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Ф Е Д Е Р А Л Ь Н Ы Й
У Н И В Е Р С И Т Е Т

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U N I V E R S I T Y



27th International Conference on Finite and Infinite Dimensional Complex Analysis and Applications

12th-16th of August, 2019, Krasnoyarsk, Russia

The conference continues the series of annual conferences started in Korea, 1993. For more than 20 years it has been one of the most significant conferences covering various topics of complex analysis. Until now it was hosted by prominent mathematical centers in Japan, Korea and China. 27th ICFIDCAA is organized in Krasnoyarsk, the city placed at the central part of Eurasia, which emphasizes the conference to be more international and bridge the traditions of different mathematical communities.

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12th of August, Monday

Plenary session chairman: Armen Sergeev

9:00-9:30	<i>Opening ceremony</i>		
9:30-10:30	Jean-Pierre Demailly		
10:30-11:00	<i>Coffee-break</i>		
11:00-12:00	Alexander Aptekarev		
12:00-13:00	Evgeniy Chirka		
13:00-14:30	<i>Lunch</i>		
Chairmen	Alain Yger, B101	Alexander Aptekarev, BFA	Stepan Orevkov, BA
14:30-15:30	Irina Antipova	Alexander Komlov	Tuen Wai Ng
15:30-16:00	Alekos Vidras	Vladimir Lysov	Liangwen Liao
16:00-16:30			Chung-Chun Yang
16:30-17:00	<i>Coffee-break</i>		
17:00-17:30	Alfonso Motes-Rodríguez	Semyon Nasyrov	
17:30-18:00		Roman Palvelev	
19:00	<i>Opening dinner</i>		

13th of August, Tuesday

Plenary session chairman: Jean-Pierre Demailly

9:00-10:00	Grigory Mikhalkin		
10:00-10:30	<i>Coffee-break</i>		
10:30-11:30	Stepan Orevkov		
11:30-12:30	Xiangyu Zhou		
12:30-14:00	<i>Lunch</i>		
Chairmen	Maria Esteban, B101	Victor Kulikov, BFA	Vladimir Dubinin, BA
14:00-15:00	Dan Popovici	Yuriy Eliyashev	Yuefei Wang
15:00-16:00	Shin Kikuta	Yoshihiro Aihara	Yang Fei
16:00-16:30	<i>Coffee-break</i>		
16:30-17:00	Tatsuhiko Honda	Alexey Shchuplev	David Shoikhet
17:00-17:30		Ji Eun Kim	

14th of August, Wednesday

Plenary session chairman: Kiyoshi Takeuchi

9:00-10:00	Franc Forstnerič		
10:00-10:30	<i>Coffee-break</i>		
10:30-11:30	Vladimir Dubinin		
11:30-12:30	Maria Esteban		
12:30-14:00	<i>Lunch</i>		
14:00-19:00	<i>Excursion to «Stolby» National Sanctuary</i>		
19:00	<i>Banquet</i>		

15th of August, Thursday

Plenary session chairman: Franc Forstnerič

10:00-10:30	<i>Coffee-break</i>		
10:30-11:30	Kiyoshi Takeuchi		
11:30-12:30	Victor Kulikov		

12:30-14:00	<i>Lunch</i>		
Chairmen	Dan Popovici, B101	Praveen Agarwal, BFA	Kang-Tae Kim, BA
14:00-15:00	Hyungwoon Koo	Junesang Choi	Gerd Schmalz
15:00-16:00	Konstantin Fedorovskiy	Jong Kyu Kim	Alexander Loboda
16:00-16:30	<i>Coffee-break</i>		
16:30-17:00	Toshiyuki Sugawa	Rakesh Kumar Parmar	Alexander Sukhov
17:00-17:30		Q-Heung Choi	
17:30-18:00	Masahiro Yanagishita	Daewook Kim	Nikolay Kruzhilin

16th of August, Friday

Plenary session chairman: August Tsikh

09:00-10:00	Alain Yger		
10:00-10:30	<i>Coffee-break</i>		
10:30-11:30	Kang-Tae Kim		
11:30-12:30	Azimbay Sadullaev		
12:30-14:00	<i>Lunch</i>		
Chairmen	Alexey Shchuplev, B101	Yuefei Wang, BFA	Rakesh Kumar Parmar, BA
14:00-14:30	Vibhuti Arora	Swadesh Kumar Sahoo	Tacksun Jung
14:30-15:00	Jugal Kishore Prajapat	Young Jae Sim	Mark Elin
15:00-15:30	Rajbala	Praveen Agarwal	Atsushi Yamamori
15:30-16:00			
16:00-16:30	<i>Coffee-break</i>		
16:30-17:30	Armen G. Sergeev		
17:30-18:00	<i>Closing ceremony</i>		

Talks abstracts

Praveen Agarwal, *Anand International College of Engineering, Agra, India*

Certain fractional integral and differential formulas involving the extended incomplete generalized hypergeometric functions

The fractional integral and differential operators involving the family of special functions have found significant importance and applications in various fields of mathematics and engineering. The goal of this chapter is to find the fractional integral and differential formulas (also known as composition formulas) of the extended incomplete generalized hypergeometric functions by using the generalized fractional calculus operators (the Marchichev-Saigo-Maeda operators). After that, we established their image formulas by using the integral transforms like: Beta transform, Laplace transform and Whittaker transform, respectively. Moreover, the reduction formulas are also considered as special cases of our main findings associated with the well known Saigo fractional integral and differential operators, Erdélyi-Kober fractional integral and differential operators, Riemann-Liouville fractional integral and differential operators and the Weyl fractional calculus operators, respectively.

Yoshihiro Aihara, *Fukushima University, Fukushima, Japan*

Holomorphic curves with deficiencies

In this talk, we discuss holomorphic curves with deficiencies in a complex projective space $\mathbf{P}^n(\mathbf{C})$. We first give some methods of making holomorphic curves with deficient hypersurfaces, We show that for a given hypersurface D in $\mathbf{P}^n(\mathbf{C})$, there is a holomorphic curve $f : \mathbf{C} \rightarrow \mathbf{P}^n(\mathbf{C})$ with $\delta_f(D) > 0$.

Next we investigate some properties of holomorphic curves with deficient divisors. In particular, we consider how the existence of deficient divisors affects the uniqueness problem of holomorphic curves.

Irina Antipova, *Siberian Federal University, Krasnoyarsk, Russia*

Multidimensional Mellin transforms

The Mellin transforms figure prominently in the complex analysis due to being the most appropriate for using the theory of residues techniques. A pair of convex domains $\Theta, U \subset \mathbb{R}^n$ encodes isomorphic functional spaces M_Θ^U, W_U^Θ which are transformed to each other by the direct and inverse Mellin transforms. Domains Θ and U predetermine the asymptotics of functions. Moreover, the asymptotics of the original function $f(x) \in M_\Theta^U$ is defined by singularities of its Mellin transform $M[f](z) \in W_U^\Theta$. It is the fundamental correspondence which determines the scope of application for Mellin transforms. In my talk, I will speak about properties of the Mellin transform for rational functions with quasi-elliptic or hypoelliptic denominators and about using the inverse Mellin transform (Mellin–Barnes integral) as a tool of getting the analytic continuation for algebraic functions. I also will focus on the role of the Mellin transforms in the realization of residue currents.

Alexander Aptekarev, *Keldysh Institute of Applied Mathematics of Russian Academy of Sciences, Moscow, Russia*

Extremal sets in the complex plane: Chebotarev, Stahl and Nuttall compacts

The classical Chebotarev problem is about a compact of the minimal logarithmic capacity, connecting a finite number of given points on the complex plane. We discuss the modern applications and generalizations of this problem, such as the «sheet»-structure of the Riemann surface of vector-analytic functions and asymptotics of the Hermite–Padé rational approximants.

Vibhuti Arora, *Indian Institute of Technology Indore, Indore, India*

Successive coefficients for spirallike and related functions

Let \mathcal{A} denote the set of all analytic functions f on the unit disk normalised so that $f(0) = f'(0) - 1 = 0$ and \mathcal{S} denote the subclass of functions $f \in \mathcal{A}$ which are univalent in the unit disk.

The problem of estimating sharp bound for successive coefficients, namely, $||a_{n+1}| - |a_n||$, is an interesting necessary condition for a function to be in \mathcal{S} . This problem was first studied by Goluzin [1] with an idea to solve the Bieberbach conjecture. Hayman [3] proved in 1963 that

$$||a_{n+1}| - |a_n|| \leq A, \quad n = 1, 2, 3, \dots, \quad (1)$$

where $A \geq 1$ is an absolute constant, for $f \in \mathcal{S}$ with the form $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$. It is still an open problem to find the minimal value of A which works for all $f \in \mathcal{S}$, however, the best known bound as of now is 3.61 which is due to Grinspan [2]. In 1978, Leung [4] proved that $A = 1$ for starlike functions implies the Bieberbach conjecture, so it is interesting to find the minimal value of A for certain other subfamilies of univalent functions.

We are considering some subclasses of the class of univalent functions, mainly, the family $\mathcal{S}_\gamma(\alpha)$ of γ -spirallike functions of order α which is defined as

$$\mathcal{S}_\gamma(\alpha) = \left\{ f \in \mathcal{A} : \operatorname{Re} \left(e^{-i\gamma} \frac{zf'(z)}{f(z)} \right) > \alpha \cos \gamma \text{ for } z \in \mathbb{D} \right\},$$

where $\alpha \in [0, 1)$ and $\gamma \in (-\pi/2, \pi/2)$. Note that $\mathcal{S}_0(\alpha) =: \mathcal{S}^*(\alpha)$ is the usual class of starlike functions of order α , and $\mathcal{S}^*(0) = \mathcal{S}^*$. A function $f \in \mathcal{A}$ is called convex of order α if and only if $zf'(z)$ belongs to $\mathcal{S}^*(\alpha)$ for some $\alpha \in [0, 1)$. The class of all convex functions of order α is denoted by $\mathcal{C}(\alpha)$.

Our objective is to obtain results related to successive coefficients for the classes $\mathcal{S}^*(\alpha)$, $\mathcal{C}(\alpha)$, $\mathcal{S}_\gamma(\alpha)$ and other related classes of functions.

- [1] G. Goluzin. On distortion theorems and coefficients of univalent functions. *Rec. Math. [Mat. Sbornik] N.S.*, 19(61):183–202, 1946.
- [2] A. Z. Grinšpan. The sharpening of the difference of the moduli of adjacent coefficients of schlicht functions. In *Some problems in modern function theory (Proc. Conf. Modern Problems of Geometric Theory of Functions, Inst. Math., Acad. Sci. USSR, Novosibirsk, 1976) (Russian)*, pages 41–45, 1976.
- [3] W. K. Hayman. On successive coefficients of univalent functions. *J. London Math. Soc.*, 38:228–243, 1963.
- [4] Yuk Leung. Successive coefficients of starlike functions. *Bull. London Math. Soc.*, 10(2):193–196, 1978.

Evgeniy Chirka, *Steklov Mathematical Institute of Russian Academy of Sciences, Moscow, Russia*

Potentials on a compact Riemann surface

The notions of logarithmic potential theory in the plane domains are generalized for domains on a compact Riemann surface. An approach with application of de Rham currents to Green functions, Dirichlet problem, Poisson kernel e.t.c. will be discussed.

Junesang Choi, *Dongguk University, Gyeongju, South Korea*

Remarks on a type of fractional kinetic equations

During the last several decades, a great variety of fractional kinetic equations involving diverse special functions have been broadly and usefully employed in describing and solving several important problems of physics and astrophysics. In this paper, we aim *firstly* to make a brief survey of as many as papers involving a chosen type of fractional kinetic equations, and, with a view to demonstrating main employed techniques and important identities which have been employed in the chosen type of fractional kinetic equations, *secondly* to find solutions of this type of fractional kinetic equations associated with the (p, q) -extended τ -hypergeometric function and the (p, q) -extended τ -confluent hypergeometric function. Also, in terms of two main employed

techniques, Laplace transform and Sumudu transform, the surveyed papers involving this type of fractional kinetic equations are classified into three classes.

This is a joint work with Nabiullah Khan (Aligarh Muslim University, Aligarh, India), Owais Khan (Aligarh Muslim University, Aligarh, India), and Kottakkaran Sooppy Nisar (Prince Sattam bin Abdulaziz University, Wadi Al dawaser, Saudi Arabia).

Q-Heung Choi, *Inha University*, Incheon, South Korea

On the Asymmetric Beam Equation

We investigate the nonlinear beam equation in an interval $(\pi/2; \pi/2)$, with Dirichlet Boundary Condition,

$$u_{tt} + u_{xxxx} + bu^+ = f(x, t), \quad \text{in} \quad (\pi/2; \pi/2) \times R \quad (1)$$

u is periodic in t and even in x and t ; here the nonlinearity (bu^+) crosses the eigenvalue λ_{10} . This equation represents a bending beam supported by cables under a load f .

The constant b represents the restoring force if the cables stretch. The nonlinearity (bu^+) models the fact that cables resist expansion but do not resist compression. McKenna and Walter [1] showed by degree theory that equation (1) with constant load $1 + h$ (h is bounded) has at least two solutions.

This is a joint work with Tacksun Jung (Kunsan National University, Kunsan, South Korea).

- [1] P. J. McKenna and W. Walter. Nonlinear oscillations in a suspension bridge. *Arch. Rational Mech. Anal.*, 98(2):167–177, 1987.

Jean-Pierre Demailly, *University of Grenoble*, Grenoble, France

On the existence of global orbifold jet differentials

We use holomorphic Morse inequalities to derive an existence theorem for global jet differentials on projective complex manifolds, in the logarithmic and orbifold contexts. The proof, which exhibits new phenomena in the orbifold case, also produces explicit bounds for cohomology groups of (orbifold) jet bundles. These results can in turn be used to investigate some fundamental conjectures such as the Green-Griffiths-Lang conjecture on entire holomorphic curves. This is joint work with F. Campana, L. Darondeau and E. Rousseau.

Vladimir Dubinin, *Far Eastern Federal University*, Vladivostok, Russia

A capacity look at the Schwarzian derivative

In the talk we will give a brief survey of results involving the Schwarzian derivative and depending on the geometry of the image of a domain under a holomorphic map [1]. The author's results obtained previously by using the theory of condenser capacity and symmetrization constitute the core of the talk [2]. Inequalities for univalent and multivalent holomorphic functions are considered both at interior and at boundary points of the domain of definition.

- [1] V. N. Dubinin. Geometric estimates for the Schwarzian derivative. *Uspekhi Mat. Nauk*, 72(3(435)):97–130, 2017.
- [2] Vladimir N. Dubinin. *Condenser capacities and symmetrization in geometric function theory*. Springer, Basel, 2014. Translated from the Russian by Nikolai G. Krushilin.

Mark Elin, *ORT Braude College*, Karmiel, Israel

Linearization of holomorphic semicycles in Banach spaces

We consider holomorphic semicycles on the open unit ball of a Banach space taking values in a Banach algebra (introduced and studied in our previous works). We establish criteria for a semicycle to be linearizable, that is, cohomologically equivalent to one independent of the spatial variable. Based on joint work with G. Katriel and F. Jacobson.

Yuriy Eliyashev, *Higher School of Economics, Moscow, Russia*
Tropical Hodge Theory

The Hodge theory on complex manifolds is a classical example of application of analytical methods in algebraic geometry. One of the main ideas of the tropical geometry is that there is should be a tropical analog of an object from the complex geometry. Following this idea Lagerberg introduced tropical currents and differential forms, Itenberg, Katzarkov, Mikhalkin, Zharkov introduced tropical cohomology theory. Based on these works, Jell, Shaw, Smacka constructed tropical de Rham cohomology theory. In my talk I will discuss how to construct a Hodge theory on Tropical varieties and how it resembles classical Hodge theory.

Maria Esteban, *Paris Dauphine University, Paris, France*
Flows, functional inequalities and spectral estimates

Functional inequalities are a very important tool in global analysis, analysis on manifolds, in the study of nonlinear partial differential equations, in mathematical physics, in spectral theory, etc. There are many kinds of inequalities which arise in different contexts. Together with the classical ones, there has been recently a large effort to find new ones and to show how they can help to study very interesting problems in various areas.

In this talk I will discuss the general topic of functional inequalities, the existence of extremal functions for them, and their qualitative properties. I will present various methods that are useful in this topic and in particular I will show how the use of ad-hoc flows and entropy-entropy production methods helps immensely both in proving the inequalities, as in the study of their extremal functions.

Last I will discuss how a duality argument allows to obtain very accurate spectral estimates for differential operators from the optimal version of well adapted functional inequalities.

Konstantin Fedorovskiy, *Bauman Moscow State Technical University and Saint Petersburg State University, Moscow (Saint Petersburg), Russia*

On Chui's conjecture and approximation by simplest fractions

In 1971 C. K. Chui conjectured that the average field strength in the unit disk $\mathbb{D} = \{|z| < 1\}$ in the complex plane due to unit point masses on the unit circle $\mathbb{T} = \{|z| = 1\}$ is minimal for the uniform distribution of masses. Formally the Chui's conjecture says that for all $\{z_1, \dots, z_N\} \subset \mathbb{T}$, $N = 1, 2, \dots$, the following is satisfied

$$\left\| \sum_{k=1}^N \frac{1}{z - z_k} \right\|_{L^1(\mathbb{D})} \geq \left\| \sum_{k=1}^N \frac{1}{z - \omega_N^k} \right\|_{L^1(\mathbb{D})},$$

where ω_N is the principal root of unity of degree N , so that $\omega_N = \exp(2\pi i/N)$, and the space $L^1(\mathbb{D})$ is considered with respect to the planar Lebesgue measure in \mathbb{D} .

This conjecture remains open, and in the talk we will consider its analogue for the weighted Bergman spaces $A_{\alpha}^2 = A_{\alpha}^2(\mathbb{D})$, $\alpha > 0$. Recall, that the space A_{α}^2 consists of all holomorphic function f in \mathbb{D} such that

$$\|f\|_{2,\alpha}^2 := \frac{\alpha + 1}{\pi} \int |f(z)|^2 (1 - |z|^2)^{\alpha} dx dy < \infty.$$

It will be shown that the statement analogous to Chui's conjecture is true for the spaces A_{α}^2 for all $\alpha \in (0, 1]$. In other words, for all such α and for all $z_1, \dots, z_N \in \mathbb{T}$, $N = 1, 2, \dots$, one has

$$\left\| \sum_{k=1}^N \frac{1}{z - z_k} \right\|_{2,\alpha} \geq \left\| \sum_{k=1}^N \frac{1}{z - \omega_N^k} \right\|_{2,\alpha}.$$

It is planned to consider also the problem about completeness in the space A_{α}^2 , $\alpha > 0$, of the

system of ‘simplest fractions’, that is functions of the form

$$\sum_{k=1}^N \frac{1}{z - z_k},$$

where $z_1, \dots, z_N \in \mathbb{T}$, $N = 1, 2, \dots$

This is a joint work with E. Abakumov (University Paris Est, Marne-la-Vallée, France) and A. Borichev (Aix-Marseille University, France).

Franc Forstnerič, *University of Ljubljana*, Ljubljana, Slovenia

Mergelyan’s and Arakelian’s theorem for manifold-valued maps

I will first discuss Mergelyan’s approximation theorem for maps from open Riemann surface into complex manifolds. I will then show how these results, along with techniques of gluing holomorphic sprays, give a version of Arakelian’s theorem on uniform approximation of holomorphic maps from closed subsets of plane domains to any compact complex homogeneous manifold.

Tatsuhiko Honda, *Senshu University*, Tokyo, Japan

Characterizations of Bloch functions on the homogeneous unit ball in a complex Banach space

There are many equivalent conditions for Bloch functions on the unit disc $\mathbb{U} = \{\zeta \in \mathbb{C} \mid |\zeta| < 1\}$. The concept of a Bloch function has been extended to various complex domains in finite or infinite dimensions. In particular, a definition of a complex-valued Bloch function on a bounded homogeneous domain was given by R. Timoney, who has shown that several equivalent conditions for complex-valued Bloch functions on the complex unit disc \mathbb{U} can be extended to finite dimensional bounded homogeneous domains.

In this talk, we begin by defining a Bloch function on a possibly infinite dimensional bounded symmetric domain and extend further to this setting several equivalent conditions for Bloch functions given by R. Timoney for finite dimensional domains.

Tacksun Jung, *Kunsan National University*, Kunsan, South Korea

We investigate multiplicity of solutions for one dimensional p -Laplacian Dirichlet boundary value problem with jumping nonlinearities. We obtain three theorems: The first one is that there exists exactly one solution when nonlinearities cross no eigenvalue. The second one is that there exist exactly two solutions, exactly one solutions and no solution depending on the source term when nonlinearities cross one first eigenvalue. The third one is that there exist at least three solutions, exactly one solutions and no solution depending on the source term when nonlinearities cross the first and second eigenvalues. We obtain the first theorem and the second one by eigenvalues and the corresponding normalized eigenfunctions of the p -Laplacian Dirichlet eigenvalue problem, and the contraction mapping principle on p -Lebesgue space (when $p \geq 2$). We obtain the third result by Leray-Schauder degree theory.

This is a joint work with Q-Heung Choi (Inha University, Incheon, South Korea).

Alexander Komlov, *Steklov Mathematical Institute of Russian Academy of Sciences*, Moscow, Russia

Reconstruction of the values of an algebraic function via the system of Hermite-Padé polynomials

In the talk we suggest a method of reconstruction of the values of an algebraic function from its initial germ on all sheets of its Riemann surface, except for the “last” (see below), with the help of systems of linear algebraic equations. More precisely, for a given germ f_0 of an algebraic function f of order $(m + 1)$, for each natural number n we define a system of m tuples of polynomials. These tuples are numbered by the number $k = 1, \dots, m$, and we call them “ k -th polynomials of Hermite-Padé m -system (of order n)”. All these polynomials are found constructively, as solutions of linear homogeneous systems, and coefficients of these systems are some linear combinations of the Taylor coefficients of the original germ f_0 . It turns out that the ratio of some polynomials from

the k -th set asymptotically (as $n \rightarrow \infty$) reconstructs the sum of the values of the original function f on the first k sheets of the so-called Nuttall partition of Riemann surface of f into sheets. We note that 1-th polynomials of Hermite-Padé m -system are well-known Hermite-Padé polynomials of the second type and m -th polynomials of Hermite-Padé m -system are well-known Hermite-Padé polynomials of the first type.

Hyungwoon Koo, *Kore University*, Seoul, South Korea

Difference of weighted composition operator

We obtain complete characterizations in terms of Carleson measures for bounded/compact differences of weighted composition operators acting on the standard weighted Bergman spaces over the unit disk. Unlike the known results, we allow the weight functions to be non-holomorphic and unbounded.

As a consequence we obtain a compactness characterization for differences of unweighted composition operators acting on the Hardy spaces in terms of Carleson measures and, as a nontrivial application of this, we show that compact differences of composition operators with univalent symbols on the Hardy spaces are exactly the same as those on the weighted Bergman spaces. As another application, we show that an earlier characterization due to Acharyya and Wu for compact differences of weighted composition operators with bounded holomorphic weights does not extend to the case of non-holomorphic weights. We also include some explicit examples related to our results.

Shin Kikuta, *Kogakuin University*, Tokyo, Japan

Boundary behavior of Kähler-Einstein metric and positivity for log-canonical bundle We would like to discuss some relations between the positivity of the log-canonical bundle and the boundary behavior of the Kähler-Einstein metric over a quasi-projective manifold. In this talk, we will propose a conjecture about them, and present supporting examples and our work in progress toward the resolution.

Daewook Kim, *Seowon University*, Cheongju, South Korea

Asymptotic behavior of solutions for the nonlinear hyperbolic equation with an external time delay

In this talk, we introduce some behavior of solutions to the nonlinear hyperbolic equation with an external time delay. The system is also considered some complex space concerning electricity. First, we introduce the main theorem and some lemmas. We also the smallness condition and taking into account the appropriate Lyapunov functional. Finally, we get the energy decay rate of the main energy for the system.

Ji-Eun Kim, *Dongguk University*, Gyeongju, South Korea

Solving quaternionic differential equations with the renewal mathematical methods

The recent results on quaternionic differential operators which can be applied to quantum mechanics, in particular tunneling effects and robotic dynamics interested in the study of resolution methods for quaternionic differential equations. In this paper, by using the real matrix representation of quaternionic operators, we propose the reduction of order for quaternionic homogeneous differential equations and extend to the noncommutative case the method of variation of parameters. Also, we show that the modified complex Wronskian admits a noncommutative extension for quaternionic functions of a quaternionic variable. Specially, we present quaternionic second order differential equations and obtain the result that linear dependence and independence of solutions of homogeneous linear differential equations.

This work is supported by the Dongguk University Research Fund and the National Research Foundation of Korea (NRF) (2017R1C1B5073944).

Jong Kyu Kim, *Kyungnam University*, Changwon, South Korea

Krasnoselski-Mann iterative method for finding a common solution of hierarchical fixed point problems and split mixed equilibrium problems

In this talk, we introduce a modified Krasnoselski-Mann type iterative method for finding a common solution of a split mixed equilibrium problem and a hierarchical fixed point problem of a

finite family of k -strictly pseudocontractive non-self mappings. Many of the algorithms for solving the split mixed equilibrium problem involves step size which depends on the norm of a bounded linear operator. Since the computation of the operator norm is very difficult, we formulate our iterative algorithm in such a way that the implementation of the proposed algorithm does not require any prior knowledge of operator norm. Weak convergence results are established under mild conditions. We also establish strong convergence results for a certain class of hierarchical fixed point and split equilibrium problem.

Kang-Tae Kim, *Pohang University of Science and Technology*, Pohang, South Korea
Infinite dimensional holomorphic homogeneous regular domains

In a collaboration with C.-H. Chu (London), S. Kim (Postech), I have shown that the infinite dimensional HHR(=holomorphic homogeneous regular) domains are domains of holomorphy, and determined completely which infinite dimensional bounded symmetric domains are HHR. This extends the study of HHR domains in the finite dimensional complex geometry. In this talk, I will report these results with (hopefully) comprehensive introduction of this direction of study, which is relatively new.

Nikolay Kruzhilin, *Steklov Mathematical Institute of Russian Academy of Sciences*, Moscow, Russia

Holomorphic maps between tube domains

Holomorphic maps between tube domains in \mathbb{C}^n and the equivalence problem for tube domains and tubular real hypersurfaces are under consideration. Situations when holomorphic equivalence implies affine equivalence are the focus of interest. The case of degenerate Levi form is discussed in greater detail.

Victor Kulikov, *Steklov Mathematical Institute of Russian Academy of Sciences*, Moscow, Russia

On deformations of germs of finite morphisms of smooth surfaces

In the talk, questions related to deformations of germs of finite morphisms of smooth surfaces will be discussed. As example, a classification up to smooth deformations of the three-sheeted and four-sheeted germs, the singularity types of branch curves and the local monodromy groups of these germs will be given.

This work is supported by the Russian Science Foundation under grant no. 19-11-00237.

Liangwen Liao, *Nanjing University*, Nanjing, China

Nevalinna theory and complex nonlinear differential equations

In this talk, we will introduce how to study complex nonlinear differential equations by using Nevanlinna theory and Wiman-Valiron theory and study value distribution theory of meromorphic functions. We will introduce some new results in complex differential equations obtain by us and other researchers.

Alexander Loboda, *Voronezh State Technical University*, Voronezh, Russia

Homogeneous real hypersurfaces in \mathbb{C}^3 : completion of classification

The complete classification of holomorphically homogeneous real hypersurfaces in two-dimensional complex spaces (in local and global forms) was proposed by E. Cartan in 1932.

Final classification of locally homogeneous hypersurfaces in complex spaces of the following dimension essentially uses the properties of the degeneracy or non-degeneracy of Levi form of the surfaces under study. An important role is also played by the dimensions estimate $5 \leq \dim g \leq 8$ (Loboda-2000) for Lie algebras of holomorphic vector fields on non-degenerate Levi non-spherical homogeneous surfaces.

In accordance with this estimate, all homogeneous nondegenerate surfaces with 8- and 7-dimensional (Loboda 2001), and then with 6-dimensional Lie algebras (Dubrov-Medvedev-The-2017) were described. Levi-degenerate homogeneous surfaces were completely studied in 2008 by Fels and Kaup.

The report discusses the final part of classification associated with non-degenerate homogeneous surfaces having only trivial stabilizers. Individual blocks of the volumetric classification of

abstract 5-dimensional Lie algebras (Mubarakzyanov-1961) were associated with the problem of homogeneity by studying holomorphic realizations of such algebras.

In addition to the previously known homogeneous manifolds, there are only three (cited in the report) new types of holomorphically homogeneous real hypersurfaces. All these new examples are indefinite (non-degenerate) surfaces. In general, the studied family of homogeneous hypersurfaces splits into 40 types of manifolds; many of them, but not all, are holomorphically equivalent to tubular manifolds. The maximum dimension of the moduli spaces for the individual components of this family is 2.

The presented results were obtained jointly with Akopyan R.S., Atanov A.V., Kossovskiy I.G. The study was supported by the RFBR grant № 17-01-00592-a.

Vladimir Lysov, *Keldysh Institute of Applied Mathematics of Russian Academy of Sciences, Moscow, Russia*

Discrete multiple orthogonal polynomials on shifted lattices

There are many ways to define multiple orthogonal polynomials with respect to the classical continuous weights. The approach as in [1–3] preserves a kind of the Rodrigues formula, which is a very useful property. We focus on adapting this approach for the discrete case — bearing in mind the deep connection between the classical discrete and continuous orthogonality.

The talk is devoted to a new class of polynomials of multiple orthogonality with respect to the product of classical discrete weights on integer lattices with noninteger shifts. We obtain explicit representations in the form of the Rodrigues formulas. The case of two weights will be presented in more detail.

This is a joint work with A. Dyachenko (UCL Department of Mathematics, Gower St, London, United Kingdom).

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Grigory Mikhalkin, *University of Geneva, Geneva, Switzerland*

Noncommutative amoebas in the hyperbolic space and coamoebas in the 3-dimensional sphere

One of the simplest examples of noncommutative complex Lie groups is the group $SL_2(\mathbb{C})$ of nonsingular 2×2 matrices with nonzero determinant. From the topological viewpoint it can be identified with a cotangent bundle of a 3-dimensional sphere. Matrix analogues of the logarithm of a norm and an argument define an amoeba-type map into the hyperbolic space and a coamoeba-type map into the 3-dimensional sphere. Properties of curves and surfaces under these mappings will be discussed in this talk. The talk is based on a joint work with Mikhail Shkolnikov.

Alfonso Montes-Rodríguez, *University of Seville, Seville, Spain*

Sampling along characteristics for solutions of the telegraph system

For each function $a : \mathbb{R} \mapsto \mathbb{C}$ with integrable modulus on \mathbb{R} , we define the *exponential telegraphic* function as

$$a_{\mathbb{T}}(x, y) := \int_{\mathbb{R}} a(t) \exp(ixt + iy/t) dt \quad x, y \in \mathbb{R}$$

Every exponential telegraphic function is a continuous solution on \mathbb{R}^2 of the partial differential equation $U_{xy} + U = 0$ with two independent real variables x, y . Conversely, for each continuous solution w of the equation $U_{xy} + U = 0$ on a convex compact subset K in \mathbb{R}^2 with nonempty interior,

there exists an exponential telegraphic function $a_{\mathbb{T}} = a_{\mathbb{T}}(w, K)$ which coincides with w on K whenever w_x and w_y are continuous on K . Exponential telegraphic functions have first been studied in 2011, see [2] where it is proved that each such function can be recoverable sampled at the points $(0, \pi n), (\pi n, 0), n \in \mathbb{Z} := \{\dots, -1, 0, 1, \dots\}$, lying on two characteristics $x = 0$ and $y = 0$ of the equation $U_{xy} + U = 0$. In other words, it follows from $a_{\mathbb{T}}(\pi n, 0) = a_{\mathbb{T}}(0, \pi n) = 0, n \in \mathbb{Z}$, that $a_{\mathbb{T}}(x, y) = 0$ for every $x, y \in \mathbb{R}$. In this work, we provide a new proof of the fact that $a_{\mathbb{T}}(\pi n, 0) = a_{\mathbb{T}}(0, -\pi n) = 0$ for all $n \in \mathbb{N}_0 := \{0, 1, 2, \dots\}$, implies $a_{\mathbb{T}}(x, -y) = 0$ for each $x, y \geq 0$ (cp[1]), which means possibility to restore each exponential telegraphic function in the quadrant $[0, +\infty) \times (-\infty, 0]$ by its values at the points $(0, -\pi n), (\pi n, 0), n \in \mathbb{N}_0$. We apply these results to continuously differentiable one time by each variable solutions $v(t, x)$ and $i(t, x)$ of the telegraph system

$$\begin{cases} i_x(t, x) + C \cdot v_t(t, x) + G \cdot v(t, x) = 0, & R - \text{resistance}, & L - \text{inductance}, & D := LG - CR \neq 0, \\ v_x(t, x) + L \cdot i_t(t, x) + R \cdot i(t, x) = 0, & C - \text{capacitance}, & G - \text{leakance}, & t \geq 0, x \in \mathbb{R}, \end{cases}$$

with the additional restriction of the existence $T > 0$ satisfying $v(t, 0) = i(t, 0) = 0, t \geq T$. It follows that such v and i in the angle $|x| \leq t/\sqrt{LC}, t \geq 0, x \in \mathbb{R}$ between the two characteristics $x = \pm t/\sqrt{LC}$ are uniquely determined by the values of v or i at the points $(2\pi nLC/|D|, \pm 2\pi n\sqrt{LC}/|D|), n \in \mathbb{N}_0$, lying on these characteristics.

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Semyon Nasyrov, *Kazan Federal University, Kazan, Russia*

One-parametric families of ramified coverings of the sphere and uniformization

We consider smooth one-parametric families of ramified coverings of the sphere by compact Riemann surfaces of a fixed genus ρ . In the case $\rho = 0$, these coverings are realized by rational functions and for $\rho = 1$ the functions are elliptic. The main problem is to describe trajectories of critical points and poles of the uniformizing functions if trajectories of critical values are known.

In the simply-connected case ($\rho = 0$), the solution to the problem is given in [1].

In the case of complex tori ($\rho = 1$), the problem is more complicated, since, besides of trajectories of critical points and poles, we also need to describe change of modules of the tori. Earlier, the author investigated the problem under the condition that the uniformizing elliptic functions have a unique pole. The case of simple branch-points lying over finite points of the Riemann sphere is described in [2]; the case of arbitrary multiplicities is studied in [3]. Here we give a solution for the general case, when the uniformizing functions of the family can have a few poles of arbitrary multiplicity.

The desired trajectories of critical points and poles are described with the help of a system of ODEs. Solving the Cauchy problem for the system we can approximately find the uniformizing functions for one-parametric families of ramified coverings.

We also give applications our method to some problems of geometric function theory and potential theory.

This work was supported by the Russian Foundation for Basic Research and the government of the Republic of Tatarstan, project No.18-41-160003.

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Tuen Wai Ng, *The University of Hong Kong*, Pokfulam Road, Hong Kong

Ax-Schanuel type inequalities for functional transcendence via Nevanlinna theory

The Ax-Schanuel Theorem implies that for any \mathbb{Q} -linearly independent modulo \mathbb{C} entire functions of one complex variable f_1, \dots, f_n , the transcendence degree over \mathbb{C} of $f_1, \dots, f_n, e(f_1), \dots, e(f_n)$ is at least $n+1$ where $e(z) = e^{2\pi iz}$. It is natural to ask what happens if one replaces the exponential map e by some other meromorphic functions. In this talk, we will apply Nevanlinna theory to obtain several inequalities of the transcendence degree over \mathbb{C} of $f_1, \dots, f_n, F(f_1), \dots, F(f_n)$ when f_j 's are entire functions with some growth restrictions and F is a transcendental meromorphic function. The results are joint work with Jiaying Huang.

Stepan Orevkov, *Steklov Mathematical Institute of Russian Academy of Sciences*, Moscow, Russia

Algorithmic problem of recognition of quasipositive braids and its relation to plane algebraic curves

A braid is called quasipositive if it is a product of conjugates of the standard generators of the braid group. Quasipositive braids naturally appear in the study of plane algebraic curves. The algorithmic problem to decide whether a given braid is quasipositive is still open. Some partial results in this direction will be presented in the talk.

Roman Palvelev, *Moscow State University and Steklov Mathematical Institute of Russian Academy of Sciences*, Moscow, Russia

Hermite–Padé polynomials for meromorphic functions on a compact Riemann surface

Let \mathfrak{X} be a compact Riemann surface and $\pi: \mathfrak{X} \rightarrow \widehat{\mathbb{C}}$ be a $(m+1)$ -fold branched covering of the Riemann sphere $\widehat{\mathbb{C}}$, $m \geq 1$. Suppose that f_1, f_2, \dots, f_m are meromorphic functions on the Riemann surface \mathfrak{X} such that the functions $1, f_1, f_2, \dots, f_m$ are independent over the field $\mathbb{C}(z)$ of rational functions on $\widehat{\mathbb{C}}$. Fix a point $\circ \in \mathfrak{X}$ that is not critical for the projection π . Without loss of generality we can suppose that $\circ \in \pi^{-1}(\infty)$ and denote $\infty^{(0)} := \circ$. If we choose a small enough neighborhood of $\infty^{(0)}$, then the restriction π_0 of the projection π to this neighborhood is biholomorphic. For $j = 1, \dots, m$ set $f_{j,\infty}(z) := f_j(\pi_0^{-1}(z))$ in the neighborhood of $\infty \in \widehat{\mathbb{C}}$. For convenience we suppose that the germs $f_{j,\infty}$ are holomorphic at ∞ .

The *Hermite–Padé polynomials of the first kind* $Q_{n,0}, \dots, Q_{n,m}$ of order $n \in \mathbb{N}$ for the tuple of germs $[1, f_{1,\infty}, \dots, f_{m,\infty}]$ at the point $\infty \in \widehat{\mathbb{C}}$ are defined as the polynomials of degree not greater than n such that at least one $Q_{n,j} \not\equiv 0$ and the following asymptotic relation at ∞ holds true:

$$Q_{n,0}(z) + \sum_{j=1}^m Q_{n,j}(z) f_{j,\infty}(z) = O\left(\frac{1}{z^{m(n+1)}}\right) \text{ as } z \rightarrow \infty.$$

In the talk we discuss asymptotic behaviour of the ratios $\frac{Q_{n,j}(z)}{Q_{n,k}(z)}$, $k, j = 0, \dots, m$ as $n \rightarrow \infty$. Our research uses the approach of J. Nuttall that is based on a special “Nuttall’s partition” of the Riemann surface \mathfrak{X} into sheets. In particular, our results allow us to asymptotically reconstruct the values of a meromorphic function f on \mathfrak{X} on m Nuttall’s sheets (all except one) from the initial germ of f at $\infty^{(0)}$ as roots of some algebraic equation of degree m . For this one should take $f_j := f^j$, $j = 1, \dots, m$.

The talk is based on the joint work with E.M.Chirka, A.V.Komlov, and S.P.Suetin.

Rakesh Kumar Parmar, *Bikaner Technical University*, Bikaner, India

On certain extended Exton’s triple hypergeometric functions and associated bounding inequalities

Motivated by certain recent extensions of Euler’s beta function, hypergeometric and confluent hypergeometric functions, we extend Exton’s triple hypergeometric functions and investigate to present its properties such as various integral representations of Euler and Laplace type, Mellin transforms, Laguerre polynomial representation, transformation formulae and a recurrence relation.

Also, by means of Luke's bounds for hypergeometric functions and various bounds upon the Bessel functions appearing in the kernels of the newly established integral representations, we deduce a set of bounding inequalities for the extended Exton's triple hypergeometric functions.

This is a joint work with Junesang Choi (Dongguk University, Gyeongju, South Korea).

Dan Popovici, *University of Toulouse, Toulouse, France*

Adiabatic Limit and Deformations of Complex Structures

We prove that if all the fibres, except possibly one, in a holomorphic family of compact complex manifolds are Moishezon (i.e. bimeromorphically equivalent to projective manifolds), then the remaining, limiting, fibre is again Moishezon. Two new ingredients are introduced for this purpose. The first one is the Frölicher Approximating Vector Bundle (FAVB) that displays the degenerating page of the Frölicher spectral sequence as the limit, when a complex constant h tends to 0, of what we call the d_h -cohomology, where $d_h = h\partial + \bar{\partial}$. The second ingredient is the introduction of E_r -sG metrics, for $r \geq 1$, that generalise the strongly Gauduchon metrics we introduced in 2009.

Jugal Kishore Prajapat, *Central University of Rajasthan, Ajmer, India*

Harmonic mappings with fixed analytic part

In this talk, we discuss a family of sense-preserving harmonic mappings whose analytic part is convex in one direction. We prove that functions in this family are close-to-convex for certain values of parameters. Also, we discuss bounds on pre-Schwarzian derivatives and bounds on the Bloch's constant. In addition, we discuss the coefficient estimates, growth and distortion results.

Rajbala, *Central University of Rajasthan, Ajmer, India*

Certain geometric properties of close-to-convex harmonic mappings

In this article, we introduce a new family of sense preserving harmonic mappings $f = h + g$ in the open unit disk and prove that functions in this family are close-to-convex. We give some basic properties such as coefficient bounds, growth estimates, convolution and determine the radius of convexity for the functions belonging to this family. In addition, we construct certain harmonic univalent polynomials belonging to this family.

Azimbay Sadullaev, *National University of Uzbekistan, Tashkent, Uzbekistan*

Oscillatory integrals and Weierstrass polynomials

The well-known Weierstrass theorem states that if $f(z, w)$ is holomorphic at a point $(z^0, w^0) \in \mathbb{C}_z^n \times \mathbb{C}_w$ and $f(z^0, w^0) = 0$, but $f(z^0, w) \not\equiv 0$, then in some neighborhood $U = V \times W$ of this point f is represented as

$$f(z, w) = \left[(w - w^0)^m + c_{m-1}(z)(w - w^0)^{m-1} + \dots + c_0(z) \right] \varphi(z, w), \quad (1)$$

where $c_k(z)$ are holomorphic in V and $\varphi(z, w)$ is holomorphic in U , $\varphi(z, w) \not\equiv 0$, $(z, w) \in U$.

In recent years, the Weierstrass representation (1) has found a number of applications in the theory of oscillatory integrals. Using a version of Weierstrass representation the first author (see [2]) obtained a solution of famous Sogge-Stein problem (see [6]). He obtained also close to a sharp bound for maximal operators associated to analytic hypersurfaces.

In the obtained results the phase function is an analytic function at a fixed critical point without requiring the condition $f(z^0, w) \not\equiv 0$. It is natural to expect the validity of Weierstrass theorem without requiring a condition $f(z^0, w) \not\equiv 0$, in form

$$f(z, w) = \left[c_m(z)(w - w^0)^m + c_{m-1}(z)(w - w^0)^{m-1} + \dots + c_0(z) \right] \varphi(z, w). \quad (2)$$

Such kind of results may be useful to studying of the oscillatory integrals and in estimates for maximal operators on a Lebesgue spaces. However, the well-known Osgood counterexample [3], p.90 (see also [1], p. 68) shows that when $n > 1$ it is not always possible.

In the talk we will discuss, that there is a global option (see [4], [5]), also a global multidimensional (in w) analogue of (2) is true without requiring the condition $f(z^0, w) \neq 0$. In addition, for an arbitrary germ of a holomorphic function, we will prove one representation, that is useful in the study of oscillatory integrals.

This is a joint work with I. Ikramov (Samarkand State University, Samarkand, Uzbekistan).

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Swadesh Kumar Sahoo, *Indian Institute of Technology Indore*, Indore, India **A Gromov hyperbolic metric vs the hyperbolic and other related metrics**

We mainly consider two metrics: a Gromov hyperbolic metric and a scale invariant Cassinian metric. A metric space (D, d) is called Gromov hyperbolic if and only if there exist a constant $\beta > 0$ such that

$$d(x, z) + d(y, w) \leq (d(x, w) + d(y, z)) \vee (d(x, y) + d(z, w)) + 2\beta$$

for all points $x, y, z, w \in D$.

For a domain $D \subsetneq \mathbb{R}^n$ equipped with the Euclidean metric, the u_D -metric [1] is defined by

$$u_D(x, y) = 2 \log \frac{|x - y| + \max\{\text{dist}(x, \partial D), \text{dist}(y, \partial D)\}}{\sqrt{\text{dist}(x, \partial D) \text{dist}(y, \partial D)}}, \quad x, y \in D.$$

Ibragimov proved in [1] that the u_D -metric is Gromov hyperbolic and it coincides with the Vuorinen's distance ratio metric [3, 4] in punctured spaces $\mathbb{R}^n \setminus \{p\}$, for $p \in \mathbb{R}^n$.

A scale invariant version of the Cassinian metric has been studied by Ibragimov in [2] which is defined by

$$\tilde{\tau}_D(x, y) = \log \left(1 + \sup_{p \in \partial D} \frac{|x - y|}{\sqrt{|x - p||p - y|}} \right), \quad x, y \in D \subsetneq \mathbb{R}^n.$$

The interesting part of this metric is that many properties in arbitrary domains are revealed in the setting of once-punctured spaces. For example, $\tilde{\tau}_D$ is a metric in an arbitrary domain $D \subsetneq \mathbb{R}^n$ if it is a metric on once-punctured spaces. The $\tilde{\tau}_D$ -metric is comparable with the Vuorinen's distance ratio metric in arbitrary domains $D \subsetneq \mathbb{R}^n$ if they are comparable in the punctured spaces (see [2]).

Our purpose is to compare the u_D -metric with the hyperbolic and the $\tilde{\tau}$ -metrics.

This is a joint work with Manas Ranjan Mohapatra.

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Gerd Schmalz, *University of New England, Armidale, Australia*
Embeddability of CR-manifolds and shearfree congruences

We introduce a CR-invariant class of Lorentzian metrics on a circle bundle over a 3-dimensional CR-structure, which we call FRT-metrics. These metrics generalise the Fefferman metric, allowing for more control of the Ricci curvature, but are more special than the shearfree Lorentzian metrics introduced by Robinson and Trautman. Our main result is a criterion for embaddability of 3-dimensional CR-structures in terms of the Ricci curvature of the FRT-metrics in the spirit of the results by Lewandowski, Nurowski and Tafel and also Hill, Lewandowski, Nurowski.

This is joint work with Masoud Ganji.

Armen Sergeev, *Stelkov Mathematical Institute of Russian Academy of Sciences, Moscow, Russia*

Adiabatic limit in Yang-Mills equation on \mathbb{R}^4

Harmonic spheres conjecture establishes a correspondence between Yang–Mills G -fields on \mathbb{R}^4 and harmonic maps of the Riemann sphere S^2 into the loop space ΩG of the group G . It is an extension to general Yang–Mills G -fields of the Atiyah–Donaldson theorem establishing a correspondence between the moduli space of G -instantons on \mathbb{R}^4 and holomorphic maps $S^2 \rightarrow \Omega G$.

In our talk we present an approach to the proof of this conjecture based on the adiabatic limit construction proposed by Popov. His construction uses a nice parametrization of the sphere $S^4 \setminus S^1$ with one deleted circle found by Jarvis and Norbury. With the help of this construction one can associate in a natural way with arbitrary Yang–Mills G -field on S^4 a harmonic map of the sphere S^2 to the loop space ΩG .

Alexey Shchuplev, *Siberian Federal University, Krasnoyarsk, Russia*

We say that a complex space (X, \mathcal{O}) admits the Hartogs phenomenon if for any compact subset K of X such that $X \setminus K$ is connected, a restriction homomorphism

$$H^0(X, \mathcal{O}) \rightarrow H^0(X \setminus K, \mathcal{O})$$

is an isomorphism.

In toric varieties this phenomenon has been explored by M. Marciniak [1] who related it to properties of corresponding fans:

Theorem. *If X_Σ is a smooth toric surface with a strictly convex fan Σ then X_Σ admits the Hartogs phenomenon.*

She has also formulated a conjecture for toric varieties of higher dimensions: *A smooth toric variety X_Σ admits the Hartogs phenomenon if the complement of its fan Σ has at least one concave connected component.*

We were able to prove it not only for smooth but also for normal toric varieties. Let X_Σ be a normal toric variety corresponding to a fan $\Sigma \subset \mathbb{R}^d = \mathbb{Z}^d \otimes_{\mathbb{Z}} \mathbb{R}$. We shall say that a connected component of $\mathbb{R}^d \setminus |\Sigma|$ is concave if its convex hull coincides with \mathbb{R}^d .

Theorem. *Let X_Σ be a normal toric variety. If the complement of its fan Σ has at least one concave connected component then X_Σ admits the Hartogs phenomenon.*

The proof follows from the study of Dolbeault cohomology with compact support of a smooth toric variety where X_Σ can be equivariantly embedded.

This is a joint work with S. Feklistov.

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David Shoikhet, *Holon Institute of Technology*, Holon, Israel

Trends and Problems in Complex Dynamics and Geometric Function Theory

Complex dynamical systems and nonlinear semigroup theory are not only of intrinsic interest, but are also important in the study of evolution problems. In recent years many developments have occurred, in particular, in the area of nonexpansive semigroups in Banach spaces. As a rule, such semigroups are generated by accretive operators and can be viewed as nonlinear analogs of the classical linear contraction semigroups. Another class of nonlinear semigroups consists of those semigroups generated by holomorphic mappings in complex finite and infinite dimensional spaces. Such semigroups appear in several diverse fields, including, for example, the theory of Markov stochastic branching processes, Krein spaces and the geometry of complex Banach spaces. In this talk based on the joint work with M.Elin and T.Sugawa we concentrate on trends and problems related to the nonlinear resolvent method and its connections to the classical geometric function theory. Also some applications to complex Banach algebras will be presented.

Young Jae Sim, *Kyungsung University*, Busan, South Korea

On coefficient problems for close-to-star functions

Let \mathcal{A} be the class of analytic functions in the unit disk \mathbb{D} which have the form $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$. And let \mathcal{CST} be the subclass \mathcal{A} consisting close-to-star functions. For given $q, n \in \mathbb{N}$ and $f \in \mathcal{A}$, the Hankel determinants $H_{q,n}(f)$ is defined as

$$H_{q,n}(f) := \begin{vmatrix} a_n & a_{n+1} & \cdots & a_{n+q-1} \\ a_{n+1} & a_{n+2} & \cdots & a_{n+q} \\ \vdots & \vdots & \vdots & \vdots \\ a_{n+q-1} & a_{n+q} & \cdots & a_{n+2(q-1)} \end{vmatrix}.$$

And, for given $m, n \in \mathbb{N} \setminus \{1\}$, the Zalcman functional $J_{n,m}(f)$ of $f \in \mathcal{A}$ is defined by

$$J_{n,m}(f) := a_{n+m-1} - a_n a_m.$$

In this talk, we discuss the sharp estimates of the second Hankel determinants such as $H_{2,1}$ and $H_{2,2}$ and the Zalcman functional $J_{2,3}$ over several subclasses of \mathcal{CST} . Also, the sharp bounds of early logarithmic coefficients and coefficients of the inverses of close-to-star functions are investigated.

This is a joint work with Oh Sang Kwon (Kyungsung University, Busan, Korea) and Nak Eun Cho (Pukyong National University, Busan, Korea).

Toshiyuki Sugawa, *Tohoku University*, Sendai, Japan ***Separation theorems of Teichmüller type***

A ring domain (annulus) in the complex plane contains a round (genuine) subring of the form $r_1 < |z - a| < r_2$ if the ring domain has a large enough modulus m . Moreover, the subring can be taken so that $\log(r_2/r_1) \geq m - C$, where C is an absolute constant. This sort of result was first proved by Teichmüller. In [1], we introduced a notion of semi-annulus and its modulus and applied it to study boundary continuity of homeomorphisms of a disk or a half-plane.

In the present talk, we extend these result into the n -dimensional case. Indeed, we have similar results for rings and semi-rings in \mathbb{R}^n . This is joint work with Anatoly Golberg.

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Alexander Sukhov, *University of Lille*, Lille, France

Local geometry of Levi-flat singularities

We discuss recent results on the local theory of singular real analytic Levi-flat manifolds of the hypersurface type

Kiyoshi Takeuchi, *University of Tsukuba*, Tsukuba, Japan

On irregularities of Fourier transforms of regular holonomic D-modules

Fourier transforms of regular holonomic D-modules are not regular in general. In this talk, we introduce our recent results on their irregularities. First, by using the irregular Riemann-Hilbert correspondence of D'Agnolo-Kashiwara and the theory of Fourier-Sato transforms for enhanced indsheaves of Kashiwara-Schapira etc., we obtain a formula for their enhanced solution complexes. Then we show that the irregularities of the Fourier transforms are expressed by the geometries of the original D-modules. The result can be applied to A-hypergeometric functions.

This is a joint work with Yohei Ito.

Alekos Vidras, *University of Cyprus*, Nicosia, Cyprus

Locally explicit version of Fundamental Principle for homogeneous convolution equations Using a weighted Koppelman integral representation formula with a complex parameter a division formula in the space of weighted entire function is proved. This formula is used to derive a locally explicit version of Ehrenpreis's Fundamental Principle for a system of homogeneous convolution equations $\hat{f} * \mu_j = 0$, $j = 1, \dots, m$, $f \in \mathcal{E}(\mathbb{R}^n)$, $\mu_j \in \mathcal{E}'(\mathbb{R}^n)$, when the Fourier Transforms $\hat{\mu}_j$, $j = 1, \dots, m$ are slowly decreasing and form a complete intersection in \mathbb{C}^n .

Yuefei Wang, *Institute of Mathematics, Chinese Academy of Sciences*, Beijing, China

On dynamics of entire maps with symmetry

We will talk about recent results on the dynamics of holomorphic maps with certain symmetries, including families of Baker's wandering maps, exponential maps and hyperbolic maps, cosine maps etc. Such a map either contains a dynamical ray in its Julia set or has a wandering domain. Moreover, any connected compact set containing a point on the ray and its forward image point intersects its Julia set.

Atsushi Yamamori, *Yamaguchi University*, Yamaguchi, Japan

Two variations of Boas-Fu-Straube's deflation identity

We study deflation type identities of the Bergman kernel functions. The first identity of this kind was obtained by Boas, Fu and Straube in 1999. In this talk, we introduce the Roos domains and establish two variations of deflation type identities.

Masahiro Yanagishita, *Yamaguchi University*, Yamaguchi, Japan

Weil-Petersson metric on square integrable Teichmüller space

Let Γ be a Fuchsian group acting on the upper half-plane \mathbb{H} . The square integrable Teichmüller space is a subset of the Teichmüller space $T(\Gamma)$ which consists of the Teichmüller equivalence classes with p -integrable Beltrami coefficients as their representatives. Here, a Beltrami coefficient is square integrable if it is square integrable with respect to the hyperbolic metric on the Riemann surface \mathbb{H}/Γ .

When Γ is of analytically finite type, $T(\Gamma)$ has a finite dimensional Hilbert manifold structure. The Weil-Petersson metric is an Hermitian metric on this structure. It is known that this metric is Kähler and is not complete. Moreover, it has the negative holomorphic sectional curvature, the negative Ricci curvature, and the negative Scalar curvature. In this talk, we will extend the concept of the Weil-Petersson metric to the square integrable Teichmüller space of Fuchsian groups of analytically infinite type.

Chung-Chun Yang, *Hong Kong University of Science and Technology*, Hong Kong

Applications of Nevanlinna's value distribution theory to functional equations and related conjectures

It's well known that Nevanlinna's value distribution theory is the most useful, powerful and effective tool in dealing with the functional equations of meromorphic functions. In the talk, some unsolved old and new conjectures (mainly posed by the speaker over past decades) that related to admissible meromorphic solutions of functional equations of various types, including differential and difference equations as well as Fermat type of equations, along with the background and progresses of these conjectures will be surveyed or reported, for further studies.

Fei Yang, *Nanjing University*, Nanjing, China
Sierpinski carpet Julia sets of holomorphic maps

Sierpinski carpet is one of the most famous fractals. As a special class of Julia sets, they appeared after the work of Milnor and Tan Lei in 1993. In this talk, we will give some examples of holomorphic maps (including rational maps and transcendental entire functions) whose Julia sets are Sierpinski carpets and study their quasisymmetric geometry, Hausdorff dimension, Lebesgue measure and etc.

Alain Yger, *University of Bordeaux 1*, Bordeaux, France

Generalized cycles and Bochner-Martinelli currents: intersection and division

I will first explain in this talk why one needs to enlarge the geometric notion of complex cycle in order to conciliate local and global intersection numbers in improper intersection theory, focusing in particular on the situation in $\mathbb{P}^n(\mathbb{C})$. This part of my talk will summarize the results obtained jointly with M. Andersson, D. Eriksson, H. Samuelsson Kalm, E. Wulcan in the last five years. I will then discuss the recent result obtained by my former student R. Gualdi, leading to a closed formula (in terms of the so-called Ronkin function) for the logarithmic height of an hypersurface (defined over \mathbb{Q}) in a complete toric variety and explain why the difficulties that arise when one tries to extend this result to cycles with higher codimension motivate the necessity to transpose the notion of generalized cycle to the arithmetic setting. Thinking about division instead of intersection, the Bochner-Martinelli residue currents play in improper intersection theory a role similar to that played by representants of generalized cycles in intersection theory. I will emphasize in this lecture the central role played by Crofton's formula since such objects can be interpreted as averaged over a product of projective spaces (with respect to Fubini-Study metric on the coordinate spaces) of Coleff-Herrera currents. After recalling the effectivity results obtained with M. Sombra with respect to multivariate residue calculus in the complete intersection setting, it will be natural to conclude this lecture addressing questions (still open) about the effectiveness of the realization of Briançon-Skoda theorem within an arithmetic frame. My talk intends also to be a tribute to Jan Erik Björk, who unfortunately left us very recently; he indeed inspired many of the topics I will discuss here.

Xiangyu Zhou, *Institute of Mathematics, Chinese Academy of Sciences*, Beijing, China
Recent results in several complex variables

We'll report some new results on optimal L^2 extension and multiplier ideal sheaves.