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Asymptotic Behavior at Infinity of the Dirichlet Problem Solution of the 2k Order Equation in a Layer

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For the operator $(-\Delta)^k u(x) + \nu^{2k} u(x)$ with $x \in R^n (n \geqslant 2, k \geqslant 2)$ an explicit fundamental solution is obtained, and for the equation $(-\Delta)^k u(x) + \nu^{2k} u(x) = f(x)$ (for $f \in C^\infty(R^n)$ with compact support) the leading term of an asymptotic expansion at infinity of a solution is computed. The same result is obtained for the solution of the Dirichlet problem in a layer in R^{n+1}.

Keywords: asymptotic behavior, elliptic equation, fundamental solution, estimation of solution, G-Meyer function.

A fundamental solution for the operator $(-\Delta)^k u(x) + \nu^{2k} u(x)$, $x \in R^n (n \geqslant 2, k \geqslant 2)$, is obtained in [1, Section 3, 2.8.3]. The leading term of an asymptotic expansion at infinity exponentially decreases and does not contain a rapidly oscillating factor.

In [2] the general form of all solutions of the equation

$$\Delta^k U + a_1 \Delta^{k-1} U + ... + a_k U = 0,$$

in a domain is deduced. Here $a_1, a_2, ..., a_k$ are complex constants.

In [3] a fundamental solution for the operator

$$(-\Delta)^k u(x) + \mu u(x),$$

satisfying the radiation condition, is considered.

Let $\tilde{h}(\xi)$ be the Fourier transform of the function $h(x) \in L_1(R^n)$

$$\tilde{h}(\xi) = F[h](\xi) = \int_{R^n} e^{-2\pi \imath x \xi} h(x) dx.$$

If $g \in L_1(R^n)$, then the inverse Fourier transform is

$$F^{-1}[g](x) = \int_{R^n} e^{2\pi \imath x \xi} g(\xi) d\xi.$$

Lemma. *The equation*

$$(-\Delta)^k E(x) + \nu^{2k} E(x) = \delta(x) \quad x \in R^n \quad (n \geq 2, \quad \nu > 0) \tag{1}$$

has a radially symmetric solution $E(r)$ of the form

$$E(r) = \frac{(2k)^{(n-4)/2}}{(2\pi)^{n/2} \nu^{2k-2} r^{n-2}} \times G_{0,2k}^{k+1,0} \left(\left(\frac{\nu r}{2k} \right)^{2k} \Big|_{\frac{n-2}{2k}, \frac{n-2}{2k} + \frac{1}{k}, ..., \frac{n-2}{2k} + \frac{k-1}{k}, \frac{k-1}{k}, \frac{k-2}{k}, ..., \frac{1}{k}, 0} \right), \tag{2}$$

here $r = \sqrt{x_1^2 + x_2^2 + ... + x_n^2}$, $G(\cdot)$ is the G-Meyer function (in the notation of [4]).

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Proof. The Fourier transform $\tilde{E}(\xi)$ of the fundamental solution $E(x)$ of the equation (1) is a solution of the equation

$$((2\pi\rho)^{2k} + \nu^{2k})\tilde{E}(\xi) = 1.$$

Hence,

$$\tilde{E}(\xi) \equiv \tilde{E(\rho)} = \frac{1}{(2\pi\rho)^{2k} + \nu^{2k}},$$

where $\rho = \sqrt{\xi_1^2 + \xi_2^2 + ... + \xi_n^2}$.

So

$$E(x)) = \int_{R^n} \frac{e^{2\pi \imath x\xi}}{(2\pi\rho)^{2k} + \nu^{2k}} d\xi.$$

Using the formulas for the inverse Fourier transform of the radially symmetric function $\Psi(\rho)$ (see [5, (2.113), (2.114)]), we have:

for $n = 2p$

$$E(x)) = (-2\pi)^{-(p-1)} \lim_{\varepsilon\to+0} \left(\frac{1}{r}\frac{\partial}{\partial r}\right)^{p-1} \left[2\pi \int_0^{+\infty} e^{-\varepsilon\rho}\Psi(\rho)\rho J_0(2\pi r\rho)d\rho\right], \tag{3}$$

for $n = 2p + 1$

$$E(x)) = (-2\pi)^{-(p-1)} \lim_{\varepsilon\to+0} \left(\frac{1}{r}\frac{\partial}{\partial r}\right)^{p-1} \left[\frac{2}{r} \int_0^{+\infty} e^{-\varepsilon\rho}\Psi(\rho)\rho \sin(2\pi r\rho)d\rho\right]. \tag{4}$$

Notice that in our case for $\varepsilon = 0$ the integrals in square brackets in formulas (3), (4) converge. Therefore, we can consider formally the following expressions, although it is not yet proved that they define a solution of (1):

for $n = 2p$

$$E(x) = (-2\pi)^{-(p-1)} \left(\frac{1}{r}\frac{\partial}{\partial r}\right)^{p-1} \left[2\pi \int_0^{+\infty} \frac{\rho}{(2\pi\rho)^{2k} + \nu^{2k}} J_0(2\pi r\rho)d\rho\right], \tag{5}$$

for $n = 2p + 1$

$$E(x) = (-2\pi)^{-(p-1)} \left(\frac{1}{r}\frac{\partial}{\partial r}\right)^{p-1} \left[\frac{2}{r} \int_0^{+\infty} \frac{\rho}{(2\pi\rho)^{2k} + \nu^{2k}} \sin(2\pi r\rho)d\rho\right]. \tag{6}$$

Consider the even dimension case. We calculate the integral on the right hand side of (5) at $k = 1, 2, 3, 4, ...$, using Mathematica (the licence L 3298-0846), and construct for it the following expression for any k:

$$\int_0^{+\infty} \frac{\rho}{(2\pi\rho)^{2k} + \nu^{2k}} J_0(2\pi r\rho)d\rho = \frac{1}{8k\pi^2\nu^{2k-2}} G_{0,2k}^{k+1,0}\left(\frac{\nu^{2k}r^{2k}}{(2k)^{2k}}\bigg|_{\frac{k-1}{k},\frac{k-2}{k},...,0,\frac{k-1}{k},\frac{k-2}{k},...,\frac{1}{k},0}\right). \tag{7}$$

Substituting it in (5) and using the formula for differentiation of the G-Meyer function (see [6, 8.2.2.32]) we obtain the function on the right hand side of the expression (2). Let us prove that this is the expression for the fundamental solution.

Using the well-known asymptotic expansion of the G-Meyer function, we put

$$G_{0,q}^{m,0}(z) \equiv G_{0,q}^{m,0}\left(z|_{b_1,b_2,...,b_q}\right).$$

From Theorem 2 (see [7, 5.9.2]) it follows that (for $m \leqslant q-1, \arg z = 0, z \to +\infty$):

$$G_{0,q}^{m,0}(z) \sim A_q^{m,0} H_{p,q}(ze^{\imath\pi(q-m)}) + \bar{A}_q^{m,0} H_{p,q}(ze^{-\imath\pi(q-m)}).$$

In the case under consideration $m = k+1, p = 0, q = 2k$.

$$H_{0,2k}(\zeta) = \frac{(2\pi)^{\frac{2k-1}{2}}}{(2k)^{\frac{1}{2}}} \exp\big(-2k\zeta^{\frac{1}{2k}}\big)\zeta^{\frac{2k-1}{4k}} \sum_{j=0}^{\infty} M_j \zeta^{\frac{-j}{2k}},$$

where $M_0 = 1$ ([7, (5.9.1.13)]),

$$A_{2k}^{k+1,0} = (-1)^{k-1}(2\pi\imath)^{-(k-1)} \exp\big(-\imath\pi \sum_{j=k+1}^{2k} b_j\big)$$

([7, 5.8.2.2]). Finally, for the G-Meyer function in (7) we obtain

$$G_{0,2k}^{k+1,0}\left(\frac{\nu^{2k}r^{2k}}{(2k)^{2k}}\Big|_{\frac{k-1}{k},\frac{k-2}{k},\dots,0,\frac{k-1}{k},\frac{k-2}{k},\dots,\frac{1}{k},0}\right) \sim \frac{2\sqrt{2\pi}}{\sqrt{\nu r}} \exp\big(-\nu r \sin\frac{\pi}{2k}\big)\cos\big(\frac{3\pi(k-1)}{4k} - \nu r\cos\frac{\pi}{2k}\big).$$

The exponential decrease ensures the convergence of the integrals and the applicability to the right hand side of (2) of the formula ([5, (2.108)]) for the Fourier transform of the radially symmetric function:

$$\Psi(\rho) = \frac{2\pi}{\rho^{(n-2)/2}} \int_0^{+\infty} \Phi(r) r^{n/2} J_{(n-2)/2}(2\pi\rho r)dr. \tag{8}$$

In our case it looks like

$$\Psi(\rho) = \frac{(2k)^{\frac{n}{2}-2}}{(2\pi\rho)^{\frac{n}{2}-1}\nu^{2k-2}} \int_0^{+\infty} r^{1-\frac{n}{2}} J_{\frac{n-2}{2}}(2\pi\rho r)\times$$

$$\times G_{0,2k}^{k+1,0}\left(\left(\frac{\nu r}{2k}\right)^{2k}\Big|_{\frac{n-2}{2k},\frac{n-2}{2k}+\frac{1}{k},\dots,\frac{n-2}{2k}+\frac{k-1}{k},\frac{k-1}{k},\frac{k-2}{k},\dots,\frac{1}{k},0}\right)dr.$$

This integral (see [6, 2.24.4.1]) is expressed via the G-Meyer function and simplified to the form

$$\Psi(\rho) = \frac{1}{(2\pi\rho)^2\nu^{2k-2}} G_{1,1}^{1,1}\left(\left(\frac{\nu}{2\pi\rho}\right)^{2k}\Big|_{\frac{k-1}{k}}^{\frac{k-1}{k}}\right).$$

Using the integral representation of the G-Meyer function (([6, Definition 8.2.1]), where the integration contour L is the straight line $(\frac{k-2}{2k} - \imath\infty, \frac{k-2}{2k} + \imath\infty))$ and formula ([8, 3.981(3)]), we find

$$G_{1,1}^{1,1}\left(z\Big|_{\frac{k-1}{k}}^{\frac{k-1}{k}}\right) = \frac{z^{\frac{2k-2}{2k}}}{1+z}.$$

From here it follows that

$$\frac{1}{(2\pi\rho)^2\nu^{2k-2}} G_{1,1}^{1,1}\left(\left(\frac{\nu}{2\pi\rho}\right)^{2k}\Big|_{\frac{k-1}{k}}^{\frac{k-1}{k}}\right) = \frac{1}{(2\pi\rho)^{2k} + \nu^{2k}}. \tag{9}$$

Finally, the statement is proved for even n.

Let us prove the lemma for odd n. First, for $n = 3$ we substitute (9) into the inverse Fourier transform formula (formula (8), where ρ and r are interchanged). We obtain the integral (see [6, 2.24.4.1]) and substitute the result in (6). Now we proceed similarly to the case of even n. The lemma is proved. □

Let us specify the leading term of the asymptotic expansion of $u(x)$ at infinity.

Theorem 1. *Let $x \in R^n$ $(n \geqslant 2,\ \nu > 0)$, $f(x)$ be a smooth function with compact support. Let the solution $u(x)$ of the equation*

$$(-\Delta)^k u(x) + \nu^{2k} u(x) = f(x) \tag{10}$$

exponentially decrease at infinity. Then the following representation holds

$$u(x) = r^{(1-n)/2} \sin\left(\nu r \cos\frac{\pi}{2k}\right) e^{-\nu r \sin\frac{\pi}{2k}} \Phi_1(\theta_1, \ldots, \theta_{n-1}) +$$

$$+ r^{(1-n)/2} \cos\left(\nu r \cos\frac{\pi}{2k}\right) e^{-\nu r \sin\frac{\pi}{2k}} \Phi_2(\theta_1, \ldots, \theta_{n-1}) + O(r^{-(n+1)/2} e^{-\nu r \sin\frac{\pi}{2k}}), \tag{11}$$

where $\Phi_1(\theta_1, \ldots, \theta_{n-1}), \Phi_2(\theta_2, \ldots, \theta_{n-1})$ are differentiable functions on the unit sphere.

Proof. Let $f(x)$ have its support in a ball Q_R of radius R . Then for the solution $u(x)$ of the equation (10) we have the following representation

$$u(x) = \int_{R^n} E(|x-y|) f(y) dy.$$

Introduce the following notation $x = (x_1, x_2, ..., x_n)$. Suppose that x runs along a ray, we can turn the coordinate system so that this ray coincides with $x_1 > 0$, $x_2 = \ldots = x_n = 0$. Then

$$u(x_1, 0, \ldots, 0) = \int_{R^n} E\left(\sqrt{(x_1 - y_1)^2 + y_2^2 + \ldots + y_n^2}\right) f(y) dy =$$

$$= \int_{Q_R} E\left(|(x_1 - y_1|\right) f(y) dy + \int_{Q_R} (E\left(\sqrt{(x_1 - y_1)^2 + y_2^2 + \ldots + y_n^2}\right) - E(|(x_1 - y_1|)) f(y) dy.$$

Denote the integrals on the right hand side of the this equality by J_1 and J_2.

For $E(r)$ we employ the asymptotic expansion

$$E(r) \sim \frac{2\nu^{\frac{n+1-4k}{2}}}{(2\pi r)^{\frac{n-1}{2}}} e^{-\nu r \sin\frac{\pi}{2k}} \cos\left(\frac{\pi(n+1)(k-1)}{4k} - \nu r \cos\frac{\pi}{2k}\right) \qquad r \to +\infty, \tag{12}$$

and obtain, as $x_1 \to +\infty$,

$$J_1 = \int_{Q_R} (|x_1 - y_1|^{(1-n)/2} \sin\left(\nu |x_1 - y_1| \cos\frac{\pi}{2k}\right) e^{-\nu |x_1 - y_1| \sin\frac{\pi}{2k}} c_1 +$$

$$+ |x_1 - y_1|^{(1-n)/2} \cos\left(\nu |x_1 - y_1| \cos\frac{\pi}{2k}\right) e^{-\nu |x_1 - y_1| \sin\frac{\pi}{2k}} c_2 +$$

$$+ O(|x_1 - y_1|^{-(n+1)/2} e^{-\nu |x_1 - y_1| \sin\frac{\pi}{2k}})) f(y) dy =$$

$$= x_1^{(1-n)/2} \sin\left(\nu x_1 \cos\frac{\pi}{2k}\right) e^{-\nu x_1 \sin\frac{\pi}{2k}} c_3 + x_1^{(1-n)/2} \cos\left(\nu x_1 \cos\frac{\pi}{2k}\right) e^{-\nu x_1 \sin\frac{\pi}{2k}} c_4 +$$

$$+ O(x_1^{-(n+1)/2} e^{-\nu x_1 \sin\frac{\pi}{2k}}),$$

where c_1, c_2, c_3, c_4 are constants.

Turn now to the estimation of J_2.

Using (12) and the mean value theorem we arrive at the following inequalities for some $0 < \Theta < 1$:

$$\left| E\left(\sqrt{(x_1 - y_1)^2 + y_2^2 + \ldots + y_n^2}\right) - E\left(|x_1 - y_1|\right) \right| =$$

$$= \left| E'\left(|x_1 - y_1| + \Theta\left(\sqrt{(x_1 - y_1)^2 + y_2^2 + \ldots + y_n^2} - |x_1 - y_1|\right)\right) \right| \times$$

$$\times \left(\sqrt{(x_1-y_1)^2 + y_2^2 + \ldots + y_n^2} - |x_1-y_1| \right) \leqslant$$

$$\leqslant C_0 e^{-\nu |x_1-y_1| \sin\frac{\pi}{2k}} |x_1-y_1|^{-\frac{n+1}{2}} \leqslant C_1 e^{-\nu x_1 \sin\frac{\pi}{2k}} x_1^{-\frac{n+1}{2}}$$

if $|y| \leqslant R,\ \ x_1 \geqslant 2R$.

Therefore

$$|J_2| \leqslant C e^{-\nu x_1 \sin\frac{\pi}{2k}} x_1^{-\frac{n+1}{2}},$$

where C_0, C_1, C are constants.

Going back to initial coordinates, we get the representation (11). □

We shall apply the obtained results to the Dirichlet problem in a layer.

Denote

$$\Pi = \{(x, x_{n+1}) \in R^{n+1} | x \in R^n, x_{n+1} \in (a,b)\}, \quad -\infty < a < b < +\infty,\ n \geqslant 2.$$

Consider the problem

$$\left\{ \begin{array}{rcl} (-1)^k \left(\dfrac{\partial^{2k}}{\partial x_{n+1}^{2k}} + \left(\sum\limits_{j=1}^{n} \dfrac{\partial^2}{\partial x_j^2} \right)^k \right) v + \alpha v &=& h, \quad (x, x_{n+1}) \in \Pi, \\ \dfrac{\partial^j v}{\partial x_{n+1}^j}\Big|_{x_{n+1}=a} = \dfrac{\partial^j v}{\partial x_{n+1}^j}\Big|_{x_{n+1}=b} &=& 0, \quad j = 0, ..., k-1. \end{array} \right. \tag{13}$$

Let $0 < \lambda_1 < \lambda_2 < ...$ be the eigenvalues and $\varphi_l, (l = 1, 2, ...)$ be the eigenfunctions of the problem

$$\left\{ \begin{array}{rcl} y^{(2k)}(t) + (-1)^{k+1} \lambda^{2k} y(t) &=& 0, \quad t \in (a,b), \\ y^{(j)}(a) = y^{(j)}(b) &=& 0, \quad j = 0, ..., k-1. \end{array} \right. \tag{14}$$

Put $\mu_l = \sqrt[2k]{\alpha + \lambda_l^{2k}} \quad (l = 1, 2, ...)$.

In[9] (Theorem 6) the solvability of the problem (13) and uniqueness of the solution v was proved for $h(x, x_{n+1}) \in C^\infty(\bar{\Pi})$ with compact support and $\alpha + \lambda_l^{2k} > 0$, as well as the estimate

$$|v(x, x_{n+1})| \leqslant C e^{-(\mu_1 \sin\frac{\pi}{2k} - \varepsilon)|x|}, \quad (x, x_{n+1}) \in \Pi, \tag{15}$$

(here $\varepsilon > 0$ is sufficiently small).

Study the proof of the estimate (15) more closely. Denote by $\tilde{v} \equiv \tilde{v}(\xi_1, ..., \xi_n, x_{n+1})$ the Fourier transform with respect to x of the function $v(x, x_{n+1})$. Then $\tilde{v}$ is a solution of the one-dimensional boundary value problem in x_{n+1} on $[a,b]$ with the parameters $\xi_1, ..., \xi_n$:

$$\left\{ \begin{array}{rcl} (-1)^k \tilde{v}^{2k} + \alpha \tilde{v} + (2\pi)^{2k} (\xi_1^2 + ... + \xi_n^2)^k \tilde{v} &=& F[h], \quad x_{n+1} \in (a,b), \\ \tilde{v}^{(j)}(a) = \tilde{v}^{(j)}(b) &=& 0, \quad j = 0, ..., k-1. \end{array} \right.$$

The singular sets in this case are given by the conditions

$$-\mu_l^{2k} = (2\pi)^{2k} (\xi_1^2 + ... + \xi_n^2)^k, \quad l = 1, 2, 3, \ldots. \tag{16}$$

Put

$$\zeta_j = \mathrm{Re}\,\xi_j, \quad \tau_j = \mathrm{Im} \quad \xi_\mathrm{j} \quad (\mathrm{j} = 1, ..., \mathrm{n}), \quad \zeta = (\zeta_1, ..., \zeta_\mathrm{n}), \quad \tau = (\tau_1, ..., \tau_\mathrm{n}).$$

To apply Theorem 6 of [9], the intersection of the cylinder $(2\pi)^2 |\tau|^2 = \gamma^2$ with the singular sets (16) must be empty. We shall find γ, for which this condition is fulfilled, that is the following system has no solutions:

$$\left\{ \begin{array}{rcl} (2\pi)^2 |\tau|^2 &=& \gamma^2, \\ (2\pi)^{2k} (\xi_1^2 + ... + \xi_n^2)^k &=& -\mu_l^{2k} \quad (l = 1, 2, 3, ...). \end{array} \right. \tag{17}$$

This system splits into k systems of the form

$$\begin{cases} (2\pi)^2|\tau|^2 &= \gamma^2, \\ (2\pi)^2(\xi_1^2+...+\xi_n^2) &= \mu_l^2\left(\cos\frac{(1+2s)\pi}{k}+\imath\sin\frac{(1+2s)\pi}{k}\right) \quad (l=1,2,3,...), \end{cases}$$

where $s=0,1,...,k-1$.
For each s the system consists of the real equations

$$\begin{cases} (2\pi)^2|\tau|^2 &= \gamma^2, \\ (2\pi)^2(|\zeta|^2-|\tau|^2) &= \mu_l^2\cos\frac{(1+2s)\pi}{k} \quad (l=1,2,3,...), \\ (2\pi)^2 2(\zeta,\tau) &= \mu_l^2\sin\frac{(1+2s)\pi}{k} \quad (l=1,2,3,...). \end{cases}$$

The Cauchy-Schwarz inequality implies that the contradiction is achieved if the following condition is fulfilled:

$$4\gamma^2\left(\mu_l^2\cos\frac{(1+2s)\pi}{k}+\gamma^2\right)<\mu_l^4\sin^2\frac{(1+2s)\pi}{k}.$$

Solving this inequality for γ, we obtain $\gamma<\mu_l\sin\dfrac{(1+2s)\pi}{2k}$.

Thus, if $\gamma<\mu_1\sin\dfrac{\pi}{2k}$, then the system (17) is inconsistent.

Theorem 2. *Let $h(x,x_{n+1})\in C^\infty(\bar\Pi)$ in the problem (13) have compact support, the constant α satisfy the condition $\alpha+\lambda_1^{2k}>0$, where λ_1 is the first eigenvalue, and φ_1 be the corresponding eigenfunction of the problem (14). Let the solution $v(x,x_{n+1})$ of the problem (13) exponentially decrease at infinity. Then*

$$v(x,x_{n+1})=\Big(\sin\big(\sqrt[2k]{\alpha+\lambda_1^{2k}}r\cos\frac{\pi}{2k}\big)\Phi_1(\theta_1,\dots,\theta_{n-1})+$$
$$+\cos\big(\sqrt[2k]{\alpha+\lambda_1^{2k}}r\cos\frac{\pi}{2k}\big)\Phi_2(\theta_1,\dots,\theta_{n-1})\Big)\times r^{(1-n)/2}e^{-\sqrt[2k]{\alpha+\lambda_1^{2k}}r\sin\frac{\pi}{2k}}\varphi_1(x_{n+1})+$$
$$+O(r^{-(n+1)/2}e^{-\sqrt[2k]{\alpha+\lambda_1^{2k}}r\sin\frac{\pi}{2k}}),$$

where $\Phi_1(\theta_1,\dots,\theta_{n-1}),\Phi_2(\theta_2,\dots,\theta_{n-1})$ are differentiable functions on the unit sphere.

Proof. Put

$$h_1(x)=\int_a^b h(x,x_{n+1})\varphi_1(x_{n+1})dx_{n+1},$$

$$v_1(x)=\int_a^b v(x,x_{n+1})\varphi_1(x_{n+1})dx_{n+1}.$$

Note that $h_1(x)$ has compact support and $h_1(x)\in C^\infty(R^n)$.

The function $v_1(x)$ is a solution of

$$(-\Delta)^k v_1(x)+\mu_1^{2k}v_1(x)=h_1(x)\quad x\in R^n.$$

Then for the solution $v(x,x_{n+1})$ of (13) we have the representation

$$v(x,x_{n+1})=v_1(x)\varphi_1(x_{n+1})+\hat v(x,x_{n+1}), \tag{18}$$

and for $\hat v(x,x_{n+1})$, by Theorem 6 of [9], we have the estimate

$$|\hat v(x,x_{n+1})|\leqslant Ce^{-(\mu_2\sin\frac{\pi}{2k}-\varepsilon)|x|},$$

($\varepsilon>0$ is sufficiently small).

The asymptotic expansion for $v(x,x_{n+1})$ follows from (18) and Theorem 1. □

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Об асимптотике решения задачи Дирихле для уравнения порядка 2k в слое

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Для оператора $(-\Delta)^k u(x) + \nu^{2k} u(x)$ *в* R^n $(n \geqslant 2, k \geqslant 2)$ *получен явный вид фундаментального решения, а для уравнения* $(-\Delta)^k u(x) + \nu^{2k} u(x) = f(x)$ *(с финитной бесконечно дифференцируемой функцией* f*) — первый член асимптотики решения на бесконечности. Изучается также задача Дирихле в слое из* R^{n+1}*.*

Ключевые слова: асимптотика, эллиптическое уравнение, фундаментальное решение, оценки решений, G-функция Мейера, слой.