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The effect of interfacial heat transfer energy on a two-layer creeping flow in a flat channel

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Abstract. The study of convective currents in the channel received substantial attention for potential use in numerous engineering and manufacturing processes. The paper considers the two-dimensional motion of an immiscible incompressible binary mixture and a viscous heat-conducting fluid in a flat layer. The fluid system is bounded by solid walls. The solution of the equations of thermal diffusion convection is described in a special form: one of the velocity components linearly depends on the longitudinal coordinate, the temperature and concentration fields have a quadratic dependence on the horizontal coordinate. This velocity field corresponds to the well-known solution of Himenz for a purely viscous fluid. At the interface of liquids, the condition of energy exchange is taken into account. An exact stationary solution of the convective problem is constructed. It allows studying the influence of interfacial heat exchange energy on the dynamics of fluid and mixture flow.

1. Introduction

Convective flows in multilayer systems play a decisive role in various natural and technological phenomena: the study of convective mixing in the fields of the chemical industry and space materials science, etc. Thermocapillary convection in the absence of thermogravity is possible in zero gravity. In the field of gravity, both types of convection usually manifest themselves, however, in thin layers of liquid the thermocapillary mechanism plays the main role. Much attention is paid to the study of currents in two-layer systems [1–3]. The description of the initial-boundary-value problem of the motion of the multi-layer flow of the system of the unmixed viscous fluids with a common interface surface, exact solution of a stationary problem and the solution of the nonstationary problem is presented in [4–5].

In this work, we study the two-layer flow of a binary mixture system and a viscous fluid with a common interface with local heating of solid walls according to a parabolic law, taking into account interfacial heat transfer. The exact solution of the stationary problem is constructed.

2. Statement of the problem

Figure 1 displays schematically the physical geometry of the chosen study. Let a binary mixture fill a layer located on a solid fixed substrate $y = 0$ and having the following dimensions $|x| < \infty$, $0 < y < l_1$. Above this layer, there is a layer of viscous heat-conducting fluid with a size $|x| < \infty$, $l_1 < y < l_2$, where $y = l_2 = \text{const}$ is a solid fixed wall. Thus, the line $y = l_1$ is the undeformable interface between two media. The system is in a zero-gravity condition.



The coefficient of surface tension depends linearly on the temperature and concentration at the interface $\sigma(\theta_1, C) = \sigma^0 - \alpha_1 \theta_1 - \alpha_2 C$ with the constants $\sigma^0 > 0$, $\alpha_1 > 0$, α_2 .

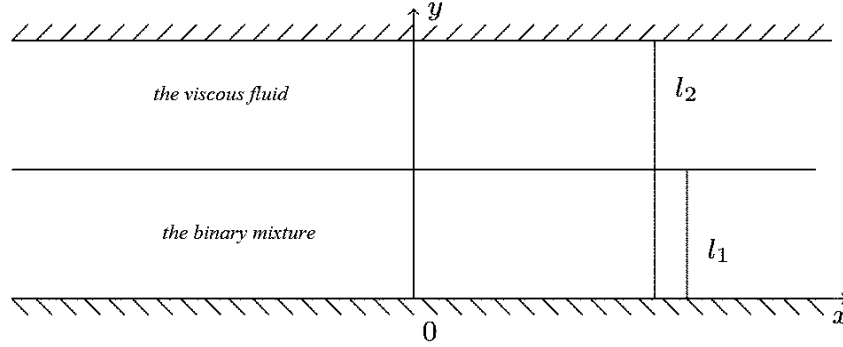


Figure 1. Physical configuration and coordinate system.

The two-dimensional stationary convective motion of a two-layer system in the absence of mass forces is described by the system

$$\begin{aligned}
 u_j u_{jx} + v_j u_{jy} + \frac{1}{\rho_j} p_{jx} &= \nu_j (u_{jxx} + u_{jyy}), \\
 u_j v_{jx} + v_j v_{jy} + \frac{1}{\rho_j} p_{jy} &= \nu_j (v_{jxx} + v_{jy}), \\
 u_j \theta_{jx} + v_j \theta_{jy} &= \chi_j (\theta_{jxx} + \theta_{jyy}), \\
 u_1 C_x + v_1 C_y &= D (C_{xx} + C_{yy} + \alpha D (\theta_{1xx} + \theta_{1yy})), \\
 u_{jx} + v_{jy} &= 0.
 \end{aligned} \tag{1}$$

where (u_j, v_j) is the velocity field, p_j is the pressure deviation from hydrostatic pressure, θ_j is the temperature, C is the concentration of the light component in the layer with the binary mixture, ρ_j is density, $\nu_j = \mu_j / \rho_j$ is the kinematic viscosity, χ_j is the thermal diffusivity, D is the coefficient diffusion and α is the thermal diffusion parameter. All media specifications are assumed to be constant and correspond to the average values temperature and concentration. Index j numbers the liquid, the first liquid is located at the bottom.

The boundary conditions on solid walls are

$$y = 0 : \quad u_1 = v_1 = 0, \quad \theta_1 = \theta_{10}(x), \quad C_y + \alpha \theta_{1y} = 0;$$

$$y = l_2 : \quad u_2 = v_2 = 0, \quad \theta_2 = \theta_{20}(x).$$

On the interface surface $y = l_1$ the conditions are as follows:

$$u_1 = u_2, \quad v_1 = v_2 = 0, \quad \theta_1 = \theta_2,$$

$$\mu_2 u_{2y} - \mu_1 u_{1y} = \sigma_x,$$

$$k_2 \theta_{2y} - k_1 \theta_{1y} = -\sigma_\theta u_x \theta,$$

$$C_y + \alpha \theta_{1y} = 0,$$

k_j is coefficient of thermal conductivity.

3. The construction of an exact solution

It is assumed that the velocity field, temperature and concentration in the layers are described as follows:

$$\begin{aligned} u_j &= U_j(y)x, & v_j &= V_j(y); \\ \theta_j &= A_j(y)x^2 + B_j(y), & c &= H(y)x^2 + K(y), \\ p_j &= P(x, y). \end{aligned} \quad (2)$$

The velocity field in (2) corresponds to the well-known Hiemenz solution for a purely viscous fluid. For $x = 0$, the horizontal component of the velocity is $u_j = 0$, therefore the motion in the system is purely vertical. The component u_j is antisymmetric with respect to x , and the terms $A_j(y)x^2, H(y)x^2$ are symmetric about the vertical axis. The quadratic specification of the temperature field on the solid walls $\theta_{10} = A_{10}x^2 + B_{10}$ and $\theta_{20} = A_{20}x^2 + B_{20}$ allows you to study the flow that occurs near the temperature extremum point (if $A_j > 0$ that the temperature has a minimum at $x = 0$ and it is the maximum temperature at $A_j < 0$).

Let there be dimensionless independent variables $\xi = x/l_2$, $\eta = y/l_2$ and velocities, pressures, temperatures and concentrations:

$$\begin{aligned} \bar{U}_j &= \frac{l_2^2}{\chi_1} U_j, & \bar{V}_j &= \frac{l_2}{\chi_1} V_j, & \bar{A}_j &= \frac{1}{A_{10}} A_j, & \bar{B}_j &= \frac{1}{A_{10}l_2^2} B_j, \\ \bar{S}_j &= \frac{l_2^4}{\chi_1^2} S_j, & \bar{H} &= \frac{H}{\alpha A_{10}}, & \bar{K} &= \frac{K}{\alpha A_{10}l_2^2}. \end{aligned}$$

Then the equation of motion of the media are written in a dimensionless form

$$V_j U_{j\eta} + U_j^2 = \frac{\nu_j}{\chi_1} U_{j\eta\eta} + S_j, \quad U_j + V_{j\eta} = 0,$$

$$2U_j A_j + V_j A_{j\eta} = \frac{\chi_j}{\chi_1} A_{j\eta\eta}, \quad V_j B_{j\eta} = \frac{\chi_j}{\chi_1} (2A_j + B_{j\eta\eta}),$$

$$2U_1 H + V_1 H_\eta = \frac{D}{\chi_1} (H_{\eta\eta} + A_{1\eta\eta}), \quad V_1 K_\eta = \frac{D}{\chi_1} (2H + K_{\eta\eta} + 2A_1 + B_{1\eta\eta})$$

(the bar is omitted below). The boundary conditions on the solid walls are written in the form

$$\eta = 0: \quad U_1 = V_1 = 0, \quad A_1 = 1, \quad B_1 = \frac{B_{10}}{A_{10}l_1^2} \equiv B_{10}, \quad H_\eta + A_{1\eta} = 0, \quad K_\eta + B_{1\eta} = 0;$$

$$\eta = 1: \quad U_2 = V_2 = 0, \quad A_2 = \frac{A_{20}}{A_{10}} = \delta, \quad B_2 = \frac{B_{20}}{A_{10}l_1^2} \equiv B_{20};$$

and the boundary conditions on the interface $y = l_1$ have the form

$$U_1 = U_2, \quad V_1 = V_2 = 0, \quad U_{2\eta} - \mu U_{1\eta} = -2M_T A_1 - 2M_K H,$$

$$A_1 = A_2, \quad B_1 = B_2, \quad A_{2\eta} - k A_{1\eta} = M_0 A_1 U_1, \quad B_{2\eta} - k B_{1\eta} = M_0 B_1 U_1,$$

$$H_\eta + A_{1\eta} = 0, \quad K_\eta + B_{1\eta} = 0.$$

Here

$$M_T = \frac{\alpha_1 l_2^3 A_{10}}{\rho_2 \nu_2 \chi_1}, \quad M_K = \frac{\alpha_2 l_2^3 A_{10}}{\rho_2 \nu_2 \chi_1}, \quad M_0 = \frac{\alpha_1 \chi_1}{k_2 l_2}, \quad k = \frac{k_1}{k_2}, \quad \mu = \frac{\rho_1 \nu_1}{\rho_2 \nu_2}.$$

It is assumed that thermocapillary and thermoconcentration effects are comparable with each other ($M_T \sim M_K$) and $U_j = M_T U_j^*$, $V_j = M_T V_j^*$, $S_j = M_T S_j^*$. Then a mathematical model describing convective motion in the system has the form

$$\begin{aligned}
 M_T (V_j U_{j\eta} + U_j^2) &= \frac{\nu_j}{\chi_1} U_{j\eta\eta} + S_j, \\
 U_j + V_{j\eta} &= 0, \\
 M_T (2U_j A_j + V_j A_{j\eta}) &= \frac{\chi_j}{\chi_1} A_{j\eta\eta}, \\
 M_T V_j B_{j\eta} &= \frac{\chi_j}{\chi_1} (2A_j + B_{j\eta\eta}), \\
 M_T (2U_1 H + V_1 H_\eta) &= \frac{D}{\chi_1} (H_{\eta\eta} + \alpha A_{1\eta\eta}), \\
 M_T V_1 K_\eta &= \frac{D}{\chi_1} (2H + K_{\eta\eta} + \alpha(2A_1 + B_{1\eta\eta})),
 \end{aligned} \tag{3}$$

with boundary conditions

$$\begin{aligned}
 \eta = 0: \quad U_1 = V_1 = 0, \quad A_1 = 1, \quad B_1 = \frac{B_{10}}{A_{10} l_1^2} \equiv B_{10}^*, \quad H_\eta + A_{1\eta} = 0, \quad K_\eta + B_{1\eta} = 0, \\
 \eta = 1: \quad U_2 = V_2 = 0, \quad A_2 = \frac{A_{20}}{A_{10}} = \delta, \quad B_2 = \frac{B_{20}}{A_{10} l_1^2} \equiv B_{20}^*, \\
 \eta = \gamma = \frac{l_1}{l_2}: \quad U_1 = U_2, \quad V_1 = V_2 = 0, \quad U_{2\eta} - \mu U_{1\eta} = -2A_1 - 2\alpha H, \\
 A_1 = A_2, \quad B_1 = B_2, \quad H_\eta + A_{1\eta} = 0, \quad K_\eta + B_{1\eta} = 0; \\
 A_{2\eta} - k A_{1\eta} = E A_1 U_1, \quad B_{2\eta} - k B_{1\eta} = E B_1 U_1.
 \end{aligned} \tag{4}$$

Here we have introduced the notation $\alpha = \alpha_2/\alpha_1$, $E = \alpha_1^2 A_{10} l_2^2 / \rho_2 \nu_2 k_2$. The parameter E determines the influence of internal interfacial energy on the dynamics of the movement of fluids inside the layers. This parameter is small for ordinary liquids at $t = 15-20^\circ C$. So in experiments in the system “air-ethyl alcohol” with $\theta = 15^\circ C$ we have $E \sim 5 \cdot 10^{-4}$.

To solve the problem (3)–(5), we consider the model problem of creeping flow when all nonlinear terms in (3) are excluded but conditions (5) are nonlinear.

To identify the features of the thermocapillary flow, we obtain an analytical solution of the conjugate problem (3)–(5) in each of the layers

$$\begin{aligned}
 A_1 &= -C_1 \eta + 1, \quad A_2 = \left[-\frac{\gamma(1-\gamma)(\alpha-1)(C_1\gamma-1)^2 E}{2(\mu(1-\gamma)+\gamma)} - k C_1 \right] (\eta-1) + \delta, \\
 U_1 &= \frac{(\alpha-1)(C_1\gamma-1)(1-\gamma)}{2\gamma(\mu(1-\gamma)+\gamma)} \eta(3\eta-2\gamma), \quad V_1 = \frac{(\alpha-1)(C_1\gamma-1)(1-\gamma)}{2\gamma(\mu(1-\gamma)+\gamma)} \eta^2(\eta-\gamma), \\
 U_2 &= \frac{(\alpha-1)(C_1\gamma-1)\gamma}{2(1-\gamma)(\mu(1-\gamma)+\gamma)} (\eta-1)(3\eta-2\gamma-1), \\
 V_2 &= -\frac{(\alpha-1)(C_1\gamma-1)\gamma}{2(1-\gamma)(\mu(1-\gamma)+\gamma)} (\eta-1)^2(\eta-\gamma). \\
 H &= C_1 \eta - 1.
 \end{aligned} \tag{6}$$

The constant C_1 is defined as the solution of a quadratic equation derived from boundary conditions (5)

$$-\frac{\gamma^3 E (\alpha-1)(1-\gamma)^2}{2(\mu(1-\gamma)+\gamma)} C_1^2 + \left[\frac{\gamma^2 (\alpha-1)(1-\gamma)^2 E}{\mu(1-\gamma)+\gamma} - k + k\gamma - \gamma \right] C_1 + 1 - \delta - \frac{\gamma(\alpha-1)(1-\gamma)^2 E}{2(\mu(1-\gamma)+\gamma)} = 0.$$

This equation can have one or two roots or have no roots. It depends on the sign of the discriminant

$$\Delta = (k(1 - \gamma) + \gamma)^2 - \frac{2\gamma^2(\varkappa - 1)(1 - \gamma)^2(k(1 - \gamma) + \delta\gamma)E}{\mu(1 - \gamma) + \gamma}.$$

The expressions for B_j , K can be obtained with help of formulas (3)–(5) and are not shown here because they are cumbersome.

Expression (6) are used to determine velocities, temperatures and concentration fields. The model scheme of the two-layer fluid flows with parameters $Pr_1 = 29$, $Pr_2 = 1.52$, $\rho = 0.945$, $\nu = 7.1$, $k = 0.42$, $\psi = 1.49$, $l_1 = 0.001$, $l_2 = 0.002$, $E = -0.328 \cdot 10^{-9}$ is used.

Figure 2 illustrates the profiles of the function $U(\eta)$ and $V(\eta)$ for the case where $A_{20} = 0$, $A_{10} < 0$, i.e. the temperatures has a maximum on lower solid wall at the point $\xi = 0$.

The vertical velocity component in the lower layer is position, the horizontal velocity component changes its sign, which means that the fluid in the lower layer at $\xi = 0$ moves vertically upward along the η axis. This conclusion agrees with figure 2.

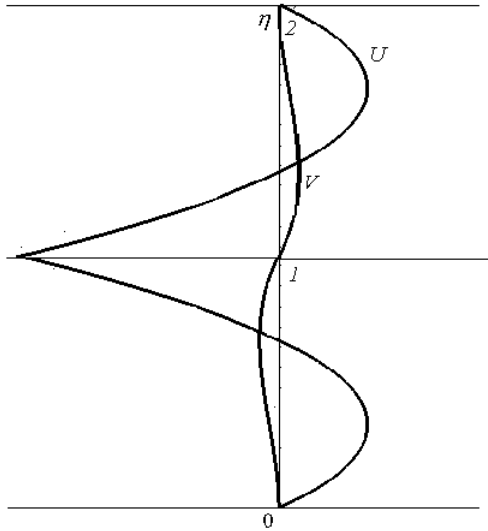


Figure 2. Velocity profiles $U(\eta)$ and $V(\eta)$ for $A_{20} = 0$, $A_{10} < 0$.

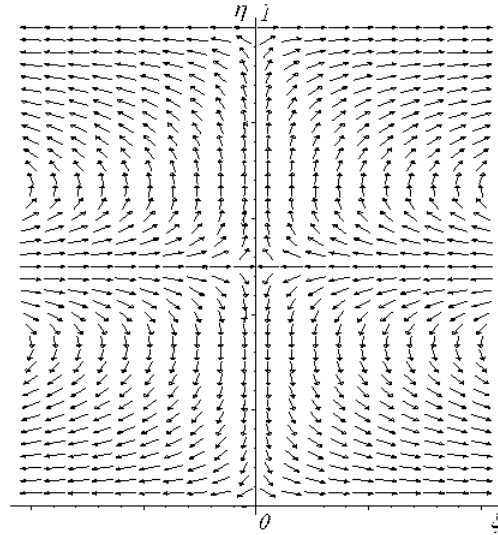


Figure 3. Stationary vector field $u = U(\eta)\xi$, $v = V(\eta)$ for $A_{20} = 0$, $A_{10} < 0$.

Suppose that the temperature and concentration coefficients at the interface are equal, then $\varkappa = 1$. In this case there is thermal diffusion equilibrium state in the system as can be seen from (6) i.e. $U = 0$, $V = 0$.

4. Conclusion

The main result of this paper is the construction of the exact solution taking into account the influence of interfacial heat transfer energy. The calculation results show that thermal-concentration forces form a complex motion in the layers, while the flow changes its direction along the layer thickness. When the temperature and concentration coefficient of surface tension are equal, the system is in thermal diffusion equilibrium.

Acknowledgments

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