DOI: 10.17516/1997-1397-2022-15-6-742-752
УДК 621.315.592

Surface Polariton Propagation in Structure with a Graphene Layer

Kirill V. Bukhensky*
Alexander B. Dubois†
Alexey N. Konukhov‡
V. F. Utkin Ryazan State Radioengineering University
Ryazan, Russian Federation
Sergey I. Kucheryavyy§
National Research Nuclear University (NRNU) MEPhI
Moscow, Russian Federation
Obninsk Institute For Nuclear Power Engineering (OINPE)
Obninsk, Russian Federation
Alexey S. Safoshkin¶
V. F. Utkin Ryazan State Radioengineering University
Ryazan, Russian Federation

Received 10.08.2022, received in revised form 04.09.2022, accepted 20.10.2022

Abstract. The paper considers the process of diffraction of an electromagnetic wave at the interface "vacuum – metal – nonlinear layer (graphene) – semiconductor" with the excitation of a surface wave. Within the framework of the theory, a modal method for calculating the process of interaction of radiation with a multilayer structure is presented, which makes it possible to calculate the energy fluxes arising in the process of diffraction.

Keywords: integro-differential equations, surface polariton, graphene, Maxwell equations.

Citation: K.V. Bukhensky, A.B. Dubois, A.N. Konukhov, S.I. Kucheryavyy, A.S. Safoshkin, Surface Polariton Propagation in Structure with a Graphene Layer, J. Sib. Fed. Univ. Math. Phys., 2022, 15(6), 742–752. DOI: 10.17516/1997-1397-2022-15-6-742-752.

The processes of surface wave (polariton) excitation as a result of electromagnetic radiation diffraction in media represent one of the most important problems of integrated optics. Compared to the processes of electromagnetic radiation propagation along multilayered structures with parallel interfaces, which have been well studied and systematized to date [1, 2], diffraction problems are much less well studied. The excitation and propagation of electromagnetic waves along planar nanostructures, primarily on graphene, are of particular interest. However, even in the simplest approximation, the solution of the Maxwell equations under the given boundary and boundary conditions leads to the cumbersome integrodifferential equations [3], which have a strictly analytical solution only for certain geometries of the structures [4].

*bukhensky.k.v@rsreu.ru https://orcid.org/0000-0003-2602-2112
†dubois.a.b@rsreu.ru https://orcid.org/0000-0002-5924-4128
‡chronos@bk.ru https://orcid.org/0000-0002-1523-7110
§kucheryavyy@iate.obninsk.ru https://orcid.org/0000-0001-6030-9286
¶safoshkin.a.s@rsreu.ru https://orcid.org/0000-0002-1419-979X

In this paper the problem for the surface wave on a planar heterostructure is solved, the attractiveness of which can be explained by the following factors:

1) relative simplicity of the used theoretical models of electromagnetic radiation propagation along such structures;

2) fundamentality of obtained results, allowing to generalize them to more complex geometries of structures;

3) planar geometry is sufficiently close to reality.

The surface polariton is a transverse magnetic TM-polarized optical wave propagating along the surface of the metal-dielectric boundary. This coupled excitation includes fluctuations of the electron charge density in metals and electromagnetic waves with a maximum at the interface and exponentially decreasing deep into both media.

The discovery of graphene and the creation of terahertz lasers on graphene [5] stimulated theoretical [6] and experimental [7] studies of the redistribution of electromagnetic waves on planar structures. In [5] the diffraction of electromagnetic waves for TE and TM polarizations for isotropic and anisotropic structures was studied. The process of reflection of the incident wave from air ($\varepsilon_1 = 1$) into a medium with permeability $\varepsilon_2 = (\varepsilon_t, \varepsilon_t, \varepsilon_n)$ (graphene) was considered. From classical electromagnetic theory [7], the reflection coefficients for TM- and TE-waves are derived as

$$r_{1|2}^{\mathrm{TM}} = \frac{\varepsilon_t \cos\theta - \sqrt{\varepsilon_t - \left(\frac{\varepsilon_t}{\varepsilon_n}\right)\sin^2\theta}}{\varepsilon_t \cos\theta + \sqrt{\varepsilon_t - \left(\frac{\varepsilon_t}{\varepsilon_n}\right)\sin^2\theta}}, \tag{1}$$

$$r_{1|2}^{\mathrm{TE}} = \frac{\cos\theta - \sqrt{\varepsilon_t - \sin^2\theta}}{\cos\theta + \sqrt{\varepsilon_t - \sin^2\theta}}, \tag{2}$$

where θ is the angle of incidence. Thus, the transmittance coefficients for TM- and TE-waves are

$$t_{1|2}^{\mathrm{TM}} = \frac{2\varepsilon_t \cos\theta}{\varepsilon_t \cos\theta + \sqrt{\varepsilon_t - \frac{\varepsilon_t}{\varepsilon_n}\sin^2\theta}}, \quad t_{1|2}^{\mathrm{TE}} = \frac{2\cos\theta}{\cos\theta + \sqrt{\varepsilon_t - \sin^2\theta}}, \tag{3}$$

respectively; in the ideal lossless case, the transmittance of TE waves is $T^{\mathrm{TE}} = 1 - \left|r_{1|2}^{\mathrm{TE}}\right|^2$. When region 2 is an isotropic medium (i.e. $\varepsilon_t = \varepsilon_n$), it follows from the analysis of the expressions for the reflection coefficients that for TM- and TE-waves the reflectivity will be different from zero, except when the incident angle is the Brewster angle for TM-waves [7]. When region 2 (graphene) is an optically uniaxial medium (i.e. $\varepsilon_t \neq \varepsilon_n$), we find that full polarization splitting can be achieved with $\left|r_{1|2}^{\mathrm{TM}}\right| = 1$ and $\left|r_{1|2}^{\mathrm{TE}}\right| = 0$, i.e., full reflectivity for TM-waves and full bandwidth for TE-waves for an arbitrary incidence angle. The corresponding condition is $\varepsilon_t = 1$ and $\varepsilon_n = 0$. In the structure considered in [5], the total polarization splitting is independent of the incidence angle due to the rotational symmetry of the structure with respect to the surface normal. In addition, complete polarization splitting can also be achieved using a thin layer of such a uniaxial medium. The drawback of this theory in this case consists, first, in the fact that the calculations were performed without taking into account surface polariton excitation, and, second, the dependence of dielectric permittivity on frequency was not taken into account. However, when considering the diffraction problem, it is possible to achieve the necessary conditions for surface polariton propagation at terahertz frequencies. Fig. 1 shows the frequency dependences of dielectric functions for the propagation of vibrations in the directions along and perpendicular to the plane of the planar structure.

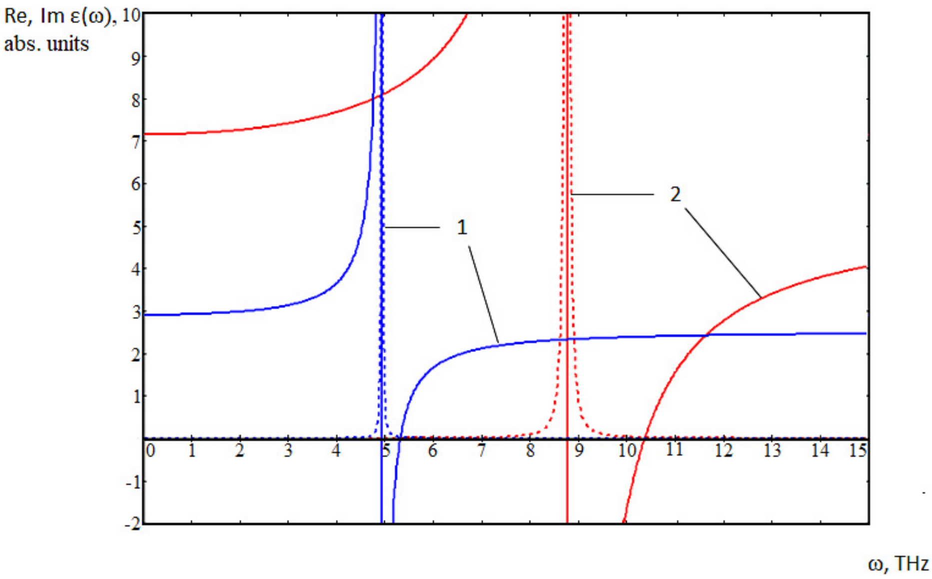

Fig. 1. Frequency dependence of the dielectric function for the graphene monolayer. The solid lines represent the real part of the dielectric function, the dotted lines represent the imaginary part. 1 — propagation of vibrations in the direction of the film plane, 2 — propagation of vibrations in the direction perpendicular to the film plane

1. Mode method

For the first time a mode method for solving the problem of electromagnetic wave diffraction was proposed in [8]. The planar structure, in which the interfaces are parallel and perpendicular planes, is easily investigated within the framework of existing theories of plane and surface wave diffraction. The main feature of the method is the reduction of rather complex integrodifferential equations to algebraic ones, which makes this method attractive from the point of view of computational procedures. For example, in [9], the problem of reflection of a surface polariton from a "semiconductor-metal" structure was solved. Consider the process of diffraction of an electromagnetic wave with a Gaussian distribution on the structure of 4 media (Fig. 2) with dielectric permittivities: ε_1 — vacuum, $\varepsilon_2(\omega)$ — metal, $\varepsilon_3(\omega)$ — nonlinear thin film, ε_4 — semiconductor. In the left half-plane (at $x < 0$) there are reflected and incident radiation, in the right half-plane ($x > 0$) there will be modes of the past radiation and, as will be shown further, modes of the surface polariton [10][10]. The essence of the method is to represent the incident, reflected, and transmitted radiation as an integral of mode overlap. Investigation of surface polariton excitation on a planar structure is an ordinary boundary value problem with given boundary conditions.

According to the harmonic character along the axis we obtain:

$$\{\mathbf{H}(x,z), \mathbf{E}(x,z)\} = \{\mathbf{H}(z), \mathbf{E}(z)\} e^{ik_x x}. \tag{4}$$

The process of electromagnetic wave diffraction with surface polariton excitation is of the greatest interest. This process is possible only in the case of an H-wave [11] (the so-called TM-polarized radiation), for which, given the chosen structure geometry, the projections are nonzero E_x, E_z, H_y and at $\varepsilon_2(\omega) < 0$. It is easy to show that these electromagnetic wave projections are

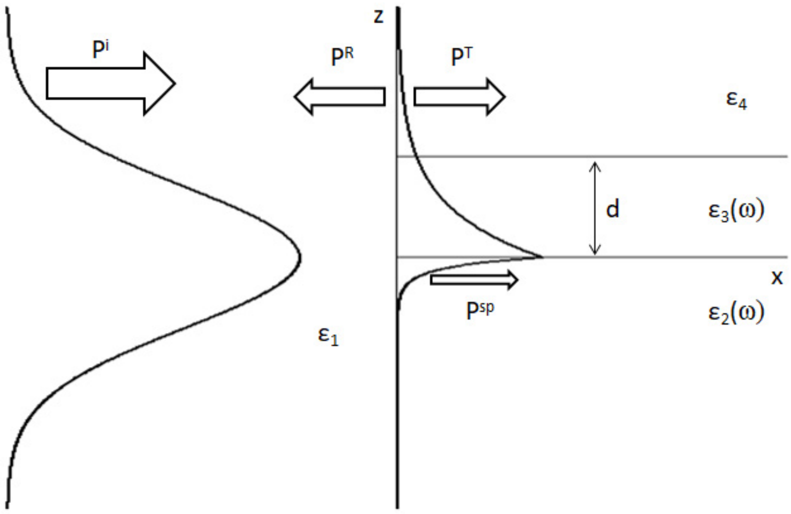

Fig. 2. Geometry of the structure and excitation process of the surface polariton. Explanations in the text

related to each other as:

$$E_x(x,z) = \frac{ic}{\omega\varepsilon(\omega)} \frac{\partial H_y(x,z)}{\partial z}; \quad E_x(z) = \frac{ic}{\omega\varepsilon(\omega)} \frac{dH_y(z)}{dz}, \tag{5}$$

$$E_z(x,z) = \frac{ic}{\omega\varepsilon(\omega)} \frac{\partial H_y(x,z)}{\partial x}; \quad E_z(z) = -\frac{c}{\omega\varepsilon(\omega)} k_x H_y(z). \tag{6}$$

Taking into account the boundary conditions

$$H_y(z=-0) = H_y(z=+0), \; E_x(z=-0) = E_x(z=+0) \tag{7}$$

the wave equation for the magnetic field and the projection of the wave vectork_x for Fig. 2 is

$$\frac{d^2 H_y}{dz^2} + \left[\frac{\omega^2}{c^2}\varepsilon_i - k_x^2\right] H_y = 0. \tag{8}$$

The solution of equation (8) at $x < 0$ has the form

$$H_{1y}(z) = B_1 \exp(i\beta z) + B_2 \exp(-i\beta z), \tag{9}$$

$$E_{1z}(z) = -\frac{ck_x^{(1)}}{\omega\varepsilon_1} \left[B_1 \exp(i\beta z) + B_2 \exp(-i\beta z)\right], \tag{10}$$

where β is the transverse wave number in media (3) and (4)

$$\beta^2 + \left(k_x^{(1)}\right)^2 = k_0^2 \varepsilon_1, \tag{11}$$

where $k_0 = \omega/c = 2\pi/\lambda$ is the wave number in vacuum, λ is the wavelength of incident radiation. In the media (3) and (4) in the framework of the proposed model there will be two harmonics each, i.e. such radiation will be degenerate (it is convenient to denote them by "+"

and "−" harmonics), and accordingly due to degeneration we obtain uncertain coefficients for the wave number k_x. Using the orthogonality and normalization conditions [12], we obtain

$$\int_{-\infty}^{\infty} E_{1z}^{\beta\pm} H_{1z}^{\beta\mp'} dz = 0 \tag{12}$$

and

$$\int_{-\infty}^{\infty} E_{1z}^{\beta\pm} H_{1y}^{\beta\pm'} dz = -\frac{c}{\omega} k_x^{(1)} \delta(\beta - \beta') . \tag{13}$$

The coefficients B_1 and B_2 of (9) are

$$B_1 = B_2^* = \frac{1}{2}\sqrt{\frac{\varepsilon_1}{2\pi}}(1 \pm i) . \tag{14}$$

Thus, in the medium (1) we get

$$H_{1y}^{\beta\pm}(z) = \frac{1}{2} B_{1\beta}\left[(1 \mp i)\exp(-i\beta z) + (1 \pm i)\exp(i\beta z)\right], \tag{15}$$

where $B_{1\beta} = (\varepsilon_1/2\pi)^{1/2}$ is the normalization constant. Then the incident and reflected radiation will be written in the form

$$H_{1y}^{i}(x,z) = \int_{0}^{\infty}\left[I_\beta^+ H_{1y}^{\beta^+} + I_\beta^- H_{1y}^{\beta^-}\right]\exp\left(ik_x^{(1)}x\right)d\beta, \tag{16}$$

$$\mathfrak{H}_{1y}^{r}(x,z) = \int_{0}^{\infty}\left[R_\beta^+ H_{1y}^{\beta^+} + R_\beta^- H_{1y}^{\beta^-}\right]\exp\left(-ik_x^{(1)}x\right)d\beta, \tag{17}$$

where I_β and $R_\beta^\pm$ are the amplitudes of the incident and reflected waves. Let's represent the magnetic field as

$$\mathfrak{H}(x,z) = G(z)\, exp(-ik_x x)$$

when approximating a Gaussian beam

$$G(z) = C_0/\left(1 + z^2/W_0^2\right),$$

where C_0 is the amplitude, and W_0 is the half-width of the beam. Then we use the properties of the delta function and after elementary transformations we obtain

$$\int_{-\infty}^{\infty} G(z)\, E_{1z}^{\beta\pm} dz = -\frac{c}{\omega} k_x^{(1)} I_\beta^\pm, \tag{18}$$

and

$$I_\beta^+ = I_\beta^- \equiv I_\beta = C_0 W_0 \sqrt{\frac{\pi}{2\varepsilon_1}}\exp(-\beta W_0) . \tag{19}$$

The projection of the Poynting vector on the Ox axis of propagation for incident radiation is [12].

$$P_x = \frac{c}{8\pi}\,\mathrm{Re}\left\{\int -\infty^{\infty}\left[\mathfrak{E}_{1z} H_{1y}^*\right]_x dz\right\} . \tag{20}$$

For incident and reflected radiation [medium (1)] we obtain [11]:

$$P_x^i = \frac{c^2}{4\pi\omega}\int_0^\infty I_\beta I_\beta^* k_x^{(1)} d\beta = \frac{1}{8\pi}\frac{1}{\sqrt{\varepsilon_1}} C_0^2 W_0 \frac{\pi}{2}, \tag{21}$$

$$P_x^R = \frac{c^2}{8\pi\omega} \int_0^\infty \left(R_\beta^+ R_\beta^{+*} + R_\beta^- R_\beta^{-*} \right) k_x^{(1)} d\beta. \tag{22}$$

In media (2), (3), (4) the volumetric radiation is represented as:

$$H_{2y}^\beta (z) = \begin{cases} D_1 \exp(-i\beta z) + D_2 \exp(i\beta z), & z > d \\ D_3 \exp(-i\beta z) + D_4 \exp(i\beta z), & 0 < z < d, \\ D_5 \exp(-\rho z), & z < 0, \end{cases} \tag{23}$$

$$E_{2z}^\beta (z) = -\frac{c}{\omega} k_x^{(2)} \begin{cases} \frac{1}{\varepsilon_4} \left[D_1 \exp(-i\beta z) + D_2 \exp(i\beta z) \right], & z > d, \\ \frac{1}{\varepsilon_3(\omega)} \left[D_3 \exp(-i\beta z) + D_4 \exp(i\beta z) \right], & 0 < z < d, \\ \frac{1}{\varepsilon_2(\omega)} D_5 \exp(-\rho z), & z < 0, \end{cases} \tag{24}$$

with the wave numbers related by the equations:

$$\beta^2 + \left(k_x^{(2)} \right)^2 = k_0^2 \varepsilon_3(\omega), -\rho^2 + \left(k_x^{(2)} \right)^2 = k_0^2 \varepsilon_2. \tag{25}$$

The normalizing constants are equal:

$$\begin{aligned} D_5 &= D_{2\beta}, \\ -D_1 = D_2^* &= \frac{1}{2} D_{2\beta} \left(1 + i \frac{\rho}{\beta} \frac{\varepsilon_4}{\varepsilon_3(\omega)} \right), \\ -D_3 = D_4^* &= \frac{1}{2} D_{2\beta} \left(1 + i \frac{\rho}{\beta} \frac{\varepsilon_3(\omega)}{\varepsilon_2(\omega)} \right), \end{aligned} \tag{26}$$

where

$$D_{2\beta} = \sqrt{\frac{(\varepsilon_2(\omega)\beta)^2 \varepsilon_3(\omega) \varepsilon_4}{\pi \left[(\varepsilon_2(\omega)\beta)^2 + (\varepsilon_3(\omega)\rho)^2 + (\varepsilon_4 \rho)^2 \right]}}.$$

Let us represent the surface polariton at the interface (2)–(3) in the form:

$$H_{2y}^\tau (z) = \begin{cases} S_1 \exp(-k_1 z), & z > 0, \\ S_2 \exp(k_2 z), & z < 0, \end{cases} \tag{27}$$

$$E_{2z}^\tau (z) = -\frac{c}{\omega} k_s \begin{cases} \frac{S_1}{\varepsilon_3(\omega)} \exp(-k_1 z), & z > 0, \\ \frac{S_2}{\varepsilon_2(\omega)} \exp(k_2 z), & z < 0. \end{cases} \tag{28}$$

If k_1 and k_2 are the projections of the wave number k_s on the propagation axes, then

$$k_1 = \sqrt{k_s - \frac{\omega^2}{c^2} \varepsilon_3(\omega)}, \quad k_2 = \sqrt{k_s + \frac{\omega^2}{c^2} |\varepsilon_2(\omega)|}. \tag{29}$$

Let the surface polariton satisfy the condition

$$\int_{-\infty}^\infty E_{2z}^\tau H_2^{\tau^*} dz = -\frac{}{\omega} k_s, \tag{30}$$

i.e., normalized along the axis Oz. Then for the coefficients in (27) and (28) it is true

$$S_1 = S_2 = S_{2\tau} = \left[\frac{1}{2k_2\varepsilon_2(\omega)} + \frac{1}{2k_1\varepsilon_3(\omega)}\right]^{-1/2}. \tag{31}$$

Hence (27) and (28) can be represented as:

$$H_{2y}^{\tau}(z) = S_{2\tau}\begin{cases}\exp(-k_1 z), & z > 0,\\ \exp(k_2 z), & z < 0,\end{cases} \tag{32}$$

$$E_{2z}^{\tau}(z) = -\frac{c}{\omega}S_{2\tau}\begin{cases}\dfrac{1}{\varepsilon_3(\omega)}\exp(-k_1 z), & z > 0,\\ \dfrac{1}{\varepsilon_2(\omega)}\exp(k_2 z), & z < 0.\end{cases} \tag{33}$$

Similar expressions are obtained for the interface (3), (4) in the case of $\varepsilon_3(\omega) < 0$. Thus, the magnetic field at $x > 0$ in the amplitude expression has the form

$$\mathfrak{H}_{2y}^{t}(x,z) = TH_{2y}^{\tau}\exp(ik_s x) + \int_0^{\infty}\left[T_\beta H_{2y}^{\beta}\right]d\beta\exp\left(ik_x^{(2)}x\right), \tag{34}$$

where volumetric and surface waves are expressed in terms of their amplitudes T_β and T, respectively. The projection of the Poynting vector along the propagation axis (flux passed) in the amplitude representation has the form [12].

$$P_x^T = \frac{c^2}{8\pi\omega}\left[TT^* k_s + \int_0^{\infty} T_\beta T_\beta^* k_x^{(2)} d\beta\right]. \tag{35}$$

Let us apply to the obtained equations (21), (22), (35) the condition of continuity of electric and magnetic fields on the boundary $x = 0$, i.e.

$$\int_0^{\infty}\left[\left(I_\beta + R_\beta^{+}\right)H_{1y}^{\beta^+} + \left(I_\beta + R_\beta^{-}\right)H_{1y}^{\beta^-}\right]d\beta = TH_{2y}^{\tau} + \int_0^{\infty}T_\beta H_{2y}^{\beta}d\beta, \tag{36}$$

$$\int_0^{\infty}\left[\left(I_\beta - R_\beta^{+}\right)E_{1z}^{\beta^+} + \left(I_\beta - R_\beta^{-}\right)E_{1z}^{\beta^-}\right]d\beta = TE_{2z}^{\tau} + \int_0^{\infty}T_\beta E_{2z}^{\beta}d\beta. \tag{37}$$

Multiplying equations (36) and (37) sequentially by $E_{1z}^{\beta^{\pm'}}$ and $H_{2y}^{\beta'}$ and taking into account the normalization and orthogonality conditions, we obtain:

$$-\frac{c}{\omega}k_x^{(1)}\left(I_\beta + R_\beta^{\pm}\right) = T\int_{-\infty}^{\infty}E_{1z}^{\beta^{\pm'}}H_{2y}^{\tau}dz + \int_0^{\infty}T_\beta\int_{-\infty}^{\infty}E_{1z}^{\beta^{\pm'}}H_{2y}^{\beta}dz d\beta, \tag{38}$$

$$-\frac{c}{\omega}k_x^{(2)}T_\beta = \int_0^{\infty}\left[\left(I_\beta - R_\beta^{+}\right)\int_{-\infty}^{\infty}H_{2y}^{\beta'}E_{1z}^{\beta^+}dz + \left(I_\beta - R_\beta^{-}\right)\int_{-\infty}^{\infty}H_{2y}^{\beta'}E_{1z}^{\beta^-}\right]d\beta, \tag{39}$$

$$-\frac{c}{\omega}k_s T = \int_0^{\infty}\left[\left(I_\beta - R_\beta^{+}\right)\int_{-\infty}^{\infty}H_{2y}^{\tau}E_{1z}^{\beta^+}dz + \left(I_\beta - R_\beta^{-}\right)\int_{-\infty}^{\infty}H_{2y}^{\tau}E_{1z}^{\beta^-}dz\right]d\beta. \tag{40}$$

Calculation of the integrals of mode overlap in (38)–(40) is not difficult:

$$\int_{-\infty}^{\infty}H_{2y}^{\beta'}E_{1z}^{\beta^{\pm}}dz = -k_x^{(1)}\frac{c}{\omega}\left\{H_{2y}^{\beta'}, E_{1z}^{\beta^{\pm}}\right\}\delta(\beta - \beta'), \tag{41}$$

where

$$\left\{H_{2y}^{\beta}, E_{1z}^{\beta^{\pm}}\right\} = B_{1\beta} D_{2\beta} \frac{\pi}{2\varepsilon_1}\left[1 \pm \frac{\varepsilon_3(\omega)}{\varepsilon_2(\omega)}\frac{\rho}{\beta}\right];$$

$$\int_{-\infty}^{\infty} E_{1z}^{\beta^{\pm}} H_{2y}^{\tau} dz = -\frac{c}{\omega} k_x^{(1)} \left\{E_{1z}^{\beta^{\pm}}, H_{2y}^{\tau}\right\}, \tag{42}$$

where

$$\left\{E_{1z}^{\beta^{\pm}}, H_{2y}^{\tau}\right\} = B_{1\beta} S_{2\tau} \frac{1}{\varepsilon_1}\left(\frac{k_2 \pm \beta}{k_2^2+\beta^2} + \frac{k_1 \mp \beta}{k_1^2+\beta^2}\right).$$

The solution of the system (38)–(41) is:

$$T_\beta = I_\beta T' - T T'', \quad R_\beta^{\pm} = I_\beta R'_{\pm} - T R''_{\pm}, \tag{43}$$

$$T = \frac{\int\limits_0^\infty k_x^{(1)} I_\beta \left[(1-R'_+)\left\{E_{1z}^{\beta^+}, H_{2y}^{\tau}\right\} + (1-R'_-)\left\{E_{1z}^{\beta^-}, H_{2y}^{\tau}\right\}\right] d\beta}{k_s + \int\limits_0^\infty k_x^{(1)} \left[\left\{E_{1z}^{\beta^+}, H_{2y}^{\tau}\right\} R''_+ + \left\{E_{1z}^{\beta^-}, H_{2y}^{\tau}\right\} R''_-\right] d\beta}, \tag{44}$$

where the notations are entered:

$$-T' = \frac{2k_x^{(1)}\left[\left\{E_{1z}^{\beta^+}, H_{2y}^{\beta}\right\} + \left\{E_{1z}^{\beta^-}, H_{2y}^{\beta}\right\}\right]}{k_x^{(2)} + k_x^{(1)}\left[\left\{E_{1z}^{\beta^+}, H_{2y}^{\beta}\right\}^2 + \left\{E_{1z}^{\beta^-}, H_{2y}^{\beta}\right\}^2\right]},$$

$$-T'' = \frac{k_x^{(1)}\left[\left\{E_{1z}^{\beta^+}, H_{2y}^{\beta}\right\}\left\{E_{1z}^{\beta^+}, H_{2y}^{\tau}\right\} + \left\{E_{1z}^{\beta^-}, H_{2y}^{\beta}\right\}\left\{E_{1z}^{\beta^-}, H_{2y}^{\tau}\right\}\right]}{k_x^{(2)} + k_x^{(1)}\left[\left\{E_{1z}^{\beta^+}, H_{2y}^{\beta}\right\}^2 + \left\{E_{1z}^{\beta^-}, H_{2y}^{\beta}\right\}^2\right]}. \tag{45}$$

It should be noted that formulas (38)–(45) are "universal" in a sense, i.e. their general form does not depend on the number of environments at $x > 0$. Thus, the number of environments will determine only the form of integrals of overlapping modes $\left\{H_{2y}^{\beta}, E_{1z}^{\beta^{\pm}}\right\}$, $\left\{E_{1z}^{\beta^{\pm}}, H_{2y}^{\tau}\right\}$.

2. Discussion of results and conclusions

Directly we can check the correspondence of obtained results to the energy conservation law: $P^i = P^R + P^T + P^{sp}$, where P^i — incident radiation, P^R — reflected volumetric radiation, P^T — passed radiation and P^{sp} — surface polariton flux and Fresnel's law.

$$\int_0^\infty \left(2I_\beta I_\beta^* - R_\beta^+ R_\beta^{+*} - R_\beta^- R_\beta^{-*}\right) k_x^{(1)} d\beta = T T^* k_s + \int_0^\infty T_\beta T_\beta^* k_x^{(2)} d\beta. \tag{46}$$

Let's choose such parameters of the Gaussian beam so that the passed and reflected waves are expressed in fractions of the incident radiation. We use the normalization

$$C_n = C_0/G\,,$$
$$W_n = (2\pi/\lambda\,)\, W_0, \tag{47}$$
$$k_x^{'} = k_x/k_0.$$

The incident flux in the amplitude representation will be

$$P_x^i = \frac{c^2}{4\pi\omega}\int_0^{\infty} I_\beta I_\beta^* k_x^{(1)} d\beta = \frac{1}{8\pi}\frac{1}{\sqrt{\varepsilon_1}}C_0^2 W_0 \frac{\pi}{2} = \left[\frac{cG^2\lambda}{16\pi^2}\right]\left(\frac{\pi C_n^2 W_n}{2\sqrt{\varepsilon_1}}\right). \tag{48}$$

It follows from (48) that it is possible to arrive at a dimensionless description of the Pointer vector flux. For example, for the incident flux we obtain:

$$P_{x,norm}^i = \frac{\pi C_n^2 W_n}{2\sqrt{\varepsilon_1}} = 1 \tag{49}$$

with $C_n = \sqrt{\frac{2\sqrt{\varepsilon_1}}{\pi W_n}}$. Let us present a model solution. Let the normalized value of the incident flux half-width is $W_n = 25$. Then the numerical calculation leads to the values shown in Tab. 1.

Table 1. Dependences of flux values on dielectric permittivity values

ε_1	ε_2	ε_3	ε_4	P^R	P^T	P^{sp}
1	-20	1	11.7	0.45	0.45	0.1
1	-50	5	12.0	0.54	0.29	0.17
1	-100	10	12.2	0.67	0.14	0.21
1	-1000	15	12.4	0.79	0.09	0.12

For small values of the half-width of the beam, the amplitude for the surface polariton is

$$T = \left[\frac{2W_n^{1/2}\,\Omega}{\sqrt{\varepsilon_3}}\right]\frac{\int\limits_0^{\infty}\exp\left(-\beta W_n\right)\left[\left(\frac{k_2}{k_2^2+\beta^2}+\frac{k_1}{k_1^2+\beta^2}\right)+\left(\frac{\beta}{k_2^2+\beta^2}-\frac{\beta}{k_1^2+\beta^2}\right)\right]d\beta}{\sqrt{\varepsilon_3}+\Omega^2\int\limits_0^{\infty}\left[\left(\frac{k_2+\beta}{k_2^2+\beta^2}+\frac{k_1-\beta}{k_1^2+\beta^2}\right)^2+\left(\frac{k_2-\beta}{k_2^2+\beta^2}+\frac{k_1+\beta}{k_1^2+\beta^2}\right)^2\right]d\beta}, \tag{50}$$

where

$$\Omega = \left[\frac{\varepsilon_3\left(\omega\right)k_1\varepsilon_2\left(\omega\right)k_2}{\pi\left(\varepsilon_3\left(\omega\right)k_1+\varepsilon_2\left(\omega\right)k_2\right)}\right]^{1/2}.$$

Thus, we get

$$T = 4\sqrt{\frac{W_n\pi}{\sqrt{\left|\varepsilon_2\left(\omega\right)\right|}}}\left[\varepsilon_3\left(\omega\right)\left(Y_1+Y_2\right)+\sqrt{\varepsilon_3\left(\omega\right)}\left(Y_4-Y_3\right)\right], \tag{51}$$

where

$$Y_1 = \cos\left(k_1W_n\right)\left[\frac{\pi}{2}-\mathrm{Si}\left(k_1W_n\right)\right]+\sin\left(k_1W_n\right)\mathrm{Ci}\left(k_1W_n\right),$$

$$Y_2 = \cos\left(k_2W_n\right)\left[\frac{\pi}{2}-\mathrm{Si}\left(k_2W_n\right)\right]+\sin\left(k_2W_n\right)\mathrm{Ci}\left(k_2W_n\right),$$

$$Y_3 = \sin\left(k_1W_n\right)\left[\frac{\pi}{2}-\mathrm{Si}\left(k_1W_n\right)\right]-\cos\left(k_1W_n\right)\mathrm{Ci}\left(k_1W_n\right),$$

$$Y_4 = \sin\left(k_2W_n\right)\left[\frac{\pi}{2}-\mathrm{Si}\left(k_2W_n\right)\right]-\cos\left(k_2W_n\right)\mathrm{Ci}\left(k_2W_n\right),$$

$$\mathrm{Si}\left(x\right) = \int_0^x \frac{\sin t}{t}dt,$$

$$\mathrm{Ci}\,(x) = -\int_x^{\infty} \frac{\cos t}{t} dt.$$

It should be noted that diffraction problems in which the geometry of the structure is not plane-parallel are of undoubted interest. In addition, certain difficulties arise in solving Maxwell's integrodifferential equations when a Gaussian beam is incident at an angle other than normal. Even a formal transition to the new coordinate system

$$\begin{cases} z = z' \cos\theta - x' \sin\theta, \\ x = z' \sin\theta - x' \cos\theta, \end{cases}$$

leads to a significant complication not only of the solution of the equations, but also of the model itself. It is necessary to consider the surface polariton propagation not only along Ox, but also along Ox'. This problem will be the object of further development of the proposed theory.

References

[1] D.Marcuse, Optical waveguides: translation from English, ed. V.V. Shevchenko, Mir, Moscow, 1974 (in Russian).

[2] E.I.Nefedov, Diffraction of electromagnetic waves on dielectric structures, Nauka, Moscow, 1978 (in Russian).

[3] V.A.Kiselev, Quantum electronics, **1**(1974), no. 2, 329 (in Russian).

[4] V.V.Shevchenko, Smooth transitions in open waveguides, Science, Moscow, 1978 (in Russian).

[5] S.Chakraborty, O.P.Marshall, T.G.Folland, Y.-J.Kim, A.N.Grigorenko, Novoselov K.S., *Science*, **351**(2016), no. 6270, 246–248. DOI: 10.1126/science.aad293

[6] Sh.Shah, X.Lin, L.Shen, M.Renuka, B.Zhang, H.Chen, *Phys. Rev. Applied.*, **10**(2018), no. 3, 034025.

[7] B.I.Afinogenov, D.S.Kopylova, K.A.Abrashitova, V.O.Bessonov, A.S.Anisimov, S.A.Dyakov, N.A.Gippius, Yu.G.Gladush, A.A.Fedyanin, A.G.Nasibulin, *Phys. Rev. Applied*, **9**(2018), no. 2, 024027.

[8] V.M.Agranovich, V.E.Kravtsov, T.A.Leskova, *ZhETF*, **81**(1981), no. 5, 1828 (in Russian).

[9] A.I.Voronko, L.G.Klimova, G.N.Shkerdin, *Solid State Comm.*, **61**(1987), no. 6, 361.

[10] Surface polaritons, ed. V.M. Agranovich, D.M. Mills, Science, Moscow, 1985 (in Russian).

[11] K.V.Bukhensky, A.B.Dubois, et al., Peculiarities of solving the integro-differential Maxwell equations and excitation of the surface polariton on the planar structure, *Zhurnal SVMO*, **19**(2017), no. 3, 73–81. DOI 10.15507/2079-6900.19.201703.73-81

[12] A.B.Dubois, R.V.Lysov, *Math. modeling*, **18**(2006), no. 3, 3–12 (in Russian).

Распространение поверхностных поляритонов в структурах с графеновым слоем

Кирилл В. Бухенский
Александр Дюбуа
Алексей Н. Конюхов
Рязанский государственный радиотехнический университет имени В. Ф. Уткина
Рязань, Российская Федерация
Сергей И. Кучерявый
Национальный исследовательский ядерный университет (НИЯУ) МИФИ
Москва, Российская Федерация
Обнинский институт ядерной энергентики (ОИНПЭ),
Обнинск, Роосcийская Федерация
Алексей С. Сафошкин
Рязанский государственный радиотехнический университет имени В. Ф. Уткина
Рязань, Российская Федерация

Аннотация. В работе рассмотрен процесс дифракции электромагнитной волны на границе раздела "вакуум – металл – нелинейная плёнка (графен) – полупроводник" с возбуждением поверхностной волны. В рамках теории представлен модовый метод расчета процесса взаимодействия излучения с многослойной структурой, позволяющий рассчитывать потоки энергий, возникающие в процессе дифракции.

Ключевые слова: интегро-дифференциальные уравнения, поверхностный поляритон, графен, уравнения Максвелла.