Model of flow propagation in a river channel taking into account disparity in the "water stage vs water volume" curves corresponding to the rise and recession of a flood wave

D A Burakov[1,2], E D Karepova[3,4], Yu V Shan'ko[3]

[1] Krasnoyarsk State Agrarian University, 90 Mira Ave., 660049 Krasnoyarsk, Russia
[2] Tomsk State University, 36 Lenin Ave., 634050, Tomsk, Russia
[3] Institute of Computational Modelling SB RAS of FRC KSC SB RAS, 660100, Akademgorodok, Krasnoyarsk, Russia
[4] Siberian Federal University, 79 Svobodny Ave., 660041 Krasnoyarsk, Russia

e.d.karepova@icm.krasn.ru

Abstract. In this paper the new family of the stream routing curves was obtained on the basis of disparity in the "water stage vs water volume" curve on the rise and the recession of the flood wave. The paper contains the results of the numerical experiments with the different stream routing curves using the row data about the artificial outflows downstream from Novotveretskaya and Ivankovo dams.

1. Introduction

One of the tasks of hydrology is the determination of water discharge (and/or water levels) in an outflow section by the known flow characteristics in an upstream inflow section. To the theoretical description of the process of accumulation and drawdown of water volume in river reaches and in other hydrological systems, the differential equations of the water balance is widely used [1–4].

$$\frac{dW}{dt} = Q(t) - q(t), \tag{1}$$

were $W(t) = W(q(t), Q(t), \boldsymbol{C})$ is a storage volume given by "water stage vs water volume" curve (hereinafter volume curve). Equation (1) determines a relation between storage volume in the reach $W(t)$ and the weighted inflow $q(t)$ and outflow $Q(t)$ discharges.

Since in linear models the volume curve is described by a linear differential expression (may be of zeroth order) with constant coefficients, the general solution (1) with zero initial data is the convolution integral:

$$Q(t) = \varphi(t, \mathbf{C}) * q(t) = \int_0^t \varphi(\tau, \mathbf{C}) q(t-\tau) \mathrm{d}\tau, \tag{2}$$

where $\varphi(t, \mathbf{C})$ is a kernel (in our case a unit hydrograph or "stream routing curve"), a "*" symbol means the convolution operator. Thus, in order to determine the water discharge in the outflow section using known water discharge in the upstream inflow section, it is necessary to solve the following problems: 1) to specify a volume curve or a stream routing curve according to some physical reasons; 2) to select values of the vector $\boldsymbol{C}$ set of parameters characterizing the hydrological system.

In this paper we discuss in detail a mathematical approach to the first problem. Thus, if the form of the volume curve is chosen, then corresponding family of stream routing curves may be constructed explicitly. Moreover, there are stream routing curves, which cannot be described by any linear differential equations, although these curves are inseparably connected with the native volume curve (for example, Gamma distribution, Brovkovich's curve, etc.).

In connection with the second problem, we briefly point out the following. If there is a sufficient amount of row data for the pair $q(t)$ and $Q(t)$, then optimization methods are used to adjust the set of parameters $\boldsymbol{C}$ (see [5, 6], etc.).

2. Families of stream routing curves

To obtain a family of stream routing curves and a physically meaningful interpretation of their parameters, it is convenient to use the Laplace transform. Thus, applying Laplace transform to the convolution integral (2) gives $\overline{Q}(p)=\overline{\varphi}(p,\boldsymbol{C})\overline{q}(p),$ where $\overline{f}(p)$ denotes a Laplace transform of the function $f(t)$. Moreover, the following relation

$$\left.\frac{\mathrm{d}^r\overline{\varphi}(p)}{\mathrm{d}p^r}\right|_{p=0}=(-1)^r\int_0^\infty\tau^r\varphi(\tau)\mathrm{d}\tau=(-1)^r m_r, \qquad (3)$$

holds. Here m_r is a r-th order moment about zero of $\varphi(t)$ stream routing curve. As D.A. Burakov first proposed, (3) allows to determine the parameters of stream routing curve through its statistical moments, taking into account the concepts of turbulent diffusion [7–9].

In tab. 1 the original function and its Laplace transform is shown for two families of stream routing curves, namely: 1) a family based on classical Kalinin-Milyukov volume curve $W(t)=kQ(t)$ [1]; 2) a new family using disparity in the volume curve corresponding to the rise and recession of a flood wave $W(t)=k_1Q(t)+k_2\dfrac{dQ}{dt}$ [9].

Since the flow velocity is always finite, the stream routing curve should take into account the minimum travel time $\tau_{\min}$ of an elementary volume of water from the inflow section to the outflow one. In order to take into account $\tau_{\min}$ mathematically, we use a time shifting in the initial $\varphi^0(t)$ stream routing curve. Thus, in tab. 1 any $\varphi^0(t)$ stream routing curve has an additional $\varphi(t)$ stream routing curve with time shifting. The Laplace transform $\overline{\varphi}(p)$ and original function $\varphi(t)$ are connected with initial curve $\overline{\varphi^0}(t)$ and $\varphi^0(t)$ by the following relations:

$$\overline{\varphi}(p)=\overline{\varphi^0}(p)\exp(-\tau_{\min}p),\quad \varphi(t)=\varphi^0(t-\tau_{\min})H(t-\tau_{\min}),$$

where $H(t)$ is Heaviside step function.

The probabilistic interpretation of the stream routing curve [7–10] allows to define the parameters $\boldsymbol{C}$ of the curves by minimum characteristics of the hydrological system with clear physical meaning. Tables 2-3 contain the main characteristics of the stream routing curves of both families. These characteristics are obtained using (3) and the formulae of the moments $m_1=\overline{\tau}$ (expected value) and $M_2=a^2\overline{\tau}$ (dispersion value) of a distribution of a travel time of an elementary water volume from inflow to outflow sections in the homogeneous and inflow free hydrological system. Here $\overline{\tau}$ is an average value of travel time and a is a coefficient of longitudinal scattering of the flow [10].

Table 1. Laplace transform and original function for two families of the stream routing curve.

The name of the stream routing curve	Laplace transform	original function
Family generated by Kalinin-Milyukov volume curve		
1. Kalinin-Milyukov, one typical reach	$\overline{\varphi^0_{\text{KM}}}(p)=\dfrac{1}{kp+1}$	$\varphi^0_{\text{KM}}(t)=\dfrac{1}{k}\exp\left(-\dfrac{t}{k}\right)$
2. Kalinin-Milyukov, $n\in N$ typical reaches	$\overline{\varphi^0_{\text{KM},n}}(p)=\dfrac{1}{(kp+1)^n}$	$\varphi^0_{\text{KM},n}(t)=\dfrac{1}{k(n-1)!}\left(\dfrac{t}{k}\right)^{n-1}\exp\left(-\dfrac{t}{k}\right)$
3. Gamma distribution, $s\in R^+$	$\overline{\varphi^0_{\Gamma}}(p)=\dfrac{1}{(kp+1)^s}$	$\varphi^0_{\Gamma}(t)=\dfrac{1}{\gamma\Gamma(s)}\left(\dfrac{t}{\gamma}\right)^{s-1}\exp\left(-\dfrac{t}{\gamma}\right)$
4. Brovkovich	$\overline{\varphi^0_{\text{Br}}}(p)=\dfrac{1}{(1+\gamma p)^s}-\dfrac{b}{6}\dfrac{(\gamma p)^3}{(1+\gamma p)^{s+3}}$	$\varphi^0_{\text{Br}}(t)=\varphi^0_{\Gamma}(t)-\dfrac{b}{6}\big(f_{s;\gamma}(t)-$ $-3f_{s+1;\gamma}(t)+3f_{s+2;\gamma}(t)-f_{s+3;\gamma}(t)\big),$ $f_{\alpha;\beta}(t)=\dfrac{1}{\beta\Gamma(\alpha)}\left(\dfrac{t}{\beta}\right)^{\alpha-1}\exp\left(-\dfrac{t}{\beta}\right)$
Family generated by Burakov volume curve		
5. Burakov, one typical reach	$\overline{\varphi^0_{\text{B}}}(p)=\dfrac{1}{k_2p^2+k_1p+1}$	$\varphi^0_{\text{B}}(t)=\dfrac{1}{k_2(b-c)}\big(\exp(-ct)-\exp(-bt)\big)$
6. Burakov, $n\in N$ typical reaches (see Remark 1)	$\overline{\varphi^0_{\text{B},n}}(p)=\dfrac{1}{(k_2p^2+k_1p+1)^n}$	$\varphi^0_{\text{B},n}(t)=\dfrac{(cb)^n}{(n-1)!}\dfrac{(-1)^n}{(c-b)^{2n-1}}\times$ $\times\left(\exp(-ct)\sum_{j=0}^{n-1}\dfrac{(2n-2-j)!((c-b)t)^j}{j!(n-1-j)!}-\right.$ $\left.-\exp(-bt)\sum_{r=0}^{n-1}\dfrac{(2n-2-j)!((b-c)t)^j}{j!(n-1-j)!}\right)$
7. Burakov, $s\in R^+$	$\overline{\varphi^0_{\text{B},s}}(p)=\dfrac{1}{(k_2p^2+k_1p+1)^s}$	$\varphi^0_{\text{B},s}(t)=\dfrac{\sqrt{\pi}}{\sqrt{k_2}\Gamma(s)}\left(\dfrac{t}{\sqrt{k_1^2-4k_2}}\right)^{s-\frac{1}{2}}\times$ $\times\exp\left(-\dfrac{k_1}{2k_2}t\right)I_{s-\frac{1}{2}}\left(\dfrac{\sqrt{k_1^2-4k_2}}{2k_2}t\right),$ where $I_s(z)$ is the modified Bessel function of the first kind [11]

Remark 1. To obtain Burakov's stream routing curves for n typical reaches ($n \in N$), we can restrict Burakov's stream routing curves for general case $s \in R^+$ using the following formula of the modified Bessel function of the first kind of half-integer index [12]:

$$I_{n+\frac{1}{2}}(z) = \frac{1}{\sqrt{2\pi z}}\left(\exp(z)\sum_{j=0}^{n}\frac{(-1)^j (n+j)!}{j!(n-j)!(2z)^j} + (-1)^{n+1}\exp(-z)\sum_{j=0}^{n}\frac{(n+j)!}{j!(n-j)!(2z)^j}\right).$$

Remark 2. The stream routing curves based on Kalinin-Milyukov volume curve are well known (for instance, [2]). The curve based on Gamma distribution wide spread in hydrological practice. We consider them here to demonstrate a unified approach to the construction of a family of different stream routing curves. We propose the new interpretation of the Brovkovich curve. The family of the stream routing curves generated by the Burakov volume curve is new.

Table 2. The parameters of the stream routing curves of the family generated by Kalinin-Milyukov volume curve.

The name of the stream routing curve	Statistic parameters	Without time shifting of original function	With time shifting of original function ($\tau_{\min}$)
1. Kalinin-Milyukov, one typical reach	m_1	$k = \bar{\tau}$, $C_v = 1$	$k = \bar{\tau} - \tau_{\min}$, $C_v \le 1$
2. Kalinin-Milyukov, $n \in N$ typical reaches	m_1, n	$k = \frac{\bar{\tau}}{n}$, $C_v = 1$	$k = \frac{\bar{\tau} - \tau_{\min}}{n}$, $C_v \le 1$
3. Gamma distribution, $s \in R^+$	m_1, M_2	$\gamma = \frac{M_2}{\bar{\tau}} = a^2$, $s = \frac{\bar{\tau}}{\gamma} = \frac{\bar{\tau}}{a^2}$ $C_v = 1$, $C_s = 2C_v$	$\gamma = \frac{M_2}{\bar{\tau} - \tau_{\min}} = \frac{a^2\bar{\tau}}{\bar{\tau} - \tau_{\min}}$, $s = \frac{\bar{\tau} - \tau_{\min}}{\gamma} = \frac{\left(\bar{\tau} - \tau_{\min}\right)^2}{a^2\bar{\tau}}$, $C_v \le 1$, $C_s = \left(2 + \frac{2\tau_{\min}}{\bar{\tau} - \tau_{\min}}\right)C_v$
4. Brovkovich	m_1, M_2, κ, $C_s = \kappa C_v$	$\gamma = a^2$, $s = \frac{\bar{\tau}}{a^2}$, $b = \frac{C_s - 2C_v}{C_v^3} = \frac{(\kappa - 2)\bar{\tau}}{a^2}$	$\gamma = \frac{a^2\bar{\tau}}{\bar{\tau} - \tau_{\min}}$, $s = \frac{\left(\bar{\tau} - \tau_{\min}\right)^2}{a^2\bar{\tau}}$, $b = \left(\frac{\bar{\tau} - \tau_{\min}}{a\bar{\tau}}\right)^2 \times$ $\times\ ((\kappa - 2)\bar{\tau} - \kappa\tau_{\min})$

Table 3. The parameters of the stream routing curves of the family generated by Burakov's volume curve.

The name of the stream routing curve	Statistic parameters	Without time shifting of original function	With time shifting of original function ($\tau_{\min}$)
1. Burakov, one typical reach	$m_1,\ M_2$	$b,c=\dfrac{k_1 \mp \sqrt{k_1^2-4k_2}}{2k_2}>0,$ $k_1=\bar{\tau},\ k_2=0.5\left(\bar{\tau}^2-a^2\bar{\tau}\right),$ $C_v \ge \dfrac{1}{\sqrt{2}}$	$k_1=\bar{\tau}-\tau_{\min},$ $k_2=0.5\left(\left(\bar{\tau}-\tau_{\min}\right)^2-a^2\bar{\tau}\right)$
2. Burakov, $n\in N$ typical reaches (see Remark 1)	$m_1,\ n,\ M_2$	$k_1=\dfrac{\bar{\tau}}{n},\ k_2=\dfrac{\bar{\tau}^2-na^2\bar{\tau}}{2n^2},$	$k_1=\dfrac{\bar{\tau}-\tau_{\min}}{n},$ $k_2=\dfrac{(\bar{\tau}-\tau_{\min})^2-na^2\bar{\tau}}{2n^2},$
3. Burakov, $s\in R^+$	$m_1,\ M_2,\ \kappa,$ $C_s=\kappa C_v$	$k_1=\dfrac{\bar{\tau}}{s},\ k_2=\dfrac{\bar{\tau}^2-sa^2\bar{\tau}}{2s^2},$ $s_{1,2}=\dfrac{\bar{\tau}}{2M_3}\left(3M_2 \pm \pm\sqrt{9M_2^2-4\bar{\tau}M_3}\right),$ $M_2=s(k_1^2-2k_2),$ $M_3=s(2k_1^3-6k_1k_2)$	$k_1=\dfrac{\bar{\tau}-\tau_{\min}}{s},$ $k_2=\dfrac{(\bar{\tau}-\tau_{\min})^2-sa^2\bar{\tau}}{2s^2},$ $s_{1,2}=\dfrac{\bar{\tau}-\tau_{\min}}{2M_3}\left(3M_2 \pm \pm\sqrt{9M_2^2-4\left(\bar{\tau}-\tau_{\min}\right)M_3}\right)$

3. Numerical experiment

To numerical experiments, we use row data about two famous Soviet full-scale observations.

1. Observation data about the artificial water pass on the Tvertsa River below Novotveretskaya dam [13]. We consider the reach with length 20.61 km. The optimization was carried out by row data from 20:00 of the fifth day from the start of the water pass to 20:00 of the eighth day. Control was performed by row data from 20:30 of the eighth day to 06:30 of the twelfth day. The time step is 30 minutes. Observations on the Tvertsa River illustrate the influence of a wide plant-filled floodplain on the parameters of an unsteady flow.
2. Observations data about the artificial water pass on the Volga River below Ivankovo dam [14]. We consider the reach with length 24.9 km. The optimization was carried out by row data from 03:30 of the second day to 20:00 of the third day. Control was performed by row data from 06:30 of the third day to 07:30 of the fourth day. The time step is 10 minutes. Floodplain on this reach has a slight effect on the flow. However, in this case, there is a lateral inflow, and optimization leads to $\tau_{\min}=0$. We can improve the criterion of quality in optimization. We use a reduction ratio to take into consideration a lateral inflow.

Table 4. Results of the numerical experiments.

The name of the stream routing curve	σ	Mathematical parameters of the stream routing curve	Statistic parameters of the stream routing curve
Tvertsa River, $\tau_{\min} = 0$			
Gamma distribution	4.504333	$s = 1.910694$, $\gamma = 12.29156$	$\bar{\tau} = 23.4854$, $a = 3.505932$, $\sqrt{M_2} = 16.99036$, $C_v = 0.7234432$, $C_s = 1.446886$, $\kappa = 2$
Brovkovich	4.504717	$s = 1.910821$, $\gamma = 12.29052$, $b = 0.503095$	$\bar{\tau} = 23.48497$, $a = 3.505783$, $\sqrt{M_2} = 16,98948$, $C_v = 0.7234192$, $C_s = 1.637306$, $\kappa = 2.263287$
Burakov	3.908327	$s = 1.397947$, $k_1 = 17.12119$, $k_2 = 28.0119$	$\bar{\tau} = 23.93452$, $a = 3.721424$, $\sqrt{M_2} = 18.20629$, $C_v = 0.7606708$, $C_s = 1.658605$, $\kappa = 2.18045$
Tvertsa River, $\tau_{\min}$ is a parameter to optimization			
Gamma distribution	3.400073	$s = 1.137918$, $\gamma = 18.13212$	$\tau_{\min} = 3.786779$, $\bar{\tau} = 24.41963$, $a = 3.914122$, $\sqrt{M_2} = 19.34211$, $C_v = 0.7920722$, $C_s = 1.874885$, $\kappa = 2.367063$
Brovkovich	3.400227	$s = 1.137985$, $\gamma = 18.13118$, $b = 0.8215682$	$\tau_{\min} = 3.786482$, $\bar{\tau} = 24.41949$, $a = 3.914046$, $\sqrt{M_2} = 19.34168$, $C_v = 0.7920592$, $C_s = 2.551595$, $\kappa = 3.22147$
Burakov	3.395426	$s = 1.133657$, $k_2 = 3.073509$, $k_1 = 18.35122$,	$\tau_{\min} = 3.62296$, $\bar{\tau} = 24.42695$, $a = 3.917156$, $\sqrt{M_2} = 19.36001$, $C_v = 0.7925676$, $C_s = 1.878163$, $\kappa = 2.369719$
Volga River, after optimization the parameter $\tau_{\min} = 0$			
Gamma distribution	6.222326	$s = 3.953681$, $\gamma = 2.190842$	$\bar{\tau} = 8.661891$, $a = 1.480149$, $\sqrt{M_2} = 4.356241$, $C_v = 0.5029203$, $C_s = 1.005841$, $\kappa = 2$
Brovkovich	6.222146	$s = 3.953927$, $\gamma = 2.190714$, $b = 2.111234$	$\bar{\tau} = 8.661924$, $a = 1.480106$, $\sqrt{M_2} = 4.356122$, $C_v = 0.5029047$, $C_s = 1.27434$, $\kappa = 2.533959$
Burakov	6.222187	$s = 1.976949$, $k_2 = 4.799272$, $k_1 = 4.381448$	$\bar{\tau} = 8.661899$, $a = 1.480109$, $\sqrt{M_2} = 4.356126$, $C_v = 0.5029065$, $C_s = 1.005813$, $\kappa = 2$

An objective function of optimization is the least mean square error $\sigma=\sqrt{\frac{1}{M}\sum_{i=1}^{M}\left(Q^{\mathrm{obs}}(t_i)-Q_i^{\mathrm{num}}\right)^2}$, where $Q^{\mathrm{obs}}(t_i)$ and Q_i^{num} are observed and calculated discharge in an outflow section in the t_i instants, M is a number of time segments for optimization.

4. Discussion

Important parameters of the stream routing curves are the skewness (a measure of the asymmetry of the distribution) $C_{\mathrm{s}}=\frac{M_3}{M_2^{3/2}}$ and the coefficient of variation $C_{\mathrm{v}}=\frac{M_2^{1/2}}{m_1}$. When the stream routing curve is considered in the form of a Gamma distribution, we always have $C_{\mathrm{s}}=2C_{\mathrm{v}}$. However, it does not correspond to the real channels (see, for example, [8, 10, 15]). The Burakov's stream routing curve extends the range of possible relations C_{s} and C_{v} to inequality $2\le\kappa\le\frac{9}{4}$. If the channel has no lateral inflow then $C_{\mathrm{s}}>2C_{\mathrm{v}}$. In this case, the using $\tau_{\min}\ne0$ significantly improves the model and $2\le\kappa\le\frac{9}{4}\frac{\bar{\tau}}{\bar{\tau}-\tau_{\min}}$ (for instance, the calculation for Tvertsa River from tab. 4). The parameter $\tau_{\min}$ may be determined both by physical considerations and by optimization.

However, for channels with a significant lateral inflow $\tau_{\min}=0$ and $C_{\mathrm{s}}<2C_{\mathrm{v}}$. In this case, in hydrological calculations, it is reasonable to use the Brovkovich curve [16]. The Brovkovich curve generalizes the Gamma distribution. In the framework of our approach, we introduce the Brovkovich stream routing curve as follows: 1) the skewness of the curve is proportional to the coefficient of variation $C_{\mathrm{s}}=\kappa C_{\mathrm{v}}$, with an additional optimization parameter ratio $\kappa\in R$; 2) the expected value and the dispersion value of the stream routing curve are independent of parameter κ; 3) when $C_{\mathrm{s}}=2C_{\mathrm{v}}$ the stream routing curve coincides with the Gamma distribution; 4) the stream routing curve is a linear combination of Gamma-distributions.

Note that under these assumptions, the Laplace transform of the stream routing curve can be specified in different ways. For example, Laplace transform $\overline{\varphi_{\mathrm{Br}}^{0}}(p)$ and original $\varphi_{\mathrm{Br}}^{0}(t)$ of Brovkovich curve is given in tab. 1 row 4 and its parameters are shown in tab. 2. Indeed, according to (3), the second term of the Laplace transform $\overline{\varphi_{\mathrm{Br}}^{0}}(p)$ does not contribute to the first and the second moments about zero, hence the correction of the Gamma distribution by Brovkovich curve does not change the expected and dispersion values of the stream routing curve. Moreover, the original $\varphi_{\mathrm{Br}}^{0}(t)$ is a linear combination of Gamma-distributions. Note that the Brovkovich curve has negative values for certain sets of parameters. Since these values do not correspond to real hydrological systems [17], the Brovkovich curve are widely used in the model of a channel flow and the hydrological forecasts.

References

[1] Kalinin G P and Milyukov G P 1958 *Approximate Calculations of the Unsteady Flow of Water Masses* (Moscow: Trudy TsIP) vol 66
[2] Kuchment L S 1980 *Models of Runoff Formation Processes* (Leningrad: Gidrometeoizdat)
[3] Koren V I 1991 *Mathematical Models in Forecasts of the River Runoff* (Leningrad: Gidrometeoizdat)
[4] Perumal M and Price R K 2016 *Handbook of Applied Hydrology, Second Edition* ed V P Singh (New York: Mcgraw-Hill Education) chapter 52
[5] Gel'fand I M and Tsetlin M L 1962 *Russ. Math. Surv.* **17**:1 95–117

[6] Koren V I and Kuchment L S 1969 *Meteorologiya i Gidrologiya* **11** 69–76
Koren V I and Kuchment L S 1971 *Meteorologiya i Gidrologiya* **11** 59–68
[7] Burakov D A 1973 *Meteorologiya i Gidrologiya* **4** 52–8
[8] Burakov D A 1978 *Vodnye Resursy* **4** 21–34
[9] Burakov D A 1989 *Meteorologiya i Gidrologiya* **10** 89–95
[10] Burakov D A 2015 *Scientific Articles Collection of the 4th Int. Scientific Conf. of Eurasian Scientific Association (2015 Moscow)* vol 2 (Moscow: ESA) pp 181–186
[11] Bateman H 1954 *Tables of Integral Transforms* (New York: McGraw-Hill Book Company) vol 1
[12] Watson G N 1995 *A Treatise on the Theory of Bessel Functions* (Cambidge: Cambidge University Press)
[13] Kondratiev N E and Uryvaev V A 1961 *Studies of Unsteady Water Movement on the Rivers Tvertsa and Oredezh* (Leningrad: Gidrometeoizdat)
[14] Gil'denblat Ya D, Makulov V V and Semikolenov A S 1948 Unsteady regime in the downstream of the hydroelectric station *Problems of Regulation of the River Runoff* (Moscow, Leningrad: USSR Academy of Sciences Publishing House) vol 2 chapter 2
[15] Nezhikhovsky R A 1971 *Channel Network of the Basin and Runoff Formation* (Leningrad: Gidrometeoizdat)
[16] Brovkovich G N and Velikanov G N 1941 *Trudy NIU GUGMS* IV **10** 217–8
[17] Rozhdestvensky A V Chebotarev A I 1974 *Statistical Methods in Hydrology* (Leningrad: Gidrometeoizdat)