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Elastic bend of twisted waveguide

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Abstract. The construction of any antenna-feeder system, in addition to providing radio technical parameters, must has the specified strength and rigidity under operational loads. This paper considers the stress and deflection of a twisted waveguide at bending as the most common and dangerous type of loading. According to the technical theory of beams, obtained equations which describing the integral characteristics of the across section as well as stresses and deflections of the twisted waveguide. It is shown that in calculating the twisted waveguide strength and stiffness, it is necessary to apply minimum values for integral characteristics of the cross section regardless of the load direction.

1. Introduction

Twisted waveguides are often used in antenna-feeder systems to turn the polarization plane of the transmitted electromagnetic signal [1-3]. Structure of a twisted waveguide is an extended thin-walled shell with a rectangular cross-section (figure 1).

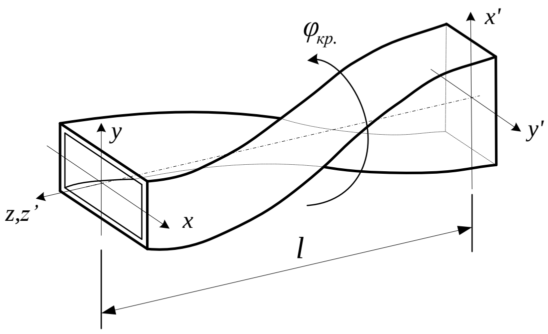

Figure 1. Twisted waveguide.

Until recently, waveguides in antenna-feeder systems did not count for strength and rigidity, as it had overly large wall thickness of 3-4 mm. This thickness provided excessive strength and rigidity to any waveguide structure. At present, in order to minimize the mass and size parameters of an antenna-feeder system, it is necessary to reduce the wall thickness of the waveguide to the minimum possible value. Consequently, works have recently appeared on methods of calculating the strength and stiffness of a waveguide [4,5], but none of them contain information about a twisted waveguide. The obvious reason for this is the fact that a twisted waveguide is rarely present in the antenna feeder system.

In this paper, we consider the bend of a twisted waveguide as the most common and not strong type of loading for any lengthy structure.

2. Twisted waveguide model

The geometry analysis of twisted waveguide design (figure 1) shows that it satisfies several approaches of mechanics: the Euler–Bernoulli beam theory [6-10], the generalized beam theory [11-13] and the theory of shells [14-17].

The most accurate approach seems to be the theory of shells [14-17]. But in this case we get a system of partial differential equations, which is a intricate problem. The generalized beam theory [11-13] was primary designed for constrained torsion, which we do not consider. Therefore, we examine the bending of twisted waveguides on the basis of the Euler–Bernoulli beam theory. For this purpose, we derive equations for inertial characteristics of a cross section from which, according to known dependencies of beam theory [6-10], stresses and deflections of twisted waveguides directly are derived.

3. Integral cross-sectional characteristics of the twisted waveguide

The following integral characteristics of the cross-section of a twisted waveguide are used in the calculations of stresses and deflections during bending: cross-section area, moments of inertia and section modulus [18-20].

3.1. Cross-sectional area

The cross section at the end of a twisted waveguide is rotated ("twisted") about the central axis of the waveguide relative to its initial position by some angle (figure 1). As a result, all longitudinal lines, being straight and parallel to the central axis of the waveguide, are simultaneously curved by the helical line. In the plane perpendicular to the axis of the waveguide, the geometry of the cross section remains rectangular (figure 2).

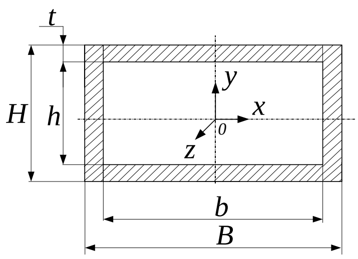

Figure 2. Cross section geometry of twisted waveguide.

Therefore, the cross-sectional area of the twisted portion is also constant along its length and equal to:

$$A = H \cdot B - h \cdot b \,. \tag{1}$$

The moments of inertia and the section modulus of the cross section for the twisted waveguide continuously change their value along its length.

3.2. Moments of inertia of twisted waveguide

Let's consider a twisted waveguide of length l, which has one end uniformly twisted relative to the other end by one revolution. In this case, the twist angle of the cross section in question is related to the longitudinal coordinate z of the section by a linear equation:

$$\phi = z\frac{\phi'_{кр}}{l} = z\frac{2\pi}{l}, \tag{2}$$

where $\phi'_{кр}$ – is a twist angle of waveguide with length *l*.

The axial moments of inertia of the twisted waveguide at the distance *z* from its beginning can be determined by the following equations [18-20]:

$$I_x(z) = \frac{BH}{12}\left[H^2\cos^2\left(z\frac{2\pi}{l}\right) + B^2\sin^2\left(z\frac{2\pi}{l}\right)\right] - \frac{bh}{12}\left[h^2\cos^2\left(z\frac{2\pi}{l}\right) + b^2\sin^2\left(z\frac{2\pi}{l}\right)\right], \tag{3}$$

$$I_y(z) = \frac{BH}{12}\left[H^2\sin^2\left(z\frac{2\pi}{l}\right) + B^2\cos^2\left(z\frac{2\pi}{l}\right)\right] - \frac{bh}{12}\left[h^2\sin^2\left(z\frac{2\pi}{l}\right) + b^2\cos^2\left(z\frac{2\pi}{l}\right)\right]. \tag{4}$$

Figure 3 shows moments of inertia (3,4) for a twisted waveguide along its length.

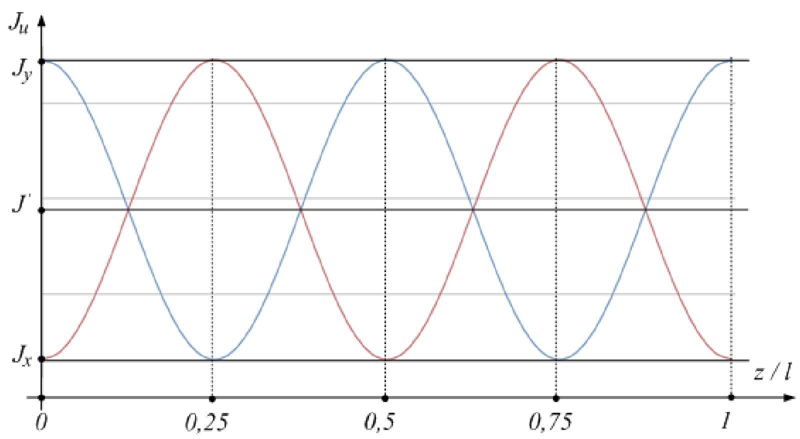

Figure 3. Moments of inertia of the twisted waveguide along its longitudinal axis.

The horizontal axis is expressed in a dimensionless form such that its total length *l* corresponds to a uniform twist of the cross section by one revolution: $\phi'_{кр} = 360^0$.

3.3. Section modulus of a twisted waveguide

The section modulus values are calculated on the basis of the dependencies (3-4) as well as the distances hx and hy from longitudinal axis of a twisted waveguide to the maximum distant points of the cross section along the axes x and y according to the equations:

$$h_x(z) = \frac{B}{2}\cos\left(z\frac{2\pi}{l}\right) - \frac{H}{2}\sin\left(z\frac{2\pi}{l}\right),\quad h_y(z) = \frac{H}{2}\cos\left(z\frac{2\pi}{l}\right) + \frac{B}{2}\sin\left(z\frac{2\pi}{l}\right). \tag{5}$$

Using the equations (3-5), the equations for the section modulus values take the forms [18-20]:

$$W_x(z) = \frac{I_x(z)}{h_y(z)},\quad W_y(z) = \frac{I_y(z)}{h_x(z)} \tag{6}$$

Figure 4 shows the graphs of the section modulus values (6) along the length of a twisted waveguide.

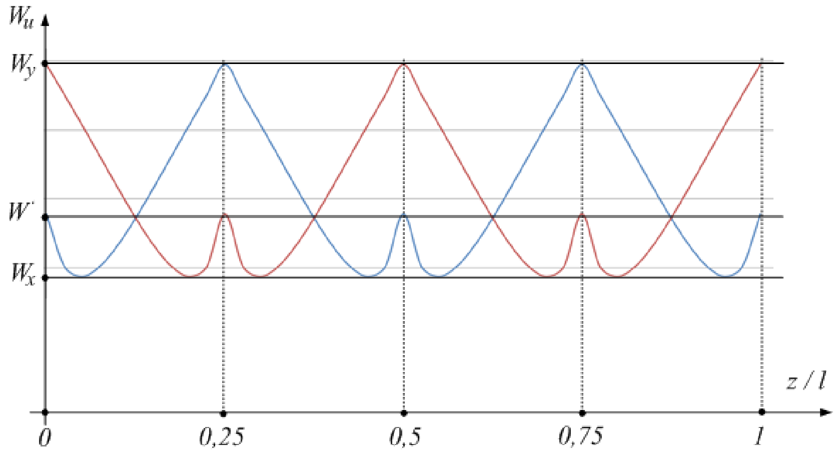

Figure 4. Section modulus values of a twisted waveguide along its longitudinal axis.

4. Bending stress and deflection distribution for a twisted waveguide

According to the Euler–Bernoulli beam theory [6-10], the stress and bending deflections of the twisted waveguide are inversely proportional to the integral characteristics of the cross section (3-6).

4.1. Bending stresses of twisted waveguide

The section modulus values (6) determine the maximum bending stresses in a twisted waveguide according to the following equations [6-10]:

$$\sigma_{Mx} = \frac{M_x}{W_x}, \qquad \sigma_{My} = \frac{M_y}{W_y}. \tag{7}$$

Figure 5 shows the graphs of the maximum normal stresses along the length of the twisted waveguide at bending by unit moments: $Mx = My = 1$.

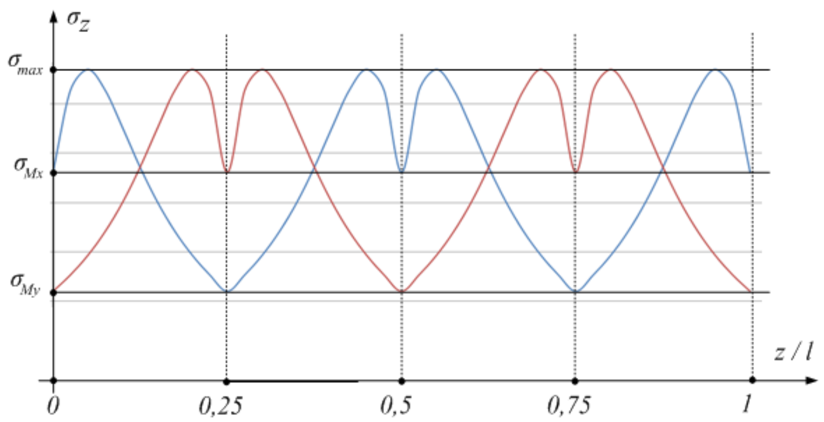

Figure 5. Maximum bending stress of twisted waveguide along its longitudinal axis.

4.2. Distribution of bending deflections for a twisted waveguide

According to the equation of the elastic line of a beam [6-10], bending deflections are inversely proportional to the moments of inertia (3,4):

$$\frac{d^2u_y}{dz^2}=\frac{M_x(z)}{EI_x},\qquad \frac{d^2u_x}{dz^2}=\frac{M_y(z)}{EI_y},\tag{8}$$

where E - Young's modulus.

Figure 6 shows the deflections (8) for single moments and Young 's module.

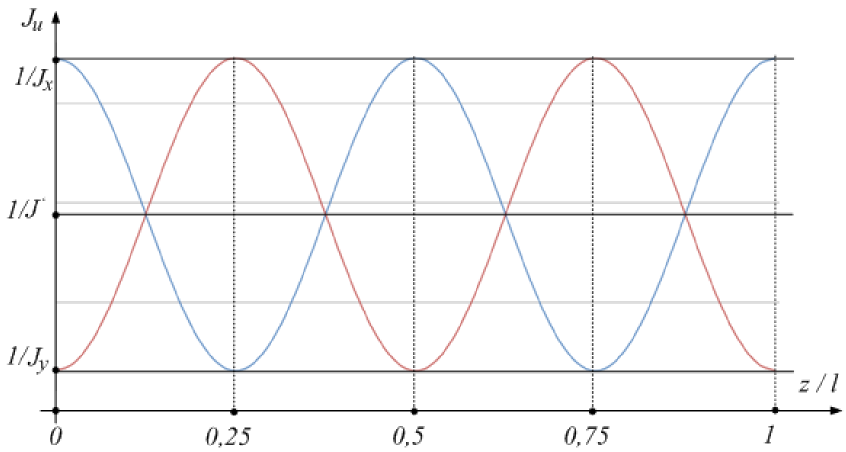

Figure 6. Twisted waveguide deflection along its longitudinal axis.

It should be noted that figure 6 shows the flexibility distribution of the twisted waveguide along its longitudinal axis rather than the deflection plots.

5. Discussion

The analysis of the graphs in figures 5,6 shows that there are two orthogonal principal axes, relative to which the strength and rigidity take maximum and minimum values. The ratio of maximum to minimum values for moments of inertia and the section modulus are equal to:

$$k_J=\frac{I_{MAX}}{I_{MIN}}\approx\frac{B}{H},\qquad k_W=\frac{W_{MAX}}{W_{MIN}}\approx\left(\frac{B}{H}\right)^2.\tag{9}$$

So stresses and deflections of a twisted waveguide appear to equivalents to straight waveguide with some averaged values J' and W', which can be taken as calculated in the solution (figures 3,4). However, in most cases of waveguide sizes, the equation (9) are in the range of:

$$k_I=2...3,\qquad k_S=4...6.\tag{10}$$

The significant differences in values (10) lead to the corresponding changes in strength and rigidity of twisted waveguides. It is rational to use in solve the worst combination of geometric parameters relative to a load direction. For example, in aircraft and spacecraft, the direction of action of external loads fails to remain constant, so the minimum values must be taken into consideration:

$$I_u = \min\left(I_x, I_y\right),\ \ W_u = \min\left(W_x, W_y\right). \tag{11}$$

In this case, deflections have the minimum values and while bending stresses have the maximum values.

6. Conclusion

In this paper, the Euler–Bernoulli beam theory has been chosen to evaluate the general distribution of stress and deflection, in a twisted waveguide at bend. This allows to clearly identify the influence of a waveguide's geometry on stresses and deflections to provide general recommendations for its design. The shell model of a twisted waveguide is expected to be developed in further research and allow to obtain a more accurate assessment of stresses and deflections.

Acknowledgements

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References

[1] Collin R E 2001 *Foundations for microwave engineering* (IEEE Press)
[2] Chang K 2005 *Encyclopedia of RF and Microwave Engineering* (John Wiley & Sons, Inc)
[3] Sophocles J 2019 *Orfanidis 2008 Electromagnetic Waves and Antennas* (Rutgers University)
[4] Silchenko P N, Mikhnev M M, Ankudinov A V and Kudryavtsev I V 2012 *J. Mach. Manuf. Reliab.* **41(1)** 91-5
[5] Kudryavtsev I V, Minakov A V and Mityaev A E 2019 *J. Mach. Manuf. Reliab.* **48(4)** 306–13
[6] Carrera E 2011 *Beam Structures. Classical and Advanced Theories* (John Wiley & Sons, Inc)
[7] Gere J M 2004 *Mechanics of materials* (CBS Publishers & Distributors)
[8] Hibbeler R C 2016 *Statics and Dynamics* (Pearson)
[9] Timoshenko S P 1940 *Strength of materials Elementary theory and problems* (Krieger Pub Co)
[10] Ugural A C 2010 *Stresses in beams, plates and shells* (CRC Press)
[11] Vlasov V Z 1961 *Thin-walled Elastic Beam* (English translation. Published for NSF and Department of Commerce by the Israel Program of Scientific Translations, Jerusalem)
[12] Meek J L and Lin W J 1990 *J. Struct. Engrg. ASCE* **116(6)** 1473-89
[13] Djugash A C 1994 *J. Construct. Steel Research* **31** 289-304
[14] Fluegge W 1973 *Stresses in shells* (Springer-Verlag Berlin Heidelberg)
[15] Altenbach H 2017 *Shell-like structures* (Springer-Verlag Berlin Heidelberg)
[16] Mansfield E H 2005 *The bending and stretching of plates* (Cambridge University Press)
[17] Altenbach H 2019 *Recent developments in the theory of shells* (Springer Int. Publ.)
[18] Norton R 2007 *Design of Machinery* (McGraw-Hill Science)
[19] Khurmi R S 2005 *A textbook of machine design* (Ram Nagar, New Delhi)
[20] Schmid S R 2014 *Fundamentals of machine elements* (CRC Press)