

# Determination of Phase Ambiguity in the Interferometer Using a Three-Frequency GLONASS Signal

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**Abstract**— The article describes the use of a three-frequency GLONASS signal to determine the phase ambiguity in the interferometer. The use of a three-frequency signal allows with high reliability to determine the phase ambiguity when using a sufficiently large distance between the antennas. In addition, it is possible to determine the phase ambiguity for each source of navigation signals separately.

**Keywords**— *phase ambiguity, three-frequency signal, GLONASS, interferometer, definition of phase ambiguity*

## I. INTRODUCTION

At present, GLONASS and GPS phase radio navigation equipment is of great interest to consumers. In this case, there are two directions for the use of such equipment - high-precision measurement of coordinates and determination of the spatial orientation of objects [1–3]. Multidimensional interferometers consisting of several antennas are used to measure the spatial orientation. As a rule, 2 or three bases are used. The equation of a single-base interferometer, by which its orientation is determined, has the following form

$$k_{xi}X + k_{yi}Y + k_{zi}Z = \Phi_i \cdot (\lambda_i / 2\pi) + n_i \lambda_i \quad (1)$$

where  $k_{xi}$ ,  $k_{yi}$ ,  $k_{zi}$  – are the direction cosines of the vector directed to the navigation spacecraft (NS);  $X$ ,  $Y$ ,  $Z$  – are the coordinates of the base vector,  $i = 1, 2, \dots, N$  – is the sequence number of the observed NS;  $\lambda_i$  – is the wavelength of the signals NS;  $\Phi_i$  – measured phase shift between antennas.

The number of unknown variables in equation (1) is three, therefore, to solve the problem, it is necessary to measure phase shifts from at least three navigation spacecraft.

The main problem with phase measurements is the presence of phase ambiguity of received signals. This problem occurs because the distance between receiving antennas  $B$  is much longer than the wavelength of the signals of the NS (Fig. 1).

Given the phase ambiguity, equation (1) is converted to the following form

$$k_{xi}X + k_{yi}Y + k_{zi}Z = \Phi_i \cdot (\lambda_i / 2\pi) + n_i \lambda_i \quad (2)$$

where  $n_i$  is the integer phase ambiguity.

If there is a phase ambiguity, there is an unknown value  $n_i$  for each measurement of the phase shift. In this case, for any number of measurements, the number of unknowns will always be greater than the number of equations, and the system of equations will always be degenerate.

Due to the phase ambiguity integrality with more than three dimensions, the system of equations will be inconsistent if the phase ambiguity is incorrectly defined. That is, when solving a system of equations by the least squares method, there will be discrepancies even in the absence of measurement error.

Thus, with excessive measurements, there is a fundamental possibility of determining the phase ambiguity. Measurement redundancy is achieved in various ways.

The first method is the redundancy of the navigation constellation. Under normal conditions in open areas, signals from 6 to 10 NS of each system (GLONASS and GPS) are usually received. This amount is sufficient to determine the phase ambiguity [1–6]. However, in conditions of rough terrain or dense urban development, there may be a shortage of NS signals.

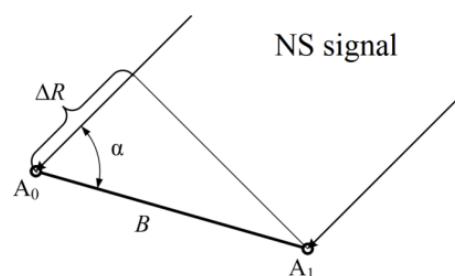


Fig. 1. Phase shift measurement ambiguity

The second method uses antenna redundancy. When measuring on a multi-element antenna array, it is possible to use the direction-finding method of measurement, and an excessive number of bases allows the resolution of phase ambiguity for each navigation satellite separately. This method is applicable in unfavorable conditions for receiving navigation signals – in conditions of interference or limited visibility. However, it requires a significant complication of navigation equipment [6, 7].

The third way is redundancy of navigation signals. In GLONASS, signals are used in three frequency ranges – L1, L2 and L3. They are intended to compensate for the ionospheric delay and can be used to determine the phase ambiguity.

This method can be used in conditions of a shortage of received signals from navigation satellites, including when measuring the spatial orientation from the signals of pseudosatellites [8].

## II. USING THE THREE-FREQUENCY SIGNAL TO DETERMINE THE PHASE AMBIGUITY

When interferometric measurements of a multi-frequency signal, there are several measured phase shifts  $\phi_i$  that are a function of one parameter  $\alpha$  (the angle between the direction to the signal source and the base vector). A continuous change in the parameter value corresponds to a hodograph of the signal vector  $\alpha$ , which in the theory of potential noise immunity is called the signal line. Each point of this line corresponds to a certain value of the parameter  $\alpha$ . Thus, the signal line serves as a scale on which the parameter values are plotted when it changes in the absence of measurement errors.

If there are measurement errors, the vector of measured values may not be on the signal line. Then the optimal estimate of the value of the parameter is found by projecting the vector of the received signal onto the signal line or as the point of the signal line closest to the measurement vector received with errors.

In the case of a two-frequency signal, the signal line in space  $(\phi_1, \phi_2)$  is a straight line. Indeed, from (1) we have:

$$\lambda_2 \phi_2 / (2\pi) = \lambda_1 \phi_1 / (2\pi) \text{ or } \phi_2 = \phi_1 \lambda_1 / \lambda_2 = \phi_1 f_2 / f_1 = \phi_1 m/n \quad (3)$$

where  $f_{1,2}$  is the navigation signal.

If there is an integer ambiguity, the signal line is displayed in the region of possible values of phase shifts, which has the shape of a square with a side equal to  $2\pi$ . When the signal reaches the boundary of the area, it jumps to the opposite side.

This corresponds to changing the value of the integer ambiguity on one of the signals. As a result, the signal line, taking into account the ambiguity, is a series of parallel lines (Fig. 2).

With the integer frequency ratio  $m/n$ , the signal line closes on itself with a path difference equal to  $\Delta R = m\lambda_1 = n\lambda_2$ . The slope of the signal line depends on the frequency ratio. The angle of inclination is  $\tan \alpha = m/n$ , and the distance between the signal lines –  $\ell = 2\pi / \sqrt{m^2 + n^2}$ .

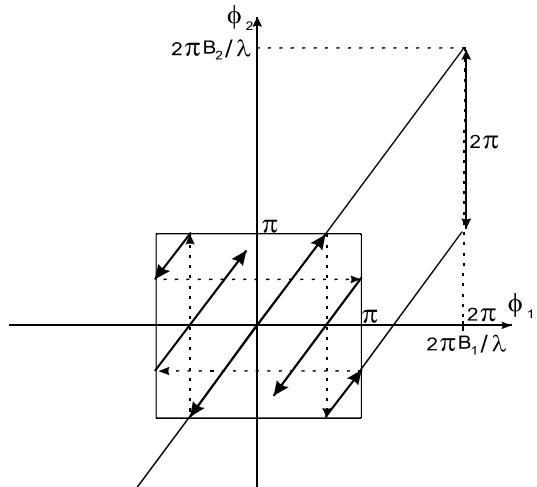


Fig. 2. Signal line

With an integer frequency ratio, the signal line is closed when it is continued and the distance between adjacent lines is the same. For a given integer frequency ratio  $m/n$ , the optimal base length is  $B = m\lambda_1 = n\lambda_2$ , while the signal line is closed, but not imposed on itself, as shown in Fig. 2

If the length of the bases is less, then there is an incomplete use of the phase space, i.e. the signal line is broken to the point of closure. If the length of the bases is longer, then the signal line itself is superimposed on itself, i.e. for some angles it is in principle impossible to unambiguously resolve the phase ambiguity.

Assessment of probability of the correct permission of phase ambiguity it is convenient to take on function of credibility in the absence of errors measurements of phase shift, or on a logarithm of this function equal to the sum of squares of not knittings.

When using two frequencies of L1 and L2 the relation of frequencies makes  $f_1/f_2 = m/n = 7/9$ . The distance between the signal lines is  $\ell = 2\pi / \sqrt{m^2 + n^2} = 2\pi / \sqrt{7^2 + 9^2} = 31.5^\circ$ . The resolution of the phase ambiguity is possible with a maximum error in measuring phase shifts  $\sigma_\phi = 10^\circ$ . In this case, the maximum length of the base of the interferometer is equal to  $B_{max} = 1.7$  m. Function of credibility with a length of base 3 m is given in Fig. 3.

In case of use of signals in the ranges of L1 and L3 the ratio of frequencies makes  $f_1/f_3 = m/n = 4/3$ . The phase distance between lines of signals  $72^\circ$ , however the maximum length of base is only 0.76 m. Function of credibility with a length of base 3 m is given in Fig. 4.

When using L2 and L3, the relation of frequencies of  $m/n = 28/27$ , phase distances between lines of signals  $9.2^\circ$  with the maximum length of base of 6.8 m. Function of credibility with a length of base 3m is given in Fig. 5.

Apparently from schedules, minima of function of credibility are multiple to distance between lines of signals.

On the one hand, the distance between lines of signals increases with reduction of  $m$  and  $n$ , but at the same time also the maximum length of base with which the line of signals becomes isolated decreases.

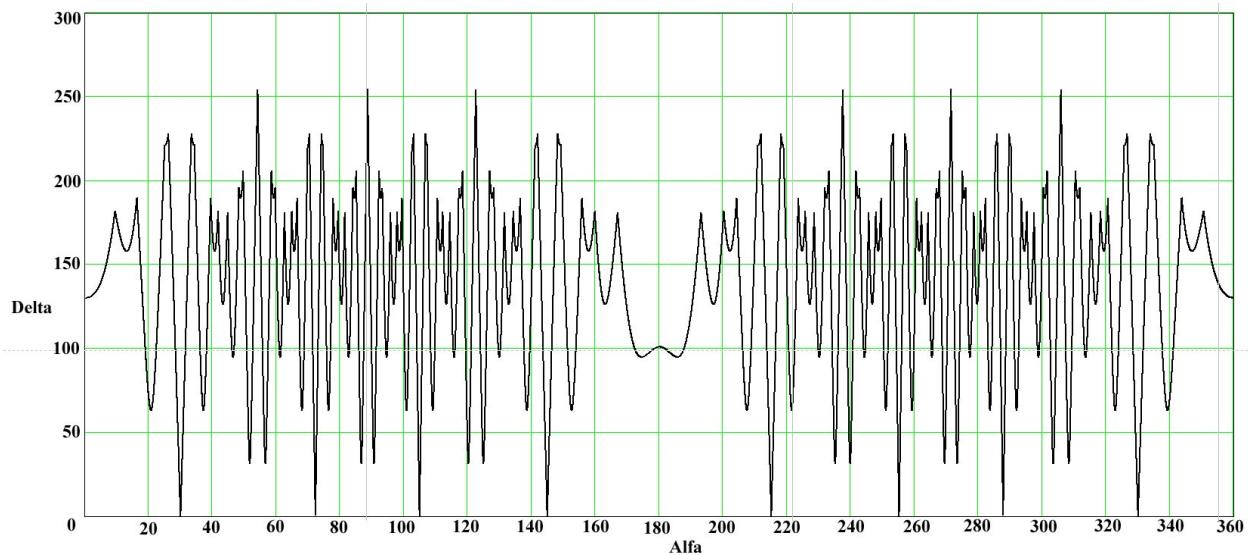


Fig. 3. Function of credibility at the double-frequency measurements of L1 and L2

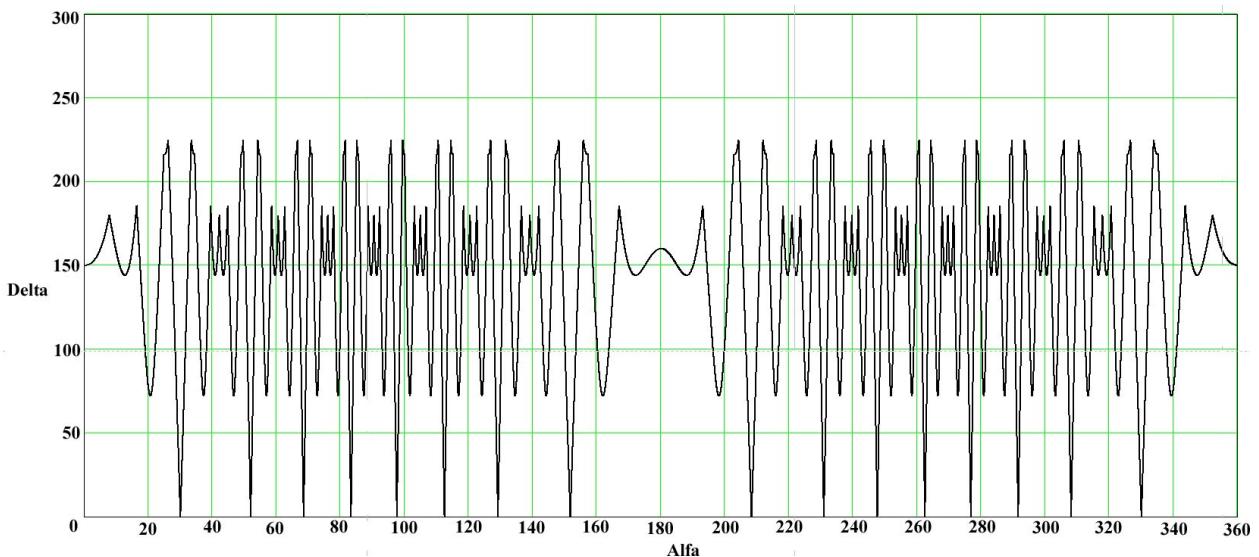


Fig. 4. Function of credibility at two-frequency measurements of L1 and L2

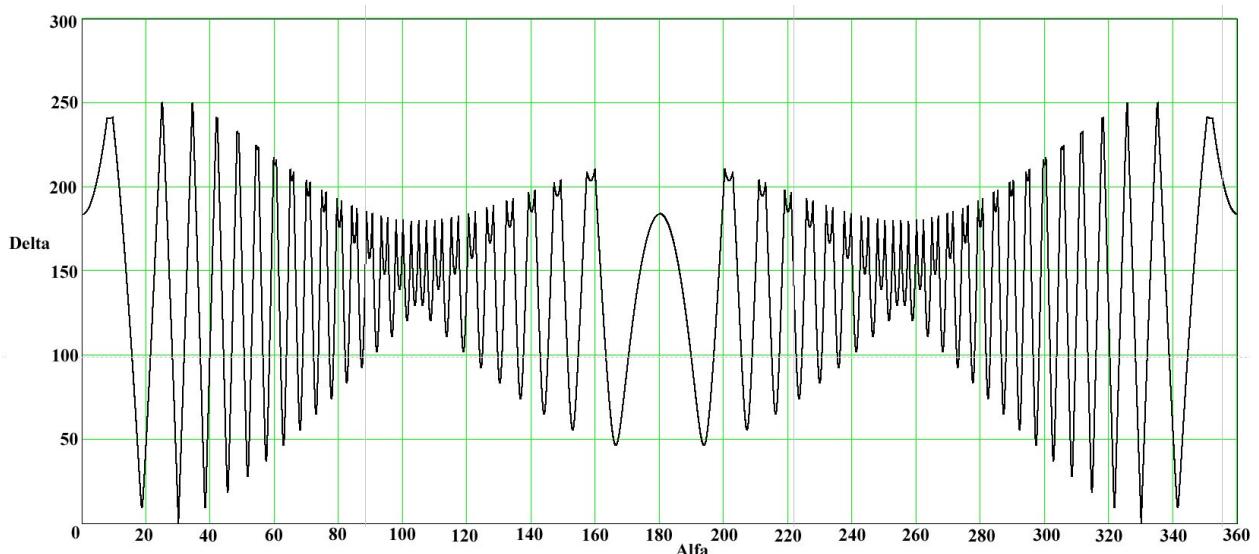


Fig. 5. Function of credibility at two-frequency measurements of L2 and L3

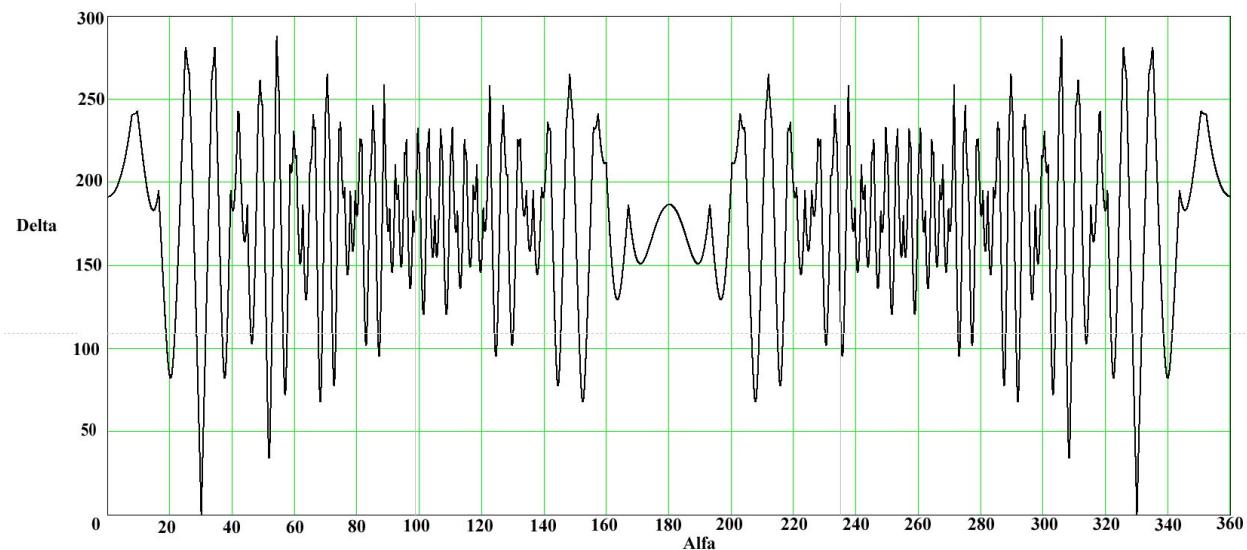


Fig. 6. Function of credibility at three-frequency measurements of L1, L2 and L3

Application of a three-frequency signal can be the decision. Ratios between signals lie from absolutely small in case of  $L1/L3 = 4/3$ , to enough great values in case of  $L2/L3 = 28/27$ . As a result rather big resulting not knitting in collateral minima remains, and at the same time the maximum length of base is several meters that it is enough for the majority of applications. Function of credibility at a three-frequency signal is given in Fig. 6.

### III. CONCLUSION

Thus, the use of a three-frequency signal provides reliable resolution of phase ambiguity, and separately for each signal source. You can also use a three-frequency signal in terrestrial radio navigation systems based on pseudosatellites.

To further increase the probability of correct resolution of phase ambiguity, you can use the signals of several beacons. When solving the problem of determining the spatial orientation, the coordinates of the object are known, which means that the cosines are guided to the signal sources.

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### REFERENCES

- [1] Yu.L. Fateev, D.D. Dmitriev, V.N. Tyapkin, E.N. Garin, and V.V. Shaidurov, "The phase ambiguity resolution in the angle-measuring navigation equipment," AIP Conference Proceedings, 2014, vol. 1611, pp. 12–14. doi: 10.1063/1.4893795.
- [2] Yu.L. Fateev, D.D. Dmitriev, V.N. Tyapkin, I.N. Ishchuk, and E.G. Kabulova, "The phase ambiguity resolution by the exhaustion method in a single-base interferometer," ARPN Journal of Engineering and Applied Sciences, 2015, vol. 10, issue 18, pp. 8264-8270.
- [3] Yu.L. Fateev, D.D. Dmitriev, V.N. Tyapkin, N.S. Kremez, and V.N. Ratushnyak, "The use of GNSS technologies for high-precision navigation geostationary spacecraft," 2015 International Siberian Conference on Control and Communications, SIBCON 2015 – Proceedings. doi: 10.1109/SIBCON.2015.7147250.
- [4] G. Giorgi, P.J.G. Teunissen, S. Verhagen, and P.J. Buijs "Improving the GNSS Attitude Ambiguity Success Rate with the Multivariate Constrained LAMBDA Method," Geodesy for Planet Earth. International Association of Geodesy Symposia, 2012, vol.136, pp. 941–948.
- [5] P.J.G Teunissen "A general multivariate formulation of the multi-antenna GNSS attitude determination problem," Artificial Satellites, 2007, vol.42, pp. 97–113.
- [6] Yu.L. Fateev, D.D. Dmitriev, V.N. Tyapkin, N.S. Kremez, and V.N. Bondarev, "Phase ambiguity resolution in the GLONASS/GPS navigation equipment, equipped with antenna arrays," 2015 International Siberian Conference on Control and Communications, SIBCON 2015 – Proceedings. doi: 10.1109/SIBCON.2015.7147251.
- [7] V.N. Tyapkin, Yu.L. Fateev, D.D. Dmitriev, I.N. Kartsan, P.V. Zelenkov, A.E. Goncharov, and I.R. Nasirov, "Using GLONASS for precise determination of navigation parameters under interference from various sources," IOP Conference Series: Materials Science and Engineering, vol. 122, issue 1, 2016. doi: 10.1088/1757-899X/122/1/012035.
- [8] Yu.L. Fateev, V.N. Ratushnyak, I.N. Kartsan, V.N. Tyapkin, D.D. Dmitriev, and A.E. Goncharov, "Analyzing measurement errors for navigation parameters in onground short-range navigation systems based on pseudolites," IOP Conference Series: Materials Science and Engineering, vol. 155, issue 1, 2016. doi: 10.1088/1757-899X/155/1/012016.