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On some Inverse Parabolic Problems with Pointwise Overdetermination

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Abstract. We examine well-posedness questions in the Sobolev spaces of inverse problems of recovering coefficients depending on time in a parabolic system. The overdetermination conditions are values of a solution at some collection of points lying inside the domain and on its boundary. The conditions obtained ensure existence and uniqueness of solutions to these problems in the Sobolev classes.

Keywords: parabolic system, inverse problem, pointwise overdetermination, convection-diffusion.

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Introduction

We consider inverse problems with pointwise overdetermination for a parabolic system of the form

$$Lu = u_t + A(t,x,D)u = f(x,t), \quad (t,x) \in Q = (0,T) \times G, \ G \subset \mathbb{R}^n, \tag{1}$$

where

$$A(t,x,D)u = -\sum_{i,j=1}^{n} a_{ij}(t,x)u_{x_j x_j} + \sum_{i=1}^{n} a_i(t,x)u_{x_i} + a_0(t,x)u,$$

G is a bounded domain with boundary $\Gamma \in C^2$, a_{ij}, a_i are matrices of dimension $h \times h$, and u is a vector of length h. The system (1) is supplemented by the initial and boundary conditions

$$u|_{t=0} = u_0, \quad Bu|_S = g, \quad S = (0,T) \times \Gamma, \tag{2}$$

where $Bu = \sum\limits_{i=1}^{n} \gamma_i(t,x)u_{x_i} + \gamma_0(t,x)u$. The overdetermination conditions are as follows:

$$< u(x_i,t), e_i >= \psi_i(t), \quad i = 1,2,\ldots,r, \tag{3}$$

where the symbol $< \cdot,\cdot >$ stands for the inner product in $\mathbb{C}^h$, $\{e_i\}$ is a collection of vectors of unit length and among the points $\{x_i\}$ as well as the vectors $\{e_i\}$ can be coinciding points and

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vectors. The right-hand side is of the form $f = \sum_{i=1}^{m} f_i(x,t)q_i(t) + f_0(x,t)$. The problems is to find the unknowns $q_i(t)$ occurring into the right-hand side and the operator A as coefficients and a solution u to the system (1) satisfying (2) and (3). The conditions (3) generalized the conventional pointwise overdetermination conditions of the form $u(x_i,t) = \psi_i(t)$. In particular, it is possible that only part of the coordinates of the vector u at a point x_i is given. These problems arise of describing heat and mass transfer, diffusion, filtration, and in many other fields (see [1–3]) and they are studied in many articles. First, we should refer to the fundamental articles by A. I. Prilepko and his followers. In particular, an existence and uniqueness theorem for solutions to the problem of recovering the source $f(t,x)q(t)$ with the overdetermination condition $u(x_0,t) = \psi(t)$ (x_0 is a point in G) is established in [4,5]. Similar results are obtained in [6] for the problem of recovering lower-order coefficient $p(t)$ in the equation (1). The Hölder spaces serve as the basic spaces in these articles. The results were generalized in the book [7, Sec. 6.6, Sec. 9.4], where the existence theory for the problems (1)–(3) was developed in an abstract form with the operator A replaced with $-L$, L is generator of an analytic semigroup. The main results employ the assumptions that the domain of L is independent of time and the unknown coefficients occur into the lower part of the equation nonlinearly. Under certain conditions, existence and uniqueness theorems were proven locally in time in the spaces of functions continuously differentiable with respect to time. We note also the article [8], where an existence and uniqueness theorem in the problem of recovering a lower-order coefficient and the right-hand was established with the overdetermination condition $u(x_i,t) = \psi(t)$ (x_i are interior points of G, $i = 1,2$). There are many articles devoted to the problems (1)–(3) in model situations, especially in the case of $n = 1$ (see, for instance, [9–14]). In these articles different collections of coefficients are recovered with the overdetermination conditions of the form (3), in particular, including boundary points x_i. In this case the boundary condition and the overdetermination condition define the Cauchy data at a boundary point. Many results in the case of $n = 1$ are exhibited in [15]. Note the book [16], where the solvability questions for inverse problems with the overdetermination conditions being the values of a solution on some hyperplanes (sections of a space domain) are studied. The problems (1)–(3) were considered in authors' articles in [17,18], where conditions on the data were weakened in contrast to those in [7, Sec. 9.4] and the solvability questions were treated in the Sobolev spaces. In contrast to the previous results, we examine the case of the points $\{x_i\}$ lying on the boundary of G as well and the special overdetermination conditions (only some combinations of the coordinate of a solution are given). These overdetermination conditions also arise in applications (see [3]). Note that numerical methods for solving the problems (1)–(3) have been developed in many articles (see [2,3,19]).

1. Preliminaries

First, we introduce some notations. Let E be a Banach space. Denote by $L_p(G;E)$ (G is a domain in $\mathbb{R}^n$) the space of strongly measurable functions defined on G with values in E and the finite norm $\|\|u(x)\|_E\|_{L_p(G)}$ [20]. We employ conventional notations for the space of continuously differentiable functions $C^k(\overline{G};E)$ and the Sobolev space $W_p^s(Q;E)$, $W_p^s(G;E)$, etc. (see [20,21]). If $E = \mathbb{C}$ or $E = \mathbb{C}^n$ then the latter space is denoted simply by $W_p^s(G)$. Therefore, the membership $u \in W_p^s(G)$ (or $u \in C^k(\overline{G})$) or $a \in W_p^s(G)$ for a given vector-function $u = (u_1, u_2, \ldots, u_k)$ or a matrix function $a = \{a_{ij}\}_{j,i=1}^k$ mean that every of the components u_i (respectively, an entry a_{ij}) belongs to the space $W_p^s(G)$ (or $C^k(\overline{G})$). Given an interval $J = (0,T)$, put $W_p^{s,r}(Q) = W_p^s(J; L_p(G)) \cap L_p(J; W_p^r(G))$, Respectively, we have $W_p^{s,r}(S) = W_p^s(J; L_p(\Gamma)) \cap L_p(J; W_p^r(\Gamma))$. The anisotropic Hölder spaces $C^{\alpha,\beta}(\overline{Q})$ and $C^{\alpha,\beta}(\overline{S})$ are defined by analogy.

The definition of the inclusion $\Gamma \in C^s$ can be found in [22, Chapter 1]. In what follows we assume that the parameter $p > n+2$ is fixed. Let $B_\delta(x_i)$ be a the ball of radius δ centered at x_i (see (3)). The parameter $\delta > 0$ will be referred to as admissible if $\overline{B_\delta(x_i)} \subset G$ for interior points $x_i \in G$, $\overline{B_\delta(x_i)} \cap \overline{B_\delta(x_j)} = \emptyset$ for $x_i \neq x_j$, $i, j = 1, 2, \ldots, r$, and, for every point $x_i \in \Gamma$, there exists a neighborhood U (the coordinate neighborhood) about this point and a coordinate system y (local coordinate system) obtained by rotation and translation of the origin from the initial one such that the y_n-axis is directed as the interior normal to Γ at x_i and the equation of the boundary $U \cap \Gamma$ is of the form $y_n = \omega(y')$, $\omega(0) = 0$, $|y'| < \delta_0$, $y' = (y_1, \ldots, y_{n-1})$; moreover, we have $\omega \in C^3(\overline{B'_\delta(0)})$ ($B'_\delta(0) = \{y' : |y'| < \delta\}$) end $G \cap U = \{y : |y'| < \delta, 0 < y_n - \omega(y') < \delta_1\}$, $(\mathbb{R}^n \setminus G) \cap U = \{y : |y'| < \delta, -\delta_1 < y_n - \omega(y') < 0\}$. The numbers δ, δ_1 for a given domain G are fixed and without loss of generality we can assume that $\delta_1 > (M+1)\delta$, with M the Lipschitz constant of the function ω. Assume that $Q^\tau = (0,\tau) \times G$, $G_\delta = \cup_i (B_\delta(x_i) \cap G)$, $Q_\delta = (0,T) \times G_\delta$, $Q_\delta^\tau = (0,\tau) \times G_\delta$, $S_\delta = (0,T) \times \cup_i (B_\delta(x_i) \cap \Gamma)$.

Consider the parabolic system

$$Lu = u_t + A(t,x,D)u = f(t,x), \quad (t,x) \in Q = (0,T) \times G, \ G \subset \mathbb{R}^n, \tag{4}$$

where

$$A(t,x,D)u = -\sum_{i,j=1}^{n} a_{ij}(t,x) u_{x_j x_j} + \sum_{i=1}^{n} a_i(t,x) u_{x_i} + a_0(t,x)u,$$

a_{ij}, a_i are matrices of dimension $h \times h$, and u is a vector of length h. The system (4) is supplemented with the initial and boundary conditions (2). We assume that there exists an admissible number $\delta > 0$ such that

$$a_{ij} \in C(\overline{Q}), \quad a_k \in L_p(Q), \quad \gamma_k \in C^{1/2,1}(\overline{S}), \quad a_{ij} \in L_\infty(0,T; W_\infty^1(G_\delta)); \tag{5}$$

$$a_k \in L_p(0,T; W_p^1(G_\delta)), \quad i,j = 1,2,\ldots,n, \ k = 0,1,\ldots,n. \tag{6}$$

The operator L is considered to be parabolic and the Lopatiskii condition holds. State these conditions. Introduce the matrix $A_0(t,x,\xi) = -\sum\limits_{i,j=1}^{n} a_{ij}(t,x)\xi_i\xi_j$ ($\xi \in \mathbb{R}^n$), and assume that there exists a constant $\delta_1 > 0$ such that the roots p of the polynomial

$$\det\big(A_0(t,x,i\xi) + pE\big) = 0$$

(E is the identity matrix) meet the condition

$$\operatorname{Re} p \leqslant -\delta_1 |\xi|^2 \quad \forall \xi \in \mathbb{R}^n \quad \forall (x,t) \in Q. \tag{7}$$

The Lopatinskii condition can be stated as follows: for every point $(t_0, x_0) \in S$ and the operators $A_0(x,t,D)$ and $B_0(x,t,D) = \sum\limits_{i=1}^{n} \gamma_i(t,x)\partial_{x_i}$, written in the local coordinate system y at this point (the axis y_n is directed as the normal to S and the axes $y_1, \ldots, y_{n-1}$ lie in the tangent plane at (x_0, t_0)), the system

$$\big(\lambda E + A_0(x_0, t_0, i\xi', \partial_{y_n})\big)v(z) = 0, \quad B_0(x_0, t_0, i\xi', \partial_{y_n})v(0) = h_j, \tag{8}$$

where $\xi' = (\xi_1, \ldots, \xi_{n-1})$, $y_n \in \mathbb{R}^+$, has a unique solution $C(\overline{\mathbb{R}}^+)$ decreasing at infinity for all $\xi' \in \mathbb{R}^{n-1}$, $|\arg\lambda| \leqslant \pi/2$, and $h_j \in \mathbb{C}$ such that $|\xi'| + |\lambda| \neq 0$.

We also assume that there exists a constant $\varepsilon_1 > 0$ such that

$$Re\,(-A_0(t,x,\xi)\eta,\eta) \geqslant \varepsilon_1|\xi|^2|\eta|^2 \quad \forall\xi \in \mathbb{R}^n,\ \eta \in \mathbb{C}^h, \tag{9}$$

where the brackets $(\cdot,\cdot)$ denote the inner product in $\mathbb{C}^h$ (see [22, Definition 7, Sec. 8, Ch. 7]). Let

$$\Big|det\Big(\sum_{i=1}^{n}\gamma_i\nu_i\Big)\Big| \geqslant \varepsilon_0 > 0, \tag{10}$$

where ν is the outward unit normal to Γ, ε_0 is a positive constant, and

$$u_0(x) \in W_p^{2-2/p}(G),\ g \in W_p^{k_0,2k_0}(S),\ B(x,0)u_0(x)|_\Gamma = g(x,0)\ \forall x \in \Gamma, \tag{11}$$

where $k_0 = 1/2 - 1/2p$. Fix an admissible $\delta > 0$. Construct functions $\varphi_i(x) \in C_0^\infty(\mathbb{R}^n)$ such that $\varphi_i(x) = 1$ in $B_{\delta/2}(x_i)$ and $\varphi_i(x) = 0$ in $\mathbb{R}^n \setminus B_{3\delta/4}(x_i)$ and denote $\varphi(x) = \sum_{i=1}^{r}\varphi_i(x)$. Additionally it is assumed that

$$\varphi(x)u_0(x) \in W_p^{3-2/p}(G),\ \ \varphi g \in W_p^{k_1,2k_1}(S)\ (k_1 = 1 - 1/2p), \tag{12}$$

$$\Gamma \in C^2,\ \gamma_k \in C^{1,2}(\overline{S_\delta})\ (k = 0,1,2,\ldots,n). \tag{13}$$

The proof of the following theorem can be found in [18].

Theorem 1. *Assume that the conditions (5)–(13) hold for some sufficiently small admissible $\delta > 0$ and the function φ, $f \in L_p(Q^\tau)$, $f\varphi \in L_p(0,\tau;W_p^1(G))$, and $\tau \in (0,T]$. Then there exists a unique solution $u \in W_p^{1,2}(Q^\tau)$ to the problem (4), (2). Moreover, $\varphi u_t \in L_p\big(0,\tau;W_p^1(G)\big)$ and $\varphi u \in L_p\big(0,\tau;W_p^3(G)\big)$. If $g \equiv 0$ and $u_0 \equiv 0$ then we have the estimates*

$$\begin{gathered}\|u\|_{W_p^{1,2}(Q^\tau)} \leqslant c\|f\|_{L_p(Q^\tau)},\\ \|u\|_{W_p^{1,2}(Q^\tau)} + \|\varphi u_t\|_{L_p(0,\tau;W_p^1(G))} + \|\varphi u\|_{L_p(0,\tau;W_p^3(G))} \leqslant c\big[\|f\|_{L_p(Q^\tau)} + \|\varphi f\|_{L_p(0,\tau;W_p^1(G))}\big],\end{gathered} \tag{14}$$

where the constant c is independent of f, a solution u, and $\tau \in (0,T]$.

2. Main results

Consider the problem (1)–(3), where

$$A = L_0 - \sum_{k=m+1}^{r} q_k(t)L_k,\ \ L_k u = -\sum_{i,j=1}^{n} a_{ij}^k(t,x)u_{x_jx_j} + \sum_{i=1}^{n} a_i^k(t,x)u_{x_i} + a_0^k(t,x)u,$$

and $k = 0, m+1, m+2, \ldots, r$. The unknowns q_i are sought in the class $C([0,T])$. Construct a matrix $B(t)$ of dimension $r \times r$ with the rows

$$< f_1(t,x_j),e_j >,\ldots,< f_m(t,x_j),e_j >,< L_{m+1}u_0(t,x_j),e_j >,\ldots,< L_r u_0(t,x_j),e_j > .$$

We suppose that

$$\psi_j \in C^1([0,T]),\ \ < u_0(x_j),e_j >= \psi_j(0)\,(j = 1,2,\ldots,r),\ \gamma_l \in C^{1/2,1}(\overline{S}) \cap C^{1,2}(\overline{S_\delta}), \tag{15}$$

$$a_{ij}^k \in C(\overline{Q}) \cap L_\infty(0,T;W_\infty^1(G_\delta)),\ \ a_l^k \in L_p(Q) \cap L_\infty(0,T;W_p^1(G_\delta))\ (i,j = 1,\ldots,n), \tag{16}$$

$$f_i \in L_p(Q) \cap L_\infty(0,T; W_p^1(G_\delta)) \ (i = 0, 1, \dots, m), \tag{17}$$

foe some admissible $\delta > 0$, $p > n + 2$, and $k = 0, m + 1, \dots, r$, $l = 0, 1, \dots n$;

$$a_i^k(t, x_l), f_i(t, x_l) \in C([0,T]) \tag{18}$$

for all possible values of i, k, l. We also need the condition

(C) there exists a number $\delta_0 > 0$ such that

$$|\det B(t)| \geqslant \delta_0 \ \text{ a.e. on } (0,T).$$

Note that the entries of the matrix B belong to the class $C([0,T])$. Consider the system

$$\psi_{jt}(0) + < L_0 u_0(0, x_j), e_j > - < f_0(0, x_j), e_j >= \\ = \sum_{k=1}^{m} q_{0k} < f_k(0, x_j), e_j > + \sum_{k=m+1}^{m_1} q_{0k} < L_k u_0(0, x_j), e_j >, \ j = 1, \dots, r, \tag{19}$$

where the vector $\vec{q}_0 = (q_{01}, q_{02}, \dots, q_{0r})$ is unknown. Under the condition (C), this system is uniquely solvable. Let $A_1 = L_0 - \sum\limits_{k=m+1}^{r} q_{0k} L_k$. Now we can state our main result.

Theorem 2. *Let the conditions* (9)–(13), (C), (15)–(18) *hold. Moreover, we assume that the conditions* (7), (8) *are fulfilled for the operator* $\partial_t + A_1$. *Then there exists a number* $\tau^0 \in (0,T]$ *such that, on the interval* $(0, \tau^0)$, *there exists a unique solution* $(u, q_1, q_2, \dots, q_r)$ *to the problem* (1)–(3) *such that* $u \in L_p(0, \tau^0; W_p^2(G))$, $u_t \in L_p(Q^{\tau^0})$, $q_i(t) \in C([0, \tau^0])$, $i = 1, \dots, r$. *Moreover,* $\varphi u \in L_p(0, \tau^0; W_p^3(G_\delta))$, $\varphi u_t \in L_p(0, \tau^0; W_p^1(G_\delta))$.

Proof. First, we find a solution to the problem

$$\Phi_t + A_1 \Phi = f_0 + \sum_{k=1}^{m} q_{0i} f_i \ \ ((x,t) \in Q), \ \ \Phi|_{t=0} = u_0(x), \ \ B\Phi|_S = g. \tag{20}$$

By Theorem 1, $\Phi \in W_p^{1,2}(Q)$, $\varphi \Phi_t \in L_p(0,T; W_p^1(G))$, $\varphi \Phi \in L_p(0,T; W_p^3(G))$. As a consequence of Theorem III 4.10.2 in [24] and embedding theorems [20, Theorems 4.6.1,4.6.2.], we infer $\varphi\Phi \in C([0,T]; W_p^{3-2/p}(G)) \subset C([0,T]; C^{3-2/p-n/p}(\overline{G}))$. Hence, $\varphi\Phi \in C([0,T]; C^2(G))$ after a possible change on a set of zero measure. The equations (20) and (18) imply that $\Phi_t(t, x_j) \in C([0,T])$. Note that this function is defined, since every summand in (20) with the weight φ belongs to $L_p(0,T; W_p^1(G)) \subset C^\alpha(\overline{G}; L_p(0,T))$ $(\alpha \leqslant 1 - n/p)$ (see the embedding theorems in [25] and the arguments below). Multiply the equation (20) scalarly by e_j and take $x = x_j$. We obtain the equality

$$< \Phi_t(0, x_j), e_j > + < L_0 u_0(0, x_j), e_j > - < f_0(0, x_j), e_j >= \\ = \sum_{k=1}^{m} q_{0k} < f_k(0, x_j), e_j > + \sum_{k=m+1}^{r} q_{0k} < L_k u_0(0, x_j), e_j >, \ j = 1, \dots, r. \tag{21}$$

The relations (19) and (21) imply that $< \Phi_t(0, x_j), e_j >= \psi_{jt}(0)$. After the change of variables $\vec{q} = \vec{q}_0 + \vec{q}_1$ and $u = w + \Phi$ in (1), we arrive at the problem

$$Lw = w_t + A_1 w - \sum_{k=m+1}^{r} q_{1k} L_k w = \sum_{i=1}^{m} f_i q_{1i} + \sum_{i=m+1}^{r} q_{1i} L_i \Phi = F, \ \ w|_{t=0} = 0, \ \ Bw|_S = 0, \tag{22}$$

$$< w(t,x_j), e_j >= \tilde{\psi}_j(t) = \psi_j(t) - < \Phi(t,x_j), e_j > \in C^1([0,T]), \quad \tilde{\psi}_j(0) = \tilde{\psi}_{jt}(0) = 0. \tag{23}$$

Fixing the vector $\vec{q}_1 = (q_{11}, \ldots, q_{1r}) \in C([0,\tau])$ and determining a solution w to the problem (22) on $(0,\tau)$, we construct a mapping $w = w(\vec{q}_1) = L^{-1}F$. Demonstrate that there exists $R_0 > 0$ such that, for $\vec{q}_1 \in B_{R_0}$, the problem

$$Lv = g, \;\; v|_{t=0} = 0, \;\; Bv|_S = 0 \tag{24}$$

for every $g \in H_\tau$ и $\tau \in (0,T]$ has a unique solution in the class $v \in W_p^{1,2}(Q^\tau)$, $\varphi v_t \in L_p(0,\tau; W_p^1(G))$, $\varphi v \in L_p(0,\tau; W_p^3(G))$ satisfying the estimate

$$\|v\|_{W_p^{1,2}(Q^\tau)} + \|\varphi v_t\|_{L_p(0,\tau;W_p^1(G))} + \|\varphi v\|_{L_p(0,\tau;W_p^3(G))} \leqslant c\|g\|_{H_\tau} \tag{25}$$

where the constant c is independent of τ and the vector $\vec{q}_1 \in B_{R_0}$ and the space H_τ is endowed with the norm

$$\|f\|_{H_\tau} = \|f\|_{L_p(Q^\tau)} + \|\varphi f\|_{L_p(0,\tau;W_p^1(Q))}.$$

In accord with Theorem 1, the problem

$$L_{01}v = v_t + A_1 v = g, \;\; v|_{t=0} = 0, \;\; Bv|_S = 0$$

for every $g \in H_\tau$ has a unique solution such that $v \in W_p^{1,2}(Q^\tau)$, $\varphi v_t \in L_p(0,\tau; W_p^1(G))$, $\varphi v \in L_p(0,\tau; W_p^3(G))$ and

$$\|v\|_{W_p^{1,2}(Q^\tau)} + \|\varphi v_t\|_{L_p(0,\tau;W_p^1(G))} + \|\varphi v\|_{L_p(0,\tau;W_p^3(G))} \leqslant c_1\|g\|_{H_\tau}, \tag{26}$$

where the constant c_1 is independent of τ. In this case the question of solvability of the problem (24) is reduced to the same question for the equation

$$f - \sum_{i=m+1}^{r} q_{1i} L_i L_{01}^{-1} f = g, \tag{27}$$

where $f = L_{01}v$. We have the estimate

$$\Big\| - \sum_{i=m+1}^{r} q_{1i} L_i v \Big\|_{H_\tau} \leqslant c\|\vec{q}_1\|_{C([0,\tau])}\big(\|v\|_{W_p^{1,2}(Q^\tau)} + \|\varphi v_t\|_{L_p(0,\tau;W_p^1(G))} + \|\varphi v\|_{L_p(0,\tau;W_p^3(G))}\big), \tag{28}$$

where the constant c depends on the coefficients of the operators L_k in Q and is independent of τ and $\vec{q}_1$. Indeed, the following estimate is obvious

$$\Big\| - \sum_{k=m+1}^{r} q_{1k} L_k v \Big\|_{H_\tau} \leqslant \|\vec{q}_1\|_{C([0,\tau])} \sum_{k=m+1}^{r} \|L_k v\|_{H_\tau}. \tag{29}$$

Estimate the quantity $\|L_k v\|_{H_\tau}$. To this aim, we estimate the norms of every of the summands in this quantity. For example, estimate the norm

$$\|a_{ij}^k v_{x_i x_j}\|_{H_\tau} \leqslant c_0\big(\|a_{ij}^k v_{x_i x_j}\|_{L_p(Q^\tau)} + \sum_{l=1}^{n} \|\varphi (a_{ij}^k v_{x_i x_j})_{x_l}\|_{L_p(Q^\tau)}\big) \leqslant$$

$$\leqslant c_1\big(\|v\|_{L_p(0,\tau;W_p^2(G))} + \|\varphi v\|_{L_p(0,\tau;W_p^3(G))}\big) + \sum_{l=1}^{n} \|\varphi a_{ijx_l}^k v_{x_i x_j}\|_{L_p(Q^\tau)}, \tag{30}$$

where the constant c_1 depends on the norms $\|a^k_{ij}\|_{L_\infty(Q)}$. The last summand here is estimated as follows:

$$\sum_{l=1}^{n}\|\varphi a^k_{ijx_l}v_{x_ix_j}\|_{L_p(Q^\tau)}\leqslant c_2\big(\|\varphi v\|_{L_p(0,\tau;W^2_\infty(G))}+\|v\|_{L_p(0,\tau;W^1_\infty(G))}\big)\leqslant$$
$$\leqslant c_3\big(\|\varphi v\|_{L_p(0,\tau;W^3_p(G))}+\|v\|_{L_p(0,\tau;W^2_p(G))}\big),\quad (31)$$

where the constant c_2 depends on the norms $\|\nabla a^k_{ij}\|_{L_p(0,T;L_\infty(G_\delta))}$. Thus, we infer

$$\|a^k_{ij}v_{x_ix_j}\|_{H_\tau}\leqslant c_4\big(\|v\|_{L_p(0,\tau;W^2_p(G))}+\|\varphi v\|_{L_p(0,\tau;W^3_p(G))}\big),\quad (32)$$

where the constant c_4 is independent of τ. Similarly, we derive that

$$\|a^k_iv_{x_i}\|_{H_\tau}\leqslant c_0\big(\|a^k_iv_{x_i}\|_{L_p(Q^\tau)}+\sum_{l=1}^{n}\|\varphi(a^k_iv_{x_i})_{x_l}\|_{L_p(Q^\tau)}\big)\leqslant$$
$$\leqslant c_1\big(\|\nabla v\|_{L_\infty(Q^\tau)}+\|\varphi v\|_{L_p(0,\tau;W^2_p(G))}\big),\quad (33)$$

where the constant c_1 depends on the norms of $a^k_i, a^k_{ix_l}$ in $L_p(Q)$ and the norms of a^k_i in $L_\infty(Q_\delta)$. However (see Lemma 3.3 in [22]), we have

$$\|\nabla v\|_{L_\infty(Q^\tau)}\leqslant c_1\|v\|_{W^{1,2}_p(Q^\tau)},$$

where the embedding constant is independent of τ. Summing the estimates obtained we justify (28). Using (28) and the estimate of Theorem 1, we conclude that

$$\Big\|\sum_{i=m+1}^{r}q_{1i}L_iL_{01}^{-1}f\Big\|_{H_\tau}\leqslant c_2\|\vec{q}_1\|_{C([0,\tau])}\|f\|_{H_\tau},\quad (34)$$

where c_2 is independent of τ and $\vec{q}_1\in B_{R_0}$. Let $R_0=1/2c_2$. In this case $c_2\|\vec{q}_1\|_{C([0,\tau])}\leqslant 1/2$ and thereby the equation (27) has a unique solution satisfying the estimate $\|f\|_{H_\tau}\leqslant 2\|g\|_{H_\tau}$, which along with Theorem 1 ensures (25).

Assume that w is a solution to the problem (22), (23). Take $x=x_j$ in (22) and multiply the equation scalarly by e_j. The traces of all function occurring into the equation exist. First, our conditions for coefficients and embedding theorems yield $\varphi w\in C([0,T];C^2(\overline{G}))$ (see the above arguments for the function Φ). Second, as we have indicated above, every of the summands in (22) with the weight φ belongs to $L_p(0,T;W^1_p(G))\subset C^\alpha(\overline{G};L_p(0,T))$ ($\alpha\leqslant 1-n/p$) (see embedding theorems in [25]). We arrive at the system

$$<\tilde{\psi}_{jt},e_j>+<A_1w(t,x_j),e_j>-\sum_{i=m+1}^{r}q_{1i}<L_iw(t,x_j),e_j>=$$
$$=\sum_{i=1}^{m}<f_i(t,x_j),e_j>q_{1i}(t)+\sum_{i=m+1}^{r}q_{1i}<L_i\Phi(t,x_j),e_j>\ \ (j=1,2,\dots,r),\quad (35)$$

which can be rewritten in the form

$$\tilde{B}\vec{q}_1=\vec{\psi}+R(\vec{q}_1),$$

where coordinates of the vectors $\vec{\psi}$ and $R(\vec{q}_1)$ agree with the functions $< \tilde{\psi}_{jt}, e_j >$ and $< A_0 w(t,x_j), e_j > - \sum_{i=m+1}^{r} q_{1i} < L_i w(t,x_j), e_j >$ ($w = w(\vec{q}_1)$); respectively, j-th row of the matrix $\tilde{B}(t)$ of dimension $r \times r$ is written as

$$< f_1(t,x_j), e_j >, \ldots, < f_m(t,x_j), e_j >, < L_{m+1}\Phi(t,x_j), e_j >, \ldots, < L_r\Phi(t,x_j), e_j >,$$

where $j = 1, \ldots, r$. This matrix differs from B by the entries $< L_i\Phi(t,x_j), e_j >$. It is easy to prove that this matrix is nondegenerate as well on some segment $[0, \tau_0]$. Indeed, the embedding theorems (see Lemma 3.3 of Chapter 1 in [22]) imply that $\nabla\Phi, \Phi_{x_ix_j} \in C^{\beta/2,\beta}(\overline{Q_{\delta/2}})$ for $\beta < 1 - (n+2)/p$ and all i, j and, therefore,

$$| < L_k\Phi(t,x_j) - L_k u_0(t,x_j), e_j > | \leqslant \sum_{i,k=1}^{n} \sup_{t\in[0,T]} \|a_{ik}^k(t,x_j)\| |\Phi_{x_kx_i}(t,x_j) - u_{0x_kx_i}(x_j)| +$$
$$+ \sum_{i=1}^{n} \sup_{t\in[0,T]} \|a_i^k(t,x_j)\| |\Phi_{x_i}(t,x_j) - u_{0x_i}(x_j)| + \sup_{t\in[0,T]} \|a_0^k(t,x_j)\| |\Phi(t,x_j) - u_0(x_j)| \leqslant ct^{\beta/2},$$

on $[0,T]$, where, by the norm of a matrix (for example, $\|a_i^k(t,x_j)\|$), we mean the norm of the corresponding linear operator $a_i^k(t,x_j) : \mathbb{C}^h \to \mathbb{C}^h$. Taking the condition (C) into account, we can say that there exists $\tau_0 > 0$ such that

$$|\det\tilde{B}(t)| \geqslant \delta_0/2 \quad \forall t \leqslant \tau_0. \tag{36}$$

We thus obtain the integral equation

$$\vec{q}_1 = \tilde{B}^{-1}\vec{\psi} + R_0(\vec{q}_1), \quad R_0(\vec{q}_1) = \tilde{B}^{-1}R(\vec{q}_1), \tag{37}$$

where the operator $R_0(\vec{q}_1) : C([0,\tau]) \to C([0,\tau])$ ($\tau \leqslant \tau_0$) is bounded. Check the conditions of the fixed point theorem. Denote $R_{0\tau} = 2\|\tilde{B}^{-1}\vec{\psi}\|_{C([0,\tau])}$. Let $\vec{q}_{01}, \vec{q}_{02}$ be two vectors of length r with coordinates q_i^j ($i = 1, 2, \ldots, r,\ j = 1, 2$) lying in the ball $B_{R_0} = \{\vec{q} : \|\vec{q}\|_{C([0,\tau])} \leqslant R_0\}$. The functions $w_1 = w(\vec{q}_{01}), w_2 = w(\vec{q}_{02})$ are solutions to the equation (22) satisfying homogeneous initial and boundary conditions. Let $v = w_1 - w_2$. We infer

$$Lv = v_t + A_1 v - \sum_{i=m+1}^{r} q_i^2 L_i v = \sum_{i=1}^{m} f_i(q_i^1 - q_i^2) + \sum_{i=m+1}^{r} (q_i^1 - q_i^2) L_i w_1, \quad v = w_1 - w_2. \tag{38}$$

In view of (23) and the definition of $R_{0\tau}$, $R_{0\tau} \to 0$ as $\tau \to 0$. Hence, there exists a parameter $\tau_1 \leqslant \tau_0$ such that, for $\tau \leqslant \tau_1$, $R_{0\tau} \leqslant R_0$. Let $R = R_{0\tau_1}$. We now derive that there exists a parameter $\tau^0 \leqslant \tau_1$ such that the equation (37) has a unique solution in the ball B_R of the space $C([0,\tau^0])$. Take $\tau \leqslant \tau_1$. Let $\vec{q}_{01}, \vec{q}_{02} \in B_R$. We have

$$\|R_0(\vec{q}_{01}) - R_0(\vec{q}_{02})\|_{C([0,\tau])} \leqslant c_1 \|R(\vec{q}_{01}) - R(\vec{q}_{02})\|_{C([0,\tau])} \leqslant$$
$$\leqslant c_2 \sum_{j=1}^{r} (\|L_0 v(t,x_j)\|_{C([0,\tau])} + \sum_{i=m+1}^{r} \|q_i^2 L_i v(t,x_j)\|_{C([0,\tau])}) \leqslant$$
$$\leqslant c_3 \sum_{j=1}^{r} (\|L_0 v(t,x_j)\|_{C([0,\tau])} + \sum_{i=m+1}^{r} \|L_i v(t,x_j)\|_{C([0,\tau])}), \tag{39}$$

where v is a solution to the problem (38). Note that

$$\|L_k v(t,x_j)\|_{C([0,\tau])} \leqslant c\tau^{\beta} (\|\varphi\nabla v\|_{W_p^{1,2}(Q^\tau)} + \|v\|_{W_p^{1,2}(Q^\tau)}), \tag{40}$$

where the constant c is independent of $\tau \in (0,T]$ and $\beta > 0$. Validate this inequality. In view of the conditions on the coefficients a_{il}^k, $a_{il}^k(t,x_j) \in C([0,T])$. Fix an arbitrary $s \in (n/p, 1-2/p)$. The embedding $W_p^s(G_{\delta/2}) \subset C(\overline{G_{\delta/2}})$ [20, Theorems 4.6.1,4.6.2.] yields

$$\|a_{il}^k(t,x_j)v_{x_ix_l}(t,x_j)\|_{C([0,\tau])} \leqslant c\|v_{x_ix_l}(t,x_j)\|_{C([0,\tau])} \leqslant c_1\|v_{x_kx_l}(t,x)\|_{L_\infty(0,\tau;W_p^s(G_{\delta/2}))} \leqslant \\ \leqslant c_2\|\nabla v(t,x)\|_{L_\infty(0,\tau;W_p^{1+s}(G_{\delta/2}))}. \quad (41)$$

Next, we employ the interpolation inequality (see [20])

$$\|v\|_{W_p^{s_0}(G)} \leqslant c\|v\|^\theta_{W_p^{s_1}(G)}\|v\|^{1-\theta}_{W_p^{s_2}(G)}, \quad s_1 < s_0 < s_2, \ \theta s_1 + (1-\theta)s_2 = s_0 \quad (42)$$

and the inequality

$$\|g\|_{L_\infty(0,\tau;E)} \leqslant \tau^{(p-1)/p}\|g_t\|_{L_p(0,\tau;E)}, \quad \forall g \in W_p^1(0,\tau;E), \ g(0) = 0, \quad (43)$$

resulting from the Newton-Leibnitz formula. Here E is a Banach space. We obtain that

$$\|\nabla v(t,x)\|_{L_\infty(0,\tau;W_p^{1+s}(G_{\delta/2}))} \leqslant c\|\nabla v(t,x)\|^\theta_{L_\infty(0,\tau;W_p^{2-2/p}(G_{\delta/2}))}\|\nabla v(t,x)\|^{(1-\theta)}_{L_\infty(0,\tau;L_p(G_{\delta/2}))} \leqslant \\ \leqslant c_1\tau^{(1-\theta)(p-1)/p}(\|\varphi\nabla v\|_{W_p^{1,2}(Q)} + \|v\|_{W_p^{1,2}(Q)}), \ (2-2/p)\theta = 1+s. \quad (44)$$

Here we have used the inequality

$$\|\nabla v(t,x)\|_{L_\infty(0,\tau;W_p^{2-2/p}(G_{\delta/2}))} \leqslant c\|\nabla v(t,x)\|_{W_p^{1,2}(G_{\delta/2}))}, \quad (45)$$

where the constant c is independent of τ (in the class of functions vanishing at $t=0$). Estimate the lower-order summands of the form $a_i^k v_{x_i}(t,x_j)$, $a_0^k v(t,x_j)$ in $L_i u(t,x_j)$. We conclude that ($s \in (n/p, 1-2/p)$, $(2-2/p)\theta_1 = 1+s$)

$$\|a_i^k v_{x_i}(t,x_j)\|_{C([0,\tau])} \leqslant c\|v_{x_i}(t,x_j)\|_{C([0,\tau])} \leqslant c_1\|v(t,x)\|_{L_\infty(0,\tau;W_p^{1+s}(G_{\delta/2}))} \leqslant \\ \leqslant \|v(t,x)\|^{\theta_1}_{L_\infty(0,\tau;W_p^{2-2/p}(G_{\delta/2}))}\|v(t,x)\|^{1-\theta_1}_{L_\infty(0,\tau;L_p(G_{\delta/2}))} \leqslant c_2\tau^{(1-\theta_1)(p-1)/p}\|v\|_{W_p^{1,2}(Q^\tau)}. \quad (46)$$

We have used the estimate (45) applied to v rather than ∇v. The second summand is estimated similarly. The estimates (39)–(46) ensure that

$$\|R_0(\vec{q}_{01}) - R_0(\vec{q}_{02})\|_{C([0,\tau])} \leqslant c_4\tau^\beta(\|\varphi\nabla v(t,x)\|_{W_p^{1,2}(Q^\tau)} + \|v(t,x)\|_{W_p^{1,2}(Q^\tau)}), \quad (47)$$

where the constant c_4 is independent of τ and $\beta = \min(1-\theta, (1-\theta_1)(p-1)/p)$. Since v is a solution to the problem (38) and $\tau \leqslant \tau_1$, we can employ (25) and obtain that

$$\|\varphi\nabla v(t,x)\|_{W_p^{1,2}(Q^\tau)} + \|v(t,x)\|_{W_p^{1,2}(Q^\tau)} \leqslant c\Big\|\sum_{i=1}^m f_i(q_i^1 - q_i^2) + \sum_{i=m+1}^r (q_i^1 - q_i^2)L_i w_1\Big\|_{H_\tau}, \quad (48)$$

where the constant c is independent of τ. Every of the functions w_1, w_2 is a solution to the problem (22), where the right-hand side contains the components of the vector $\vec{q}_{01}$ or $\vec{q}_{02}$. The estimate (25) yields

$$\|\varphi\nabla w_j(t,x)\|_{W_p^{1,2}(Q^\tau)} + \|w_j(t,x)\|_{W_p^{1,2}(Q^\tau)} \leqslant c\Big\|\sum_{i=1}^m f_i q_i^j + \sum_{i=m+1}^r q_i^j L_i\Phi\Big\|_{H_\tau}. \quad (49)$$

The estimate (48), (49) and the conditions on the coefficients imply that

$$\|\varphi\nabla w_j(t,x)\|_{W_p^{1,2}(Q^\tau)} + \|w_j(t,x)\|_{W_p^{1,2}(Q^\tau)} \leqslant c_1(R). \tag{50}$$

$$\|\varphi\nabla v(t,x)\|_{W_p^{1,2}(Q^\tau)} + \|v(t,x)\|_{W_p^{1,2}(Q^\tau)} \leqslant c_2\|\vec{q}_{01} - \vec{q}_{02}\|_{C([0,\tau])}, \tag{51}$$

where the constant c_i are independent of τ. In turn, these estimates and those in (47) validate the estimate

$$\|R_0(\vec{q}_{01}) - R_0(\vec{q}_{02})\|_{C([0,\tau])} \leqslant c_5\tau^\beta\|\vec{q}_{01} - \vec{q}_{02}\|_{C([0,\tau])} \tag{52}$$

with a constant c_5 independent of τ. Choose a parameter $\tau^0 \leqslant \tau_1$ such that $c_5(\tau^0)^\beta \leqslant 1/2$. The fixed point theorem ensures solvability of the equation (37) in the ball B_R.

Show that w satisfies the overdetermination conditions (23). Multiply the equation (22) scalarly by e_j and take $x = x_j$ in the equation. We obtain the equality

$$< w(t,x_j), e_j >_t + < L_0 w(t,x_j), e_j > - \sum_{i=m+1}^{r} q_i < L_i w(t,x_j), e_j >=$$
$$= \sum_{i=1}^{m} < f_i(t,x_j), e_j > q_i(t) + \sum_{i=m+1}^{r} q_i < L_i\Phi(t,x_j), e_j >, \quad j = 1,2,\dots,r, \tag{53}$$

Subtracting this equality from (35), we obtain that $\tilde{\psi}_{j_t} - < w(t,x_j), e_j >_t = 0$. Integrating this equality from 0 to t, we derive that $\tilde{\psi}_j(t) - < w(t,x_j), e_j >= 0$, since the agreement conditions imply that $\tilde{\psi}_j(0) = 0$, $< w(0,x_j), e_j >= 0$. Thus, we infer $\tilde{\psi}_j(t) =< w(t,x_j), e_j >$ and the equality (23) holds. □

In the case of the unknown lower-order coefficients, the results can be reformulated in a more convenient form. In this case the operator A is assumed to be representable in the form

$$A = L_0 - \sum_{i=m+1}^{r} q_i(t) l_i, \quad L_0 u = -\sum_{i,j=1}^{n} a_{ij}(t,x) u_{x_j x_j} + \sum_{i=1}^{n} a_i(t,x) u_{x_i} + a_0(t,x) u,$$
$$l_i u = \sum_{j=1}^{n} b_{ij}(t,x) u_{x_j} + b_{i0}(t,x) u. \tag{54}$$

Moreover, the rows of the matrix $B(t)$ of dimension $r \times r$ are as follows:

$$< f_1(t,x_i), e_i >, \dots, < f_m(t,x_i), e_i >, < l_{m+1} u_0(t,x_i), e_i >, \dots, < l_r u_0(t,x_i), e_i > .$$

We suppose that

$$\psi_j \in W_p^1(0,T), \quad < u_0(x_j), e_j >= \psi_j(0), \quad j = 1,2,\dots,r, \tag{55}$$

$$f_i, b_{kj} \in L_\infty(0,T;W_p^1(G_\delta)) \cap L_\infty(0,T;L_p(G)), \quad f_0 \in L_p(Q) \cap L_p(0,T;W_p^1(G_\delta)), \tag{56}$$

for some admissible $\delta > 0$, where $i = 1,\dots,m$, $j = 0,1,\dots,n$, $k = m+1,\dots,r$. The remaining coefficients satisfy the conditions

$$a_{ij} \in C(\overline{Q}), \quad a_k \in L_p(Q), \quad \gamma_k \in C^{1/2,1}(\overline{S}) \cap C^{1,2}(\overline{S_\delta}), \quad a_{ij} \in L_\infty(0,T;W_\infty^1(G_\delta)); \tag{57}$$

$$a_k \in L_p(Q) \cap L_p(0,T;W_p^1(G_\delta)), \; i,j = 1,2,\dots,n, \; k = 0,1,\dots,n. \tag{58}$$

The corresponding theorem is stated in the following form.

Theorem 3. *Assume that the parabolicity condition and the Lopatinskii condition* (7), (8) *for the operator* $\partial_t + L_0$*, the conditions* (9)–(13), (55)–(58), (C) *for some admissible* $\delta > 0$ *and* $p > n + 2$ *hold. Then, for some* $\gamma_0 \in (0, T]$*, on the interval* $(0, \gamma_0)$*, there exists a unique solution* $(u, q_1, q_2, \dots, q_r)$ *to the problem* (1)–(3) *such that* $u \in L_p(0, \gamma_0; W_p^2(G))$*,* $u_t \in L_p(Q^{\gamma_0})$*,* $\varphi u \in L_p(0, \gamma_0; W_p^3(G))$*,* $\varphi u_t \in L_p(0, \gamma_0; W_p^1(G))$*,* $q_i(t) \in L_p(0, \gamma_0)$*,* $i = 1, \dots, r$*.*

The proof is omitted, since it is quite similar to that of the previous theorem.

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О некоторых классах параболических обратных задач с точечным переопределением

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Аннотация. В работе рассматривается вопрос о корректности в пространствах Соболева обратных задач о восстановлении коэффициентов параболической системы, зависящих от времени. В качестве условий переопределения рассматриваются значения решения в некотором наборе точек области, лежащих как внутри области, так и на ее границе. Приведены условия, гарантирующие существование и единственность решений задачи в классах Соболева.

Ключевые слова: параболическая система, обратная задача, конвекция-диффузия, точечное переопределение.