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New Fixed Point Results on α_L^{ψ}-rational Contraction Mappings in b-Metric-Like Spaces

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Abstract. The purpose of this paper is to prove some new results for α_L^{ψ}-rational contractive and cyclic α_L^{ψ}-rational contractive mappings defined in d-complete b-metric-like spaces. Moreover, an example is presented to illustrate the validity of our theoretical results.

Keywords: metric like-space, b-metric like-space, cyclic contractions.

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1. Introduction and preliminaries

A vigorous research activity focuses on the research on fixed points for given mappings with certain contractive conditions in various abstract spaces.

Hence, there exist numerous generalizations of the concept of metric spaces; symmetric spaces, quasimetric spaces, fuzzy metric spaces, partial metric spaces, like metric spaces and this has pulled in the consideration of many researchers to obtain some fixed point theorem for mappings satisfying a different contractive condition, in particular, cyclic contractions and cyclic contractive type mappings, (we refer the reader to see [2–11, 13]).

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This paper points to proceed the think about of a few cyclical contractive mappings taking after many papers from this still actual subject nowadays.

Our results extend and generalize the results [11] into b-metric like-space. The obtained results extend many recent results in the literature.

The following definitions and results will be needed

Definition 1 ([1]). *Let X be a non empty set. A function $d : X \times X \longrightarrow [0, +\infty)$ such that for all $x, y, z \in X$, we have the following assertions :*

1) $d(x, y) = 0$ implies $x = y$,

2) $d(x, y) = d(y, x)$,

3) $d(x, y) \leqslant d(x, z) + d(z, y)$.

The pair (X, d) is called a metric like-space. A metric-like d on X satisfies all conditions of a metric except that $d(x, x)$ may be positive for some $x \in X$.

Definition 2 ([12]). *Let X be a non empty set, $s \geqslant 1$ a fixed real number, A function $d : X \times X \longrightarrow [0, +\infty)$ a mapping. Then, (X, d) is said to be b-metric like space if for all $x, y, z \in X$, the following statements hold true :*

1) $d(x, y) = 0$ implies $x = y$,

2) $d(x, y) = d(y, x)$,

3) $d(x, y) \leqslant s [d(x, z) + d(z, y)]$.

Then b-metric like space is a pair (X, d),and d a b-metric-like on X.

It should be noted that the class of b-metric-like spaces is larger than the class of metric-like spaces, since a b-metric-like is a metric-like with $s = 1$.

Example ([17]). Let $\Omega = \{0, 1, 2, 3, 4\}$ and let

$$\omega(\kappa, \tau) = \begin{cases} 5 & \kappa = \tau = 0 \\ \dfrac{1}{5} & \text{otherwise.} \end{cases}$$

Then, (Ω,ω) is a b-metric-like space with a coefficient $s = 5$.

For more examples in metric-like and b-metric-like spaces, see [17, 18].

Definition 3 ([12]). *Let $\{x_n\}$ be a sequence in a b-metric-like space (X, d) with the coefficient s. Then :*

i) The sequence $\{x_n\}$ is said to be convergent to x if $\lim\limits_{n\to+\infty} d(x_n, x) = d(x, x)$.

ii) The sequence $\{x_n\}$ is said to be Cauchy in (X, d) if $\lim\limits_{n,m\to+\infty} d(x_n, x_m)$ exists and is finite.

ii) A b-metric-like space (X, d) is d-complete if for every d-Cauchy sequence $\{x_n\}$ in X there exists an $x \in X$, such that

$$\lim_{n,m\to+\infty} d(x_n, x_m) = d(x, x) = \lim_{n\to+\infty} d(x_n, x).$$

Remark 1.1. *The limit of the sequence need not be unique and a convergent sequence need not be a Cauchy in the context of b-metric-like.*

Lemma 1. *Let $\{x_n\}$ be a sequence on a complete b-metric space (X, d) with $s \geqslant 1$ such that $\lim\limits_{n\to+\infty} d(x_n, x_{n+1}) = 0$.*
If $\lim\limits_{n,m\to+\infty} d(x_n, x_m) \neq 0$, there exist $\varepsilon > 0$ and two sequences $\{m_k\}_{k=1}^{+\infty}, \{n_k\}_{k=1}^{+\infty}$ of positive integers with $n_k > m_k > k$ such that

$$d(x_{n_k}, x_{m_k}) \geqslant \varepsilon, d(x_{m_k}, x_{n_k-1}) < \varepsilon, \quad \frac{\varepsilon}{s^2} \leqslant \lim_{k\to+\infty} \sup d(x_{n_k-1}, x_{m_k-1}) \leqslant s\varepsilon,$$
$$\frac{\varepsilon}{s} \lim_{k\to+\infty} \sup d(x_{n_k-1}, x_{m_k}) \leqslant \varepsilon, \quad \frac{\varepsilon}{s^2} \lim_{k\to+\infty} \sup d(x_{n_k}, x_{m_k-1}) \leqslant \varepsilon s^2.$$

Definition 4 ([13,14]). *Let (X,d) be a b-metric-like space with the coefficient s. A sequence $\{x_n\}$ is called 0-d-Cauchy sequence if $\lim_{n,m\to+\infty} d(x_n,x_m)=0$. The space (X,d) is said to be 0-d-complete if every 0-d-Cauchy sequence in X converges to a point $x\in X$ such that $d(x,x)=0$.*

Lemma 2 ([15,16]). *Let $\{x_n\}$ be a sequence in a b-metric-like space (X,d) with the coefficient $s\geqslant 1$ such that*

$$d(x_n,x_{n+1})\leqslant qd(x_{n-1},x_n),$$

for some $q\in[0,1)$ and $n\in\mathbb{N}$ then $\{x_n\}$ is a d-Cauchy sequence in (X,d) such that

$$\lim_{n,m\to+\infty} d(x_n,x_m)=0.$$

Definition 5 ([17]). *Let X be a non empty set. A mapping $T:X\to X$ is said to be an α-admissible mapping if $\alpha(x,y)\geqslant 1$ implies $\alpha(Tx,Ty)\geqslant 1$ for all $x,y\in X$ and $\alpha: X\times X\to[0,+\infty)$.*

Further T called α-continuous on X if $\lim_{n\to+\infty} x_n=x$ implies $\lim_{n\to+\infty} Tx_n=Tx$ for any sequence $\{x_n\}$ for Y which $\alpha(x_n,x_{n+1})\geqslant 1$, $n\in\mathbb{N}$.
Moreover, [17] present some new fixed point theorems for a α_L^{ψ} cyclic rational contraction self-mapping on complete metric-like spaces, subsequently the authors [11] found some doubts about some defintions and theorems, and give a new formula of it as follow:

Let denote Ω the class of all function $\Psi:[0,+\infty)\to[0,+\infty)$, satisfying the following condition:
i) Ψ non-decreasing and continuous;
ii) $\lim_{n\to+\infty}\Psi^n(t)=0$ for all $t>0$.

Definition 6. *Let (X,d) be a b-metric-like space, $p\in N$, $B_1,B_2,\ldots,B_p$ be d-closed subsets of X, $Y=B_1\cup\cdots\cup B_p$ and $\alpha: Y\times Y\to[0,\infty)$ be a mapping. We say that $T: Y\times Y\to Y$ is cyclic α_L^{Ψ}-rational contractive mapping if :*
1) $T(B_i)\subseteq B_{i+1}$, $i=1,2,\ldots,p$, where $B_{p+1}=B_1$;
2) for any $x\in B_i$ and $y\in B_{i+1}$, $i=1,2,\ldots,p$, where $B_{p+1}=B_1$ and $\alpha(x,Tx)\alpha(y,Ty)\geqslant 1$, holds

$$\Psi(d(Tx,Ty))\leqslant\Psi(M(x,y))-LM(x,y), \tag{1}$$

where $\Psi\in\Omega$, $L\in(0,1)$ and

$$M_d(x,y)=\max\left\{d(x,y),\frac{d(x,Ty)}{2s},\frac{d(x,Tx)\,d(y,Ty)}{1+d(x,y)},\frac{d(y,Ty)\,[1+d(x,Tx)]}{1+d(x,y)},\frac{d(x,Ty)+d(y,Tx)}{4s}\right\}.$$

2. Main results

This part is devoted to define a cyclic α_L^{ψ}-rational contractive mapping, and some new fixed point results through this contractive on the setting of complete b-metric-like spaces are presented.

Theorem 2.1. *Let(X,d) be a d-complete b-metric like space and $\alpha: X\times X\to[0,+\infty)$ be a mapping. Assume that $T:X\to X$ is an α_L^{ψ}-contractive mapping satisfying the following assertions:*
(i) T is an α-admissible mapping,
(ii) $\alpha(x_0,Tx_0)\geqslant 1$ for an element x_0 in X,
(iii) T is α-continuous, or;

(iv) if $\{x_n\}$ *is a sequence in* X *such that* $\alpha(x_n, x_{n+1}) \geqslant 1$
for all $n \in \mathbb{N}$ *and* $x_n \to x$ *as* $n \to +\infty$*, then* $\alpha(x_n, Tx_n) \geqslant 1$.
Then T *admits a fixed point in* X.
Moreover, if
(v) $\alpha(x, x) \geqslant 1$*, whenever* $x \in Fix(T)$*, then* T *admits a unique fixe point.*

Proof. Let start with define the sequence $x^n = T^n x_0$, where x_0 is the given point for which $\alpha(x_0, Tx_0) \geqslant 1$. Since T is an α-admissible mapping, we get that

$$\alpha(x_1, Tx_1) = \alpha(Tx_0, TTx_1) \geqslant 1.$$

Continuing this process, we get $\alpha(x_n, Tx_n) \geqslant 1$ for all $n \in \mathbb{N}$, and so,

$$\alpha(x_n, Tx_n)\,\alpha(x_{n-1}, Tx_{n-1}) \geqslant 1 \text{ for all } n \in \mathbb{N}.$$

If $x_n = x_{n-1}$ for some $n \in \mathbb{N}$, x_{n-1} is a fixed point of T.
Therefore, assume that $x_{n-1} \neq x_n$ for all $n \in \mathbb{N}$. Hence, we have that

$$d(x_{n-1}, x_n) > 0 \text{ for all } n \in \mathbb{N}.$$

In order to prove that the sequence $\{x_n\}$ is a d-Cauchy sequence. According to (1), we get

$$\Psi(d(x_n, x_{n+1})) \leqslant \Psi(M(x_{n-1}, x_n)) - LM(x_{n-1}, x_n), \tag{2}$$

where

$$\begin{aligned}
&M(x_{n-1}, x_n) = \\
&= \max\left\{\begin{array}{c} d(x_{n-1}, x_n), d(x_n, x_n), \dfrac{d(x_{n-1}, x_n)\, d(x_n, x_{n+1})}{1 + d(x_{n-1}, x_n)}, \dfrac{d(x_n, x_{n+1})\,[1 + d(x_{n-1}, x_n)]}{1 + d(x_{n-1}, x_n)}, \\ \dfrac{d(x_{n-1}, x_{n+1}) + d(x_n, x_{n+1})}{4s} \end{array}\right\} \\
&\leqslant \max\left\{d(x_{n-1}, x_n), d(x_n, x_{n+1}), \frac{3}{4} d(x_{n-1}, x_n) + \frac{1}{4} d(x_n, x_{n+1})\right\} \\
&\leqslant \max\{d(x_{n-1}, x_n), d(x_n, x_{n+1})\}
\end{aligned}$$

Hence, we get

$$\Psi(d(x_n, x_{n+1})) \leqslant \Psi(\max\{d(x_{n-1}, x_n), d(x_n, x_{n+1})\}) - L\max\{d(x_{n-1}, x_n), d(x_n, x_{n+1})\}.$$

If

$$\max\{d(x_{n-1}, x_n), d(x_n, x_{n+1})\} = d(x_n, x_{n+1}) \text{ for some } n \in \mathbb{N},$$

we have

$$\begin{aligned}
\Psi(d(x_n, x_{n+1})) &\leqslant \Psi(d(x_n, x_{n+1})) - Ld(x_n, x_{n+1}), \\
0 &\leqslant -Ld(x_n, x_{n+1}).
\end{aligned} \tag{3}$$

Which contradiction.
Hence, we get

$$d(x_n, x_{n+1}) \geqslant d(x_n, x_{n+1}).$$

So, there exists

$$\lim_{n\to+\infty} d(x_n, x_{n+1}) = d_k \geqslant 0.$$

Letting $n \to +\infty$ in (2), we obtain that

$$\begin{aligned} \lim_{n\to+\infty} \Psi(d(x_n, x_{n+1})) &\leqslant \lim_{n\to+\infty} [\Psi(d(x_n, x_{n+1})) - Ld(x_n, x_{n+1})], \\ \Psi(d_k) &\leqslant \Psi(d_k) - Ld_k. \end{aligned}$$

Thus

$$\lim_{n\to+\infty} d(x_n, x_{n+1}) = 0.$$

Now, if $\lim_{n,m\to+\infty} d(x_n, x_m) \neq 0$, we have sequences $\{m_k\}$ and $\{n_k\}$ such that

$$\lim_{k\to+\infty} d(x_{n_k}, x_{m_k}) = \varepsilon > 0.$$

Let $u = x_{n_k}$ and $v = x_{m_k}$ in (1), we get

$$\Psi(d(x_{n_k+1}, x_{m_k+1})) \leqslant \Psi(M(x_{n_k}, x_{m_k})) - LM(x_{n_k}, x_{m_k}), \tag{4}$$

where

$$\begin{aligned} M(x_{n_k}, x_{m_k}) &= \max\left\{\begin{array}{c} d(x_{n_k}, x_{m_k}), d(x_{m_k}, x_{n_k+1}), \dfrac{d(x_{n_k}, x_{m_k}) d(x_{m_k}, x_{m_k+1})}{1 + d(x_{n_k}, x_{n_k+1})}, \\ \dfrac{d(x_{m_k}, x_{m_k+1})[1 + d(x_{n_k}, x_{n_k+1})]}{1 + d(x_{n_k}, x_{m_k})}, \\ \dfrac{d(x_{n_k}, x_{m_k+1}) + d\left(x_{m_k}, x_{n_{k+1}}\right)}{4s} \end{array}\right\} \\ &\to \max\left\{\varepsilon, \frac{\varepsilon}{2s}, 0, 0, \frac{\varepsilon}{4s}\right\} \end{aligned}$$

So, as $n \to +\infty$ in (4), we have

$$\Psi(\varepsilon) \leqslant \Psi(\varepsilon) - L\varepsilon$$

which is a contradiction.

Hence, the sequence$\{x_n\}$ is a cauchy and

$$\lim_{n,m\to+\infty} d(x_n, x_m) = 0.$$

This means that there exists a unique point $x^* \in X$ such that

$$d(x^*, x^*) = \lim_{n\to+\infty} d(x_n, x^*) = \lim_{n,m\to+\infty} d(x_n, x_m) = 0.$$

Now, we will proof that x^* is fixed point of T i.e., $Tx^* = x^*$, thus is clear if T is α-continuous. Further, suppose that for any sequence x_n in X and for all $n \in \mathbb{N}$,if $\alpha(x_n, x_{n+1}) \geqslant 1$ and $\lim_{n\to+\infty} x_n = x^*$, then

$$\alpha(x^*, Tx^*) \geqslant 1.$$

Let $d(x^*, Tx^*) > 0$. Since

$$\alpha(x_n, Tx_n)\, \alpha(x^*, Tx^*) \geqslant 1.$$

According to the given contractive condition, we have

$$\Psi\left(d\left(Tx,Tx^*\right)\right) \leqslant \Psi\left(M\left(x,x^*\right)\right) - LM\left(x,x^*\right),$$

where

$$\begin{aligned} M\left(x,x^*\right) &= \max\left\{\begin{array}{c} d\left(x_n,x^*\right), d\left(x^*,x_{n+1}\right), \dfrac{d\left(x_n,x^*\right)d\left(x^*,Tx^*\right)}{1+d\left(x_n,x_{n+1}\right)}, \\ \dfrac{d\left(x^*,Tx^*\right)\left[1+d\left(x_n,x_{n+1}\right)\right]}{1+d\left(x_n,x_{n+1}\right)}, \\ \dfrac{d\left(x_n,Tx^*\right)+d\left(x^*,x_{n+1}\right)}{4s} \end{array}\right\} \\ &\leqslant \max\left\{\begin{array}{c} d\left(x_n,x^*\right), d\left(x^*,x_{n+1}\right), \dfrac{d\left(x_n,x^*\right)d\left(x^*,Tx^*\right)}{1+d\left(x_n,x_{n+1}\right)}, \\ d\left(x^*,Tx^*\right), \\ \dfrac{s\left[d\left(x_n,x^*\right)+d\left(x^*,Tx^*\right)\right]+d\left(x^*,x_{n+1}\right)}{4s} \end{array}\right\} \\ &\to \max\left\{0,0,0,d\left(x^*,Tx^*\right), \dfrac{d\left(x^*,Tx^*\right)}{4}\right\} = d\left(x^*,Tx^*\right) \text{ as } n\to+\infty. \end{aligned}$$

Now, letting the limit in for $n \to +\infty$, we get

$$\Psi\left(d\left(x^*,Tx^*\right)\right) \leqslant \Psi\left(d\left(x^*,Tx^*\right)\right) - Ld\left(x^*,Tx^*\right)$$

which is a contradiction again. This means that $d\left(x^*,Tx^*\right)=0$, that is, we prouve that $Tx^*=x^*$.

Finally, to show the uniqueness of the fixed point of the map T, suppose that u,v $(u\neq v)$ are two fixed point of T.

Then,we get $d\left(u,v\right)>0$, $\alpha\left(u,u\right)\geqslant 1$, $\alpha\left(v,v\right)\geqslant 1$.

Further, since

$$\alpha\left(u,u\right)\alpha\left(v,v\right)\geqslant 1,$$

we obtain

$$\Psi\left(d\left(u,v\right)\right) \leqslant \Psi\left(M\left(u,v\right)\right) - LM\left(u,v\right),$$

where

$$\begin{aligned} M\left(u,v\right) &= \max\left\{\begin{array}{c} d\left(u,v\right), \dfrac{d\left(v,u\right)}{2s}, \dfrac{d\left(u,v\right)d\left(v,v\right)}{1+d\left(u,u\right)}, \\ \dfrac{d\left(v,v\right)\left[1+d\left(u,u\right)\right]}{1+d\left(u,v\right)}, \\ \dfrac{d\left(u,v\right)+d\left(v,u\right)}{4s} \end{array}\right\} \\ &= \max\left\{d\left(u,v\right), \dfrac{d\left(v,u\right)}{2s}, 0, 0, \dfrac{d\left(v,u\right)}{2s}\right\} = d\left(u,v\right). \end{aligned}$$

Hence

$$\Psi\left(d\left(u,v\right)\right) \leqslant \Psi\left(d\left(u,v\right)\right) - Ld\left(u,v\right)$$

which is a contradiction. This finishe the proof. □

Remark 2.1. *It is useful to notice that the The case $s=1$ means that (X,d) is actually a complete metric-like space and we get the results of [11].*

Example. Let $X=\mathbb{R}$ be a b-metric-like space with constant $s=4$.

Define the function $d:\mathbb{R}^2\rightarrow$: $[0,+\infty)$ by $d(x,y)=(|x|+|y|)^3$.

It is clear that (X,d) is a complete b-metric-like space. Suppose that

$$B_1=(-\infty,0]\,,\ B_2=[0,+\infty)$$

and

$$Y=B_1\cup B_2.$$

Define $T:Y\rightarrow Y$ and $a:Y\times Y\rightarrow[0,+\infty)$ by

$$Tx=\begin{cases} x^2 & \text{if}\quad x\in(-\infty,1)\\ -\dfrac{x}{6} & \text{if}\quad x\in[-1,0]\\ -\dfrac{x^2}{7} & \text{if}\quad x\in[0,1]\\ -x & \text{if}\quad x\in(1,+\infty)\end{cases}\quad\text{and } a(x,y)=\begin{cases} |x|+|y|+1, & \text{if}\ \ x,y\in[-1;1]\\ 0, & \text{otherwise}\end{cases}.$$

Also, define $\Psi:[0,+\infty)\rightarrow[0,+\infty)$ by $\Psi(t)=\dfrac{1}{2}t$ and $L=\dfrac{1}{6}$. Clearly, $T(B_1)\subset B_2$ and $T(B_2)\subset B_1$.

Let $x\in B_1$, $y\in B_2$ and $a(x,Tx)\,a(y,Ty)\geqslant 1$. If $x\notin[-1,1]$ or $y\notin[-1,1]$, then $a(x,Tx)=0$ or $(y,Ty)=0$. That is, $a(x,Tx)\,a(y,Ty)=0$ which is a contradiction. Hence $x\in B_1$, $y\in B_2$ and $x,y\in[-1,1]$.

This implies that $x\in[-1,0]$ or $y\in[1,0]$.

Then

$$\begin{aligned}\Psi(d(Tx,Ty)) &= \frac{1}{2}\left(\left|-\frac{x}{6}\right|+\left|-\frac{y^2}{7}\right|\right)^3\\ &\leqslant 2\left(\left|\frac{x}{6}\right|^3+\left|\frac{y^2}{7}\right|^3\right)\\ &\leqslant \frac{1}{3}\left(|x|^3+\left|y^2\right|^3\right)\\ &\leqslant \frac{1}{3}\left(|x|^3+|y|^3\right),\ \text{since } y\in[1,0].\\ &\leqslant \frac{1}{3}(|x|+|y|)^3\\ &= \frac{1}{3}d(x,y)=\Psi(M_d(x,y))-LM_d(x,y).\end{aligned}$$

Then T is a cyclic α_L^{ψ}-rational contractive mapping. It is clear that $a(0,T0)\geqslant 1$ and so the condition (ii) of Theorem 2.1 is satisfied.

If $a(x,y)\geqslant 1$, then $x,y\in[-1,1]$ which implies that $a(Tx,Ty)\geqslant 1$, that is, T is an α-admissible mapping.

Let $\{x_n\}$ be a sequence in X such that, $a(x_n,Tx_n)\geqslant 1$ and $x_n\rightarrow x$ as $n\rightarrow+\infty$.

Then, we must have $x_n\in[-1,1]$ and so, $x\in[-1,1]$, that is $a(x,Tx)\geqslant 1$. Hence, all the conditions of Theorem 2.1 hold and T has a fixed point $x=0\in B_1\cap B_2$.

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Новые результаты с фиксированной точкой для α_L^{ψ}-рациональных отображений притяжения в b-метрических пространствах

Сухейб Мерад
Файсел Мергади
Университет Ларби Тебесси
Тебесса, Алжир
Тайеб Хамаизия
Университет Ларби Бен М'Хиди
Ум-эль-Буаги, Алжир
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Аннотация. Целью данной статьи является доказательство некоторых новых результатов для α_L^{ψ}-рациональных сжимающих и циклических α_L^{ψ}-рациональных стягивающих отображений, определенных в d-полные b-метрикоподобные пространства. Кроме того, приведен пример, иллюстрирующий справедливость наших теоретических результатов.

Ключевые слова: метрическое подобное пространство, b-метрическое подобное пространство, циклические сокращения.