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New Periodic Gibbs Measures for q-state Potts Model on a Cayley Tree

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In this paper under some conditions on parameters of the q-state Potts model on a Cayley tree of order k we prove existence of the periodic (non translation-invariant) Gibbs measures. Also we give a result about the number of such measures.

Keywords: Cayley tree, configuration, Potts model, Gibbs measure, periodic Gibbs measures, translation-invariant measures.

Introduction

The main problem for a given hamiltonian is the description of all corresponding limiting Gibbs measures (see f.e. [1,3]). This problem was fully studied for the Ising model on the Cayley tree. For example, in [4] an uncountable set of extremal Gibbs measures is constructed and in [5] a necessity and sufficient condition of extremity of unordered phase for Ising model on a Cayley tree is found.

The Potts model is a generalization of the Ising model. The Potts model is not studied to the same extent as the Ising model. For example, in [6] a ferromagnetic Potts model with three-states on a second-order Cayley tree was considered and it was proved that there exists a critical temperature $T_c > 0$ such that for $T < T_c$, there are three translation-invariant and uncountably many not translation-invariant Gibbs measures. The results of [6] on the Potts model with finitely many states were generalized to a Cayley tree of an arbitrary (finite) order in [7].

It was proved [8] that the translation-invariant Gibbs measure of the antiferromagnetic Potts model with an external field is unique. In [9] the Potts model with a countable number of states and nonzero external field on a Cayley tree was considered. It is proved that this model has a unique translation-invariant Gibbs measure.

Other properties of the Potts model on a Cayley tree were studied in [10, p. 105–121]. In [11] it were showed that the Potts model (with an external field $\alpha \in R$) admits only periodic Gibbs measure of period two; it was considered the case $\alpha = 0$, and on the base of the same invariants, is was proved that all periodic Gibbs measures are neccesarily translation-invariant; it were found conditions under which the Potts model with a nonzero external field admits periodic (non translation-invariant) Gibbs measures. In [12] it was fully describe the set of translation-invariant Gibbs measures for the ferromagnetic q-state Potts model and it is proved that the number of translation-invariant measures can be up to $2^q - 1$. In [13] for q-state Potts model (with an external field $\alpha \in R$) on the Cayley tree of order $k = 3$ and $k = 4$ under some conditions on parameters it was proved existence of periodic (non translation-invariant) Gibbs measures of period two. In [14] a ferromagnetic Potts model (with zero external field $\alpha \in R$) on a Cayley

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tree of order $k \geqslant 3$ was studied and it was proved that there exists a critical temperature T_c such that for $T < T_c$, there exist at least two of periodic (non translation-invariant) Gibbs measures.

In this paper under some conditions on parameters of the q-state Potts model on a Cayley tree of order $k \geqslant 2$ we shall prove existence of the periodic (non translation-invariant) Gibbs measures, and we give a lower bound for number of these measures.

1. Definitions and known facts

The Cayley tree $\Im^k$ of order $k \geqslant 1$ is an infinite tree, i.e., a graph without cycles, such that exactly $k+1$ edges originate from each vertex. Let $\Im^k = (V, L, i)$, where V is the set of vertices $\Im^k$, L the set of edges and i is the incidence function setting each edge $l \in L$ into correspondence with its endpoints $x, y \in V$. If $i(l) = \{x, y\}$, then the vertices x and y are called the *nearest neighbors*, denoted by $l = \langle x, y\rangle$. The distance $d(x,y), x, y \in V$ on the Cayley tree is the number of edges of the shortest path from x to y:

$$d(x,y) = \min\ \{d | \exists x = x_0, x_1, \ldots, x_{d-1}, x_d = y \in V \ \text{ such that } \ \langle x_0, x_1\rangle, \ldots, \langle x_{d-1}, x_d\rangle\}.$$

For a fixed $x^0 \in V$ we set $W_n = \ \{x \in V \ \mid \ d(x, x^0) = n\}$,

$$V_n = \ \{x \in V \ \mid \ d(x, x^0) \le n\}, \quad L_n = \ \{l = \langle x, y\rangle \in L \ \mid \ x, y \in V_n\}. \tag{1}$$

It is known that there exists a one-to-one correspondence between the set of vertices V of the Cayley tree $\Im^k$ and the group G_k that is the free product of $k+1$ cyclic groups of second order with the generators $a_1, a_2, \ldots, a_{k+1}$.

We consider the model in which the spin variables take values in the set $\Phi = \ \{1, 2, \ldots, q\}$, $q \geqslant 2$ and are located at the tree vertices. A *configuration* σ on V is then defined as a function $x \in V \to \sigma(x) \in \Phi$; the set of all configurations coincides with $\Omega = \Phi^V$.

The Hamiltonian of the Potts model is defined as

$$H(\sigma) = -J \sum_{\langle x,y\rangle \in L} \delta_{\sigma(x)\sigma(y)}, \tag{2}$$

where $J \in R$, $\langle x, y\rangle$ are nearest neighbors and δ_{ij} is the Kronecker symbol: $\delta_{ij} = \begin{cases} 0, & \text{if } \ i \neq j \\ 1, & \text{if } \ i = j. \end{cases}$
Define a finite-dimensional distribution of a probability measure μ in the volume V_n as

$$\mu_n(\sigma_n) = Z_n^{-1} \exp\left\{-\beta H_n(\sigma_n) + \sum_{x \in W_n} h_{\sigma(x),x}\right\}, \tag{3}$$

where $\beta = 1/T$, $T > 0$ is temperature, Z_n^{-1} is the normalizing factor, $\{h_x = (h_{1,x}, \ldots, h_{q,x}) \in R^q, x \in V\}$ is a collection of vectors and

$$H_n(\sigma_n) = -J \sum_{\langle x,y\rangle \in L_n} \delta_{\sigma(x)\sigma(y)}$$

is the restriction of Hamiltonian on V_n.

We say that the probability distributions (3) are compatible if for all $n \geqslant 1$ and $\sigma_{n-1} \in \Phi^{V_{n-1}}$:

$$\sum_{\omega_n \in \Phi^{W_n}} \mu_n(\sigma_{n-1} \vee \omega_n) = \mu_{n-1}(\sigma_{n-1}). \tag{4}$$

Here $\sigma_{n-1} \vee \omega_n$ is the concatenation of the configurations. In this case, there exists a unique measure μ on Φ^V such that, for all n and $\sigma_n \in \Phi^{V_n}$

$$\mu(\{\sigma|_{V_n} = \sigma_n\}) = \mu_n(\sigma_n).$$

Such a measure is called a *splitting Gibbs measure* corresponding to the Hamiltonian (2) and vector-valued function $h_x, x \in V$.

The following statement describes conditions on h_x, guaranteeing compatibility of $\mu_n(\sigma_n)$.

Theorem 1 ([8]). *The probability distributions $\mu_n(\sigma_n)$, $n = 1, 2, \ldots$ in (3) are compatible for Potts model iff, for any $x \in V$ the following equation holds:*

$$h_x = \sum_{y \in S(x)} F(h_y, \theta), \tag{5}$$

where $F : h = (h_1, \ldots, h_{q-1}) \in R^{q-1} \to F(h, \theta) = (F_1, \ldots, F_{q-1}) \in R^{q-1}$ is defined as

$$F_i = \ln \left(\frac{(\theta - 1)e^{h_i} + \sum\limits_{j=1}^{q-1} e^{h_j} + 1}{\theta + \sum\limits_{j=1}^{q-1} e^{h_j}} \right)$$

and $\theta = \exp(J\beta)$, $S(x)$ is the set of direct successors of x and $h_x = (h_{1,x}, \ldots, h_{q-1,x})$ with

$$h_{i,x} = \tilde{h}_{i,x} - \tilde{h}_{q,x}, \quad i = 1, \ldots, q-1.$$

Let $\widehat{G}_k$ be a subgroup of the group G_k.

Definition 1. *The set of vectors $h = \{h_x,\, x \in G_k\}$ is said to be $\widehat{G}_k$-periodic if $h_{yx} = h_x$ for all $x \in G_k, y \in \widehat{G}_k$.*

The G_k-periodic sets are said to be translation-invariant.

Definition 2. *The measure μ is said to be $\widehat{G}_k$-periodic if it corresponds to the $\widehat{G}_k$-periodic set of vectors h.*

The following theorem characterizes periodic Gibbs measures.

Theorem 2 ([11]). *Let K be a normal divisor of finite index in the group G_k. Then for the Potts model, all K-periodic Gibbs measures are either $G_k^{(2)}$-periodic or translation-invariant, where $G_k^{(2)} = \{x \in G_k :$ the length of x is even$\}$.*

2. Periodic Gibbs measures

We consider case $q \geqslant 3$, i.e. $\sigma : V \to \Phi = \{1, 2, 3, ..., q\}$. By Theorem 2, we have only $G_k^{(2)}$-periodic Gibbs measures corresponding to the sets of vectors $h = \{h_x \in R^{q-1} : x \in G_k\}$ of the form

$$h_x = \begin{cases} h, & \text{if } |x| \text{ is even,} \\ l, & \text{if } |x| \text{ is odd.} \end{cases}$$

Here $h = (h_1, h_2, ..., h_{q-1})$, $l = (l_1, l_2, ..., l_{q-1})$. From equality (5), we then obtain

$$\begin{cases} h_i = k \ln \dfrac{(\theta - 1)\exp(l_i) + \sum\limits_{j=1}^{q-1} \exp(l_j) + 1}{\sum\limits_{j=1}^{q-1} \exp(l_j) + \theta}, \\ l_i = k \ln \dfrac{(\theta - 1)\exp(h_i) + \sum\limits_{j=1}^{q-1} \exp(h_j) + 1}{\sum\limits_{j=1}^{q-1} \exp(h_j) + \theta}, \end{cases} \quad i = \overline{1, q-1}.$$

We introduce the notations $\exp(h_i) = x_i$, $\exp(l_i) = y_i$. We can then rewrite the last system of equations for $i = \overline{1, q-1}$ as

$$\begin{cases} x_i = \left(\dfrac{(\theta - 1)y_i + \sum\limits_{j=1}^{q-1} y_j + 1}{\sum\limits_{j=1}^{q-1} y_j + \theta} \right)^k, \\ y_i = \left(\dfrac{(\theta - 1)x_i + \sum\limits_{j=1}^{q-1} x_j + 1}{\sum\limits_{j=1}^{q-1} x_j + \theta} \right)^k. \end{cases} \tag{6}$$

Remark 1. 1. In the case $q = 2$, the Potts model coincides with the Ising model which was studied in [8].

2. In the case $k = 2$, $q = 3$ and $J < 0$, it was proved that all $G_k^{(2)}$-periodic Gibbs measures on base of invariant $I = \{(x_1, x_2, y_1, y_2) \in R^4 : x_1 = x_2,\ y_1 = y_2\}$ are translation-invariant (see [11]).

3. In the case $k \geqslant 1$, $q = 3$ and $J > 0$, it was proved that all $G_k^{(2)}$-periodic Gibbs measures are translation-invariant (see [11]).

For $q \geqslant 3$, $0 < \theta < 1$, $k \geqslant 3$, we define

$$I_m = \{z = (u, v) \in R^{q-1} \times R^{q-1} : x_i = x,\ y_i = y, i = \overline{1, m};\ \ x_i = y_i = 1, i = \overline{m+1, q-1}\},$$

i.e. $u = (\underbrace{x, x, ..., x}_{m}, 1, 1, ..., 1)$, $v = (\underbrace{y, y, ..., y}_{m}, 1, 1, ..., 1)$ and

$$I'_m = \{z = (u, v) \in R^{q-1} \times R^{q-1} : x_i = x,\ i = \overline{1, m};\ x_i = 1,\ i = \overline{m+1, q-1-m};$$

$$x_i = y, i = \overline{q-m, q-1};\ y_i = y,\ i = \overline{1, m};\ y_i = 1,\ i = \overline{m+1, q-1-m};\ y_i = x, i = \overline{q-m, q-1}\},$$

i.e. $u = (\underbrace{x, x, ..., x}_{m}, 1, 1, ..., 1, \underbrace{y, y, ..., y}_{m})$, $v = (\underbrace{y, y, ..., y}_{m}, 1, 1, ..., 1, \underbrace{x, x, ..., x}_{m})$. Here $2m \leqslant q - 1$.

We consider the map $W : R^{q-1} \times R^{q-1} \to R^{q-1} \times R^{q-1}$, defined as

$$\begin{cases} x_i^{'} = \left(\dfrac{(\theta-1)y_i + \sum\limits_{j=1}^{q-1} y_j + 1}{\sum\limits_{j=1}^{q-1} y_j + \theta} \right)^k, \\ y_i^{'} = \left(\dfrac{(\theta-1)x_i + \sum\limits_{j=1}^{q-1} x_j + 1}{\sum\limits_{j=1}^{q-1} x_j + \theta} \right)^k. \end{cases}$$

We note that the system (6) is the equation $z = W(z)$. Solving the system (6) is therefore equivalent to finding fixed points of the map $z^{'} = W(z)$, where $z = (u, v)$, $z^{'} = (u^{'}, v^{'})$.

Lemma 1. *Sets I_m and $I_m^{'}$ are invariant subsets relatively to the map W.*

The proof is similar to that of Lemma 2 in [11].

The case I_m. In the case we rewrite the system (6) as

$$\begin{cases} x = \left(\dfrac{\theta y + (m-1)y + (q-m)}{\theta + my + (q-m-1)} \right)^k, \\ y = \left(\dfrac{\theta x + (m-1)x + (q-m)}{\theta + mx + (q-m-1)} \right)^k \end{cases} \tag{7}$$

or

$$\begin{cases} x = f^k(y), \\ y = f^k(x), \end{cases} \quad \texttt{where } f(x) = \frac{\theta x + (m-1)x + (q-m)}{\theta + mx + (q-m-1)}, \tag{8}$$

and $f^k(x)$ is k-power of function $f(x)$.

Remark 2. Let $\pi \in S_{q-1}$ be a permutation. We shall define the action of π to the vector $x = (x_1, x_2, ..., x_{q-1})$ as $\pi(x) = (x_{\pi(1)}, x_{\pi(2)}, ..., x_{\pi(q-1)})$. Then $\pi(A) = \{(\pi x, \pi y) : (x, y) \in A\}$, where $A = I_m$ or $I_m^{'}$ is also invariant subset relatively to the map W but in cases $\pi(I_m)$ and $\pi(I_m^{'})$ corresponding system of equations coincides with (7) and (9) (see below), respectively. Therefore without loss of generality, we can consider sets I_m и $I_m^{'}$.

Proposition 1. *Let $k \geqslant 3$, $3 \leqslant q < k+1$, $\theta_{cr} = \dfrac{k-q+1}{k+1} < 1$. Then system of equations (6) on I_m has at least three solutions for $0 < \theta < \theta_{cr}$, it has at least one solution for $\theta = \theta_{cr}$ and it has only one solution for $\theta > \theta_{cr}$.*

Proof. By (8) we obtain

$$x = g(x) = f^k(f^k(x)).$$

We have

$$f'(x) = \frac{(\theta-1)(\theta+q-1)}{(\theta+my+q-m-1)^2};$$

$$g'(x) = k^2 f^{k-1}(f^k(x)) f'(f^k(x)) f^{k-1}(x) f'(x).$$

Consequently, for $0 < \theta < 1$ the function $f(x)$ decreases monotonically and the equation $f(x) = x$ has a unique solution $x = 1$ such that $f'(1) = \dfrac{\theta-1}{\theta+q-1}$. We note that $g(x)$ is increasing and $x = 1$ is a solution to $g(x) = x$. If $g'(1) = k^2(f'(1))^2 = \left(k\dfrac{\theta-1}{\theta+q-1}\right)^2 > 1$ then this solution is not unique, because in this case for $x > 1$ the graph of the function g lies above the bisector and the function g is bounded. Thus a critical value for θ can be found by the equation $\left(k\dfrac{\theta-1}{\theta+q-1}\right)^2 = 1$ which for $\theta < 1$ gives $\theta_{cr} = \dfrac{k-q+1}{k+1}$. Hence it follows that for $0 < \theta < \theta_{cr}$ the equation $g(x) = x$ has at least three solutions $x_0^* < x_1^* = 1 < x_2^*$, i.e. the equation $g(x) = x$ has at least two roots, which are distinct from roots of the equation $f(x) = x$. For $\theta = \theta_{cr}$ the graph of the function g tangents to the bisector in $x = 1$. This means that in this condition the equation $g(x) = x$ has at least one solution. Besides it is clear that for $\theta > \theta_{cr}$ the equation $g(x) = x$ has a unique solution $x_1^* = 1$, which it is solution to equation $f(x) = x$. $\square$

The case $I_m^{'}$. We consider the set $I_m^{'}$. We rewrite the system of equations (6) on this set as

$$\begin{cases} x = \left(\dfrac{(\theta-1)y + my + (q-2m-1) + mx + 1}{\theta + mx + my + (q-2m-1)}\right)^k, \\ y = \left(\dfrac{(\theta-1)x + mx + (q-2m-1) + my + 1}{\theta + mx + my + (q-2m-1)}\right)^k, \end{cases} \tag{9}$$

where $\exp(h_i) = x_i,\ \exp(l_i) = y_i$.

Remark 3. 1. For $m = 0$ we obtain $u = (1, 1, ..., 1),\ v = (1, 1, ..., 1)$, which corresponds to the translation-invariant Gibbs measure. Thus we consider the case $m \geqslant 1$.

2. In the case $k = 2,\ q = 3,\ m = 1$ on $I_m^{'}$ it was proved that all $G_k^{(2)}$-periodic Gibbs measures are translation-invariant (see [11]).

In the last system substituting $\sqrt[k]{x} = z,\ \sqrt[k]{y} = t$, we obtain

$$\begin{cases} z = \dfrac{(\theta+m-1)t^k + mz^k + q - 2m}{\theta + mz^k + mt^k + q - 2m - 1}, \\ t = \dfrac{(\theta+m-1)z^k + mt^k + q - 2m}{\theta + mz^k + mt^k + q - 2m - 1}. \end{cases} \tag{10}$$

From the first equation of (10) we find t^k, t:

$$t^k = \frac{mz^{k+1} - mz^k + (\theta+q-2m-1)z - q + 2m}{\theta + m - 1 - mz};$$

$$t = \left(\frac{mz^{k+1} - mz^k + (\theta+q-2m-1)z - q + 2m}{\theta + m - 1 - mz}\right)^{\frac{1}{k}}$$

and substitute to the second equation of (10). Then we obtain

$$\begin{aligned} f(z) = {} & [(\theta+2m-1)z^k - mz^{k+1} + mz + q - 2m]^k(\theta + m - 1 - mz) - \\ & -(mz^k + q - m - 1 + \theta)^k[mz^{k+1} - mz^k + (\theta+q-2m-1)z - q + 2m] = 0. \end{aligned} \quad (11).$$

We consider the function $f(z)$. We note that $f(0) = (q-2m)^k(\theta+m-1) + (q-2m)(\theta+q-m-1) > 0$ for $2m < q$. Besides $f(1) = 0$ and $f(z) \to -\infty$ for $z \to +\infty$. Consequently it is clear that if $f'(1) > 0$, then the equation (11) has at least three solutions. Therefore we consider

$$f'(1) = (k^2-1)s^2 - 2qs - q^2 = (k^2-1)\left(s - \frac{q}{k-1}\right)\left(s + \frac{q}{k+1}\right) > 0,$$

where $s = \theta - 1 < 0$. Consequently, if $s + \dfrac{q}{k+1} < 0$, i.e. $0 < \theta < 1 - \dfrac{q}{k+1} = \theta_{cr}$ then $f'(1) > 0$.

Thus we proved the following

Proposition 2. *Let* $k \geqslant 3$*,* $3 \leqslant q < k+1$*,* $\theta_{cr} = \dfrac{k-q+1}{k+1} < 1$*. Then the system of equations (6) on* $I_m^{'}$*:*

1) for $0 < \theta < \theta_{cr}$ *has at least three solutions;*

2) for $\theta = \theta_{cr}$ *has at least one solution;*

3) for $\theta > \theta_{cr}$ *has only one solution.*

Remark 4. It is clear that in Propositions 1 and 2 one of measures corresponds to the solution $x_1^* = 1$ which is translation-invariant, the remainning measures are $G_k^{(2)}$-periodic (non-translation-invariant), and in case $\theta > \theta_{cr}$ the measure corresponding to the unique solution $x_1^* = 1$.

Similarly as in [12, p. 6], it is easy to show that for $0 < \theta < \theta_{cr}$ on each I_m and $I_m^{'}$, where $m = 1, 2, ..., q$, the number of $G_k^{(2)}$-periodic (non-translation-invariant) Gibbs measures is not less than $2 \cdot \binom{q}{m}$ and $2 \cdot \binom{q}{m} \cdot \binom{q-m}{m}$, respectively. Consequently, the number on $\bigcup_{m=1}^{q} I_m$ and $\bigcup_{m=1}^{q} I_m^{'}$ is not less than

$$2 \cdot \sum_{m=1}^{q} \binom{q}{m} = 2^{q+1} - 2, \qquad \text{and} \qquad 2 \cdot \sum_{m=1}^{[q/2]} \binom{q}{m} \cdot \binom{q-m}{m},$$

respectively.

Thus we have the

Theorem 3. *For* $k \geqslant 3$*,* $3 \leqslant q < k+1$ *and* $0 < \theta < \theta_{cr}$ *for the Potts model exist at least*

$$2 \cdot \left(2^q - 1 + \sum_{m=1}^{[q/2]} \binom{q}{m} \cdot \binom{q-m}{m} \right)$$

$G_k^{(2)}$*-periodic (non translation-invariant) Gibbs measures.*

Remark 5. In [12] the number and the description of all translation-invariant Gibbs measures for the Potts model were given.

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Новые периодические меры Гиббса для модели Поттса с q-состояниями на дереве Кэли

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В данной статье изучается модель Поттса с q-состояниями на дереве Кэли порядка k и показано существование периодических (не трансляционно-инвариантных) мер Гиббса при некоторых условиях на параметры этой модели. Кроме того, указана нижняя граница количества существующих периодических мер Гиббса.

Ключевые слова: дерево Кэли, конфигурация, модель Поттса, мера Гиббса, периодические меры, трансляционно-инвариантные меры.