{n} \frac{(-n)_m(2a)_m(2b)_m(1-2b)_m(1+\frac{2}{3}a)_m 4^{-m}}{(1+2a+n)_m(\frac{2}{3}a)_m(1+a-b)_m(a+b-\frac{1}{2})_m m!}$$

by (6) and (7). Identification of the inner sum as a ${}_5F_4(\frac{1}{4})$ hypergeometric series then leads to

$$L(z) = \sum_{n=0}^{\infty} \frac{\Psi(n)z^n}{n!} {}_5F_4\left[\begin{matrix} -n, 2a, 2b, 1-2b, 1+\frac{2}{3}a \\ \frac{2}{3}a, 1+a-b, a+b+\frac{1}{2}, 1+2a+2n \end{matrix}; \frac{1}{4}\right]. \tag{9}$$

Finally, using the summation theorem (3) to evaluate the ${}_5F_4$ series on the right-hand side of (9), followed by some routine algebra, we obtain the desired result (8). □

The second identity is given by the following theorem:

Theorem 2.2. *Let $\{\Psi(\mu)\}_{\mu=1}^{\infty}$ be a bounded sequence of complex (or real) numbers such that $\Psi(0) \neq 0$. Then, the following double-series identity holds true:*

$$\sum_{n=0}^{\infty}\sum_{m=0}^{\infty} \Psi(n+m) \frac{(1+a)_{\frac{1}{2}n+\frac{1}{2}m}(a)_m(b)_m(a-b+\frac{1}{2})_m(1+\frac{2}{3}a)_m}{(1+a)_{\frac{1}{2}n+\frac{3}{2}m}(1+2a-2b)_m(2b)_m(\frac{2}{3}a)_m} \frac{(-4)^m z^{n+m}}{n!m!} =$$

$$= \sum_{n=0}^{\infty} \frac{\Psi(2n)(1+a)_n}{(1+a-b)_n(b+\frac{1}{2})_n} \frac{(z/2)^{2n}}{n!}, \tag{10}$$

provided $1+a-b$, $1+a$, $1+2a-2b$, $2b$, $b+\frac{1}{2}$, $\frac{2}{3}a \in \mathbf{C}\backslash\mathbf{Z}_0^-$ and the infinite series on both sides of (10) are absolutely convergent.

Proof of Theorem 2.2. Let

$$M(z) := \sum_{n=0}^{\infty}\sum_{m=0}^{\infty} \Psi(n+m) \frac{(1+a)_{\frac{1}{2}n+\frac{1}{2}m}(a)_m(b)_m(a-b+\frac{1}{2})_m(1+\frac{2}{3}a)_m}{(1+a)_{\frac{1}{2}n+\frac{3}{2}m}(1+2a-2b)_m(2b)_m(\frac{2}{3}a)_m} \frac{(-4)^m z^{n+m}}{n!m!}.$$

Replacing n by $n-m$ and using Cauchy's double series identity (5), together with (6), we have

$$M(z) = \sum_{n=0}^{\infty} \frac{\Psi(n)z^n}{n!} \sum_{m=0}^{n} \frac{(-n)_m(a)_m(b)_m(a-b+\frac{1}{2})_m(1+\frac{2}{3}a)_m 4^m}{(1+a+\frac{1}{2}n)_m(1+2a-2b)_m(2b)_m(\frac{2}{3}a)_m m!} =$$

$$= \sum_{n=0}^{\infty} \frac{\Psi(n)z^n}{n!} {}_5F_4\left[\begin{matrix} -n, a, b, a-b+\frac{1}{2}, 1+\frac{2}{3}a \\ 1+a+\frac{1}{2}n, 1+2a-2b, 2b, \frac{2}{3}a \end{matrix}; 4\right].$$

From the series decomposition identity

$$\sum_{n=0}^{\infty} \Phi(n) = \sum_{n=0}^{\infty} \Phi(2n) + \sum_{n=0}^{\infty} \Phi(2n+1),$$

we then obtain

$$M(z)=\sum_{n=0}^{\infty}\Psi(2n)\frac{z^{2n}}{(2n)!}\,{}_5F_4\left[\begin{matrix}-2n,a,b,a-b+\frac{1}{2},1+\frac{2}{3}a\\ 1+a+n,1+2a-2b,2b,\frac{2}{3}a\end{matrix};4\right]+$$

$$+\sum_{n=0}^{\infty}\Psi(2n+1)\frac{z^{2n+1}}{(2n+1)!}\,{}_5F_4\left[\begin{matrix}-2n-1,a,b,a-b+\frac{1}{2},1+\frac{2}{3}a\\ \frac{3}{2}+a+n,1+2a-2b,2b,\frac{2}{3}a\end{matrix};4\right].$$

Finally, use of the summation theorem (4), followed by some straightforward algebra, leads to the required result (10). □

3. Application to the Srivastava-Daoust function

In this section we establish two results concerning the reducibility of the Srivastava-Daoust double hypergeometric function defined in (2). We have

Theorem 3.1. *The following results hold true:*

$$F^{D+2:\;0;\;4}_{E+1:\;0;\;3}\left(\begin{matrix}[(d_D):1,1],[a+\frac{1}{2}:1,1],[1+a:1,1]:—;[2a:1],[2b:1],[1-2b:1],[1+\frac{2}{3}a:1];\\ [(e_E):1,1],[1+2a:2,3]:—;[\frac{2}{3}a:1],[1+a-b:1],[a+b+\frac{1}{2}:1];\end{matrix}\;4z,-z\right)=$$

$$={}_{D+2}F_{E+2}\left[\begin{matrix}(d_D),a+\frac{1}{2},a+1\\ (e_E),a+b+\frac{1}{2},1+a-b\end{matrix};z\right]\tag{11}$$

and

$$F^{D+1:\;4;\;0}_{E+1:\;3;\;0}\left(\begin{matrix}[(d_D):1,1],[a+1:\frac{1}{2},\frac{1}{2}]:[a:1],[b:1],[a-b+\frac{1}{2}:1],[1+\frac{2}{3}a:1];—;\\ [(e_E):1,1],[a+1:\frac{3}{2},\frac{1}{2}]:[2a-2b+1:1],[2b:1],[\frac{2}{3}a:1];—;\end{matrix}\;4z,-z\right)=$$

$$={}_{2D+1}F_{2E+2}\left[\begin{matrix}\frac{1}{2}(d_D),\frac{1}{2}+\frac{1}{2}(d_D),\;a+1\\ \frac{1}{2}(e_E),\frac{1}{2}+\frac{1}{2}(e_E),1+a-b,b+\frac{1}{2}\end{matrix};\frac{z^2}{4^{E-D+1}}\right],\tag{12}$$

where all numerator and denominator parameters are neither zero nor negative integers. When $D\leqslant E$ both sides of (11) and (12) are convergent for $|z|<\infty$, but when $D=E+1$ the above hypergeometric functions are convergent for suitably constrained values of $|z|$.

Proof of Theorem 3.1. Set

$$\Psi(p)=\frac{(d_1)_p(d_2)_p\dots(d_D)_p}{(e_1)_p(e_2)_p\dots(e_E)_p}\qquad(p\in\mathbf{N_0});$$

then the double-series identity (8) becomes

$$\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\frac{\prod_{j=1}^{D}(d_j)_{n+m}}{\prod_{j=1}^{E}(e_j)_{n+m}}\,\frac{(a+\frac{1}{2})_{n+m}(1+a)_{n+m}(2a)_m(2b)_m(1-2b)_m(1+\frac{2}{3}a)_m}{(1+2a)_{2n+3m}(\frac{2}{3}a)_m(1+a-b)_m(a+b+\frac{1}{2})_m}\,\frac{(-1)^m4^nz^{n+m}}{m!n!}=$$

$$=\sum_{n=0}^{\infty}\frac{\prod_{j=1}^{D}(d_j)_n}{\prod_{j=1}^{E}(e_j)_n}\,\frac{(a+\frac{1}{2})_n(1+a)_n}{(a+b+\frac{1}{2})_n(1+a-b)_n}\,\frac{z^n}{n!}.\tag{13}$$

Applying the definition of the Srivastava-Daoust function in (2) to the left-hand side of (13) and the definition of the generalised hypergeometric function in (1) to the right-hand side of (13), we obtain the desired result (11).

The proof of (12) follows exactly the same procedure and will be omitted. This completes the proof of Theorem 3.1. □

Remark: Most of the applications of similar types of hypergeometric reduction formulas (11) and (12) lie in the fields of Statistics, Physics, Operational Research and Engineering. The hypergeometric functions have potential uses in all problems concerning of large sets of differential equations with several variables and parameters. Also, there are some applications of double hypergeometric functions in asymptotic theory, boundary value problems, Celestial Mechanics (Hill's equation), heat conduction, etc. (see, for example, [2,4,9,14,17,20–22] and closely related references therein).

The authors are very grateful to the anonymous referees for their constructive and encouraging comments which improved this revised paper.

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Приложения двух теорем суммирования Госпера для гипергеометрического ряда ${}_5F_4$

М. И. Куреши
Инженерно-технологический факультет
Джамия Миллия Исламия (Центральный университет)
Нью-Дели, Индия

Р. Б. Парис
Отдел вычислительной техники и математики
Университет Абертай
Данди DD1 1HG, Великобритания

М. Кашиф Хан
Факультет наук
Интегральный университет
Лакхнау-226026 (UP), Индия

Аннотация. Используя две теоремы суммирования Р. В. Госпера для терминирующего гипергеометрического ряда ${}_5F_4$ аргументов 1/4 и 4, мы получаем два общих тождества двойных рядов, включающих ограниченную последовательность произвольных комплексных чисел. Затем эти ряды применяются для получения двух формул редукции двойной гипергеометрической функции Шриваставы–Дауста.

Ключевые слова: обобщенная гипергеометрическая функция, ограниченная последовательность, теоремы суммирования Госпера, двойная гипергеометрическая функция Шриваставы–Дауста.