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Kalman filtering of coordinates determination for local positioning systems mobile objects

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Kalman filtering of coordinates determination for local positioning systems mobile objects

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Abstract. The article is devoted to the study of Kalman filtering errors of the measured radio navigation parameters into the coordinates and elements of the objects movement in relation to the ground-based radio navigation system of local positioning. The stationary state of the mobile station, its uniform rectilinear and uniformly accelerated movement are considered. The use of optimal estimation of the Kalman filter allows reducing the standard deviation of coordinates by 1.5-3 times for the considered mobile station trajectory.

1. Introduction

One of the key factors determining the accuracy and reliability of the radio navigation systems, including local positioning systems, are the algorithms used in its software for converting measured radio navigation parameters (ranges, quasi-ranges, range differences) into coordinates and elements of the movement of objects. Inaccuracies in the implementation of location algorithms can become a source of additional methodological errors. On the other hand, the study of errors in the algorithms for determining coordinates in various location methods allows us to estimate the expected values of the accuracy of calculation of these coordinates and compare with the given requirements.

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2. Positioning accuracy

The error in determining radio navigation parameters σ_D (or $\sigma_{\Delta D}$ for difference-rangefinder) in positioning systems is generally determined by a number of factors:

- the difference between the actual speed of propagation of the radio signal on the working track from its calculated value;
- the difference between the actual signal frequency at which the phase measurements are performed from its calculated value;
- the presence of random and systematic errors in the measured values of the phase of the received signals.

Ways to reduce errors are considered in [1]. The residual component of the random and systematic errors of phase measurement determines the error of measurement of radio navigation parameters, which, in turn, leads to errors in the measurement of coordinates of the mobile station, the value of which depends on:

- the location of reference stations relative to each other, which causes the formation of the working area of the positioning system;
- the position of the mobile station relative to the reference station;
- the method of positioning the coordinates of the mobile station.

It is known that the covariance matrix of coordinate errors in the radio navigation system

$$W = H^{-1} \cdot S \cdot (H^{-1})^T, \tag{1}$$

where H is the matrix of partial derivatives, the measured radio navigation parameters; S is the covariance matrix of measurement errors of radio navigation parameters.

For equal-accurate measurements by the rangefinder method, the matrix of partial derivatives, measured by the radio navigation parameters by coordinates, is defined as

$$H = \begin{bmatrix} \frac{\partial D_1}{\partial x} & \frac{\partial D_1}{\partial y} \\ \frac{\partial D_2}{\partial x} & \frac{\partial D_2}{\partial y} \\ \frac{\partial D_3}{\partial x} & \frac{\partial D_3}{\partial y} \\ \frac{\partial D_4}{\partial x} & \frac{\partial D_4}{\partial y} \end{bmatrix}, \tag{2}$$

where $D_1,..D_4$ is the measured radio navigation parameters (range / quasi-range).

The covariance matrix of errors of measurement of radio navigation parameters has the form

$$S = \sigma_\tau^2 c^2 \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tag{3}$$

where σ_τ is the standard error of delay measurement; c – signal propagation speed.

In the case of the difference-rangefinder method, the matrix data have the form

$$H = \begin{bmatrix} \frac{\partial D_2}{\partial x} - \frac{\partial D_1}{\partial x} & \frac{\partial D_2}{\partial y} - \frac{\partial D_1}{\partial y} \\ \frac{\partial D_3}{\partial x} - \frac{\partial D_1}{\partial x} & \frac{\partial D_3}{\partial y} - \frac{\partial D_1}{\partial y} \\ \frac{\partial D_4}{\partial x} - \frac{\partial D_1}{\partial x} & \frac{\partial D_4}{\partial y} - \frac{\partial D_1}{\partial y} \end{bmatrix};$$

$$S = \sigma_{\Delta\tau}^2 c^2 \cdot \begin{bmatrix} 1 & 0{,}5 & 0{,}5 \\ 0{,}5 & 1 & 0{,}5 \\ 0{,}5 & 0{,}5 & 1 \end{bmatrix},$$

where $\sigma_{\Delta\tau}$ is the standard error of the delay measurement differences.

The above expressions allow us to estimate the covariance matrix of errors W at any point in the working area of the positioning system in the form

$$W = \begin{bmatrix} \sigma_x^2 & \sigma_{xy} \\ \sigma_{yx} & \sigma_y^2 \end{bmatrix}.$$

In the theory of radio navigation systems, the concept of a geometric factor is often used to characterize errors, which is the coefficient by which the error in measuring the range (range difference) is multiplied to obtain an estimate of the error in measuring coordinates.

For the case where the standard error of the estimates of the radio navigation parameters is not equal (when, $\sigma_{\tau 1} \neq \sigma_{\tau 2} \neq .. \neq \sigma_{\tau N}$) the covariance matrix of the errors of measurement of the radio navigation parameters (for the ranging method) has the next form

$$S = \begin{bmatrix} \sigma_{\tau 1}^2 & \sigma_{\tau 1}\sigma_{\tau 2} p_{12} & \sigma_{\tau 1}\sigma_{\tau 3} p_{13} & \sigma_{\tau 1}\sigma_{\tau 4} p_{14} \\ \sigma_{\tau 1}\sigma_{\tau 2} p_{21} & \sigma_{\tau 2}^2 & \sigma_{\tau 2}\sigma_{\tau 3} p_{23} & \sigma_{\tau 2}\sigma_{\tau 4} p_{24} \\ \sigma_{\tau 1}\sigma_{\tau 3} p_{31} & \sigma_{\tau 2}\sigma_{\tau 3} p_{32} & \sigma_{\tau 3}^2 & \sigma_{\tau 3}\sigma_{\tau 4} p_{34} \\ \sigma_{\tau 1}\sigma_{\tau 4} p_{41} & \sigma_{\tau 2}\sigma_{\tau 4} p_{42} & \sigma_{\tau 3}\sigma_{\tau 4} p_{43} & \sigma_{\tau 4}^2 \end{bmatrix}, \quad (4)$$

where p_{ij} is the correlation coefficients of measurement readings of radio navigation parameters between stations i and j.

Without taking into account the geometric factor, the determining contribution to the error in determining the coordinates of a mobile station is made by the accuracy of radio navigation parameters measuring (σ_τ or $\sigma_{\Delta\tau}$) and the error of synchronization of signals of the reference stations of the radio navigation system.

3. Recursive algorithms using

The use of filtering methods reduces the random measurement error. At present, a large number of filtering algorithms for changing parameters have been developed, among which the methods of Kalman filtering are most widely used.

A number of the radio navigation systems use a simplified model for filtering the measured values of phase shifts, the ($\alpha - \beta$)-filter. The analysis showed that with an unlimited increase in the reference number k, the coefficients α and β decrease down to zero, i.e. with a large value of k, the filtering proceeds with less weight, and, ultimately, the filter ceases to respond to changes in the input data. The filter begins to diverge, which allows it to be considered insolvent [2].

Such filters are used in various systems because of the simplicity of implementation and low requirements for computing power [3]. Such systems use fixed or pre-calculated gain factors. However, as practice shows, these parameters are not always applicable, for example, at the beginning of tracking and in the case of a maneuver, rebounds are observed, after which in some cases the trajectory does not return to true. The restoration of the trajectory in this case is possible either after a very long time (up to several tens of minutes), or by manually resetting the filtering by the operator. This makes it unsuitable for use in automated radio navigation systems [4].

4. Kalman filter using

The widespread use of the Kalman filter algorithm in modern information systems is due to a number of its advantages. This is, first of all, that the current estimate that he forms is the best in terms of the minimum variance in comparison with other linear estimates obtained only by linear transformations of observations. In addition, having somewhat complicated the Kalman algorithm, it can also be used in nonlinear filtering problems. The step-by-step (recurrent) nature of the Kalman algorithm allows you to get the current estimate by adjusting its previous value using only the next observation. This is convenient for real-time digital filter implementation, i.e. as data becomes available.

Among the main problems associated with the implementation of the Kalman filter is ensuring the convergence of filter calculations [5].

The Kalman filter algorithm is constructed using two repeatedly repeated operations: extrapolation and filtering. The essence of the first is to predict the state at the next moment in time (taking into account the inaccuracy of the measurement):

$$\hat{s}_k(t_i^-) = F_k(t_i, t_{i-1}) \cdot \hat{s}_k(t_{i-1}^+) + B_k(t_{i-1}) \cdot n(t_{i-1}); \tag{5}$$

$$P_k(t_i^-) = F_k(t_i, t_{i-1}) \cdot P_k(t_{i-1}^+) \cdot F_k^T(t_i, t_{i-1}) + Q_k(t_i). \tag{6}$$

Further, the information received from the meter corrects the predicted value (taking into account the inaccuracy of the information):

$$K_k(t_i) = P_k(t_i^-) \cdot H_k^T(t_i) \cdot \left[H_k(t_i) \cdot P_k(t_i^-) \cdot H_k^T(t_i) + R\right]^{-1}; \tag{7}$$

$$P_k(t_i^+) = P_k(t_i^-) - K_k(t_i) \cdot H_k(t_i) \cdot P_k(t_i^-); \tag{8}$$

$$\hat{s}_k(t_i^+) = \hat{s}_k(t_i^-) + K_k(t_i) \cdot \left[z(t_i) - H_k(t_i) \cdot \hat{s}_k(t_i^-)\right]. \tag{9}$$

The following notations are used in formulas (3.10)–(3.14): $F_k(t_i, t_{i-1})$ is the matrix describing the dynamics of the system (transition matrix between states); $B_k(t_{i-1})$ is the matrix that determines the application of the control action; $H_k(t_i)$ is the matrix that determines the relationship between measurements and the state of the system; $Q_k(t_i)$ the process noise covariance matrix; R is the measurement error determined by the matrix of measurement errors; $s(t_i)$ and $\hat{s}(t_i)$ is the state vector of the system and its assessment; $n(t_i)$ is the input impact vector; $z(t_i)$ is the observed measurement vector; $P(t_i)$ is the prediction of covariance errors; $K(t_i)$ is Kalman gain.

In the development of the Kalman filter for local positioning systems, the object motion model plays an important role. In General, the complete models of motion of mobile stations are dynamic, representing the differential equations of motion of the center of mass of the object and kinematic equations of velocity with angular and spatial coordinates [6]. To estimate the motion of an object on a plane, it is sufficient to apply more simplified models that would reflect the main changes in the parameters of the object's motion. In relation to the positioning system of sea vessels in the area of economic entities, the change in the course or speed of the vessel may take, depending on the maneuverability of the vessel, from a few seconds for small vessels (boats) to a few minutes for large vessels with high inertia (tankers) [7]. In a situation where an urgent change in the course or speed of the vessel is required, this delay can be critical [8]. In turn, when working with highly maneuverable objects, it is also necessary to keep tracking of the radio navigation parameters in conditions of high multipath in production facilities.

5. Modeling

As already noted, the purpose of modeling is to assess the applicability (behavior) of Kalman filtering for testing the radio navigation parameters. When performing the simulation, the following models of movement of mobile stations were considered.

- the stationary state of the object, in which the coordinates of the object are assumed unchanged. In this model, the noise of states is random changes in the speed of an object with zero mathematical expectation and dispersion σ_{υ}^2. The state vector has the form $s(t_i) = [x(t_i); y(t_i)]$;

- uniform rectilinear movement of the object. In this case, the noise of states is random variations of the acceleration of the object, having zero mathematical expectation and a given dispersion σ_a^2. The state vector has the form $s(t_i) = [x(t_i); y(t_i); v_x(t_i); v_y(t_i)]$;

- uniformly accelerated movement of the object. Noise states are random jerks of an object. The state vector has the form $s(t_i) = [x(t_i); y(t_i); v_x(t_i); v_y(t_i); a_x(t_i); a_y(t_i)]$.

The Kalman filter was modeled in the Octave environment with the following initial parameters:

- for each model, an interval of 100 units of discrete time was used;
- the navigation problem was solved by the rangefinder method;
- to simulate the noise of coordinate measurements, random variables distributed according to the normal law with zero mathematical expectation and standard deviation of 6 m were added to the radio navigation parameters (ranges);
- for the model of uniform motion of the object, a speed of 15 m / s was chosen;
- for the model of uniformly accelerated object movement, an acceleration of 0.5 m / s^2 was chosen.

6. The results

Figure 3.1 shows the trajectory of the object's movement on the plane obtained as a result of modeling: section a — stationary position, section b — uniform rectilinear motion, section c — uniform acceleration, section d — maneuvering along a curve (circle).

The simulation results, presented by the values of the standard deviations of the coordinate estimates at the input and output of the filtered coordinates, are shown in table 1.

Table 1. The consolidated table of modeling results

Section	standard deviations of the measured coordinates, m	standard deviations of the filtered coordinates, m
A	11.13	4.04
B	11.84	3.92
C	6.64	4.51
D	6.9	3.94

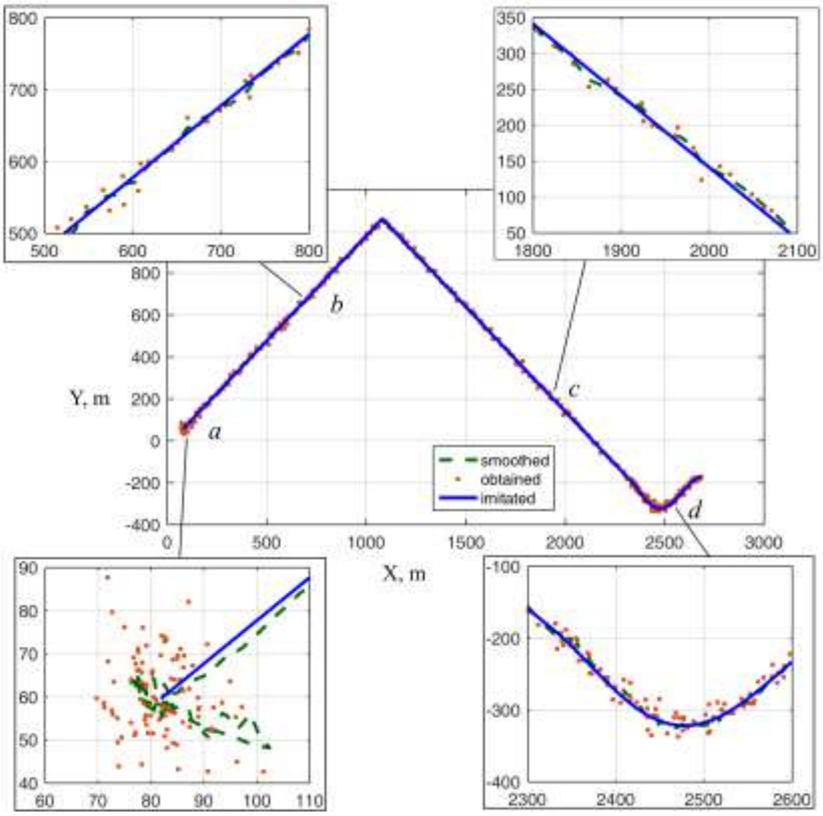

Figure 1. The studied trajectory of motion

7. Conclusion

Based on the simulation results, we can conclude that the use of optimal estimation of the Kalman filter allows reducing the standard deviation of coordinates by 1.5-3 times for the considered trajectory. The deterioration in the efficiency of the Kalman filter at the end of section *d* is associated with a sharp maneuver of the object without reducing the speed, which led to a distortion of the trajectory by 3 discrete samples, which can be explained by the insufficient filter reaction rate.

The main task in developing an adaptive Kalman filter is to fine-tune the filtering parameters for the current operating mode (motion model). On the one hand, the filter should not miss the maneuver, i.e. have sufficient sensitivity to changes in the trajectory, and on the other hand, respond as little as possible to disturbance in the radio navigation information when the object moves in a straight line.

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