EMPIRICAL ORTHOGONAL ANALYSIS OF TEMPERATURE AND VERTICAL VELOCITY IN LAKE SHIRA

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ABSTRACT

The empirical orthogonal functions method is widely used for the study of the hydrophysical characteristics in meteorology and oceanography, for example for the analysis of ocean surface currents in the North Carolina and the distribution of horizontal velocities in the Shira Lake. This method is also applied to study the distribution temperatures with depth in the Pacific Ocean and to analyze sea surface temperature in the Western North Atlantic.

The empirical orthogonal functions method gives us an optimal modal decomposition of the data and allows us to identify particular modes with relevant physical processors.

The empirical orthogonal functions analysis used in this study was performed to measure temperature and vertical velocity in Lake Shira (Southern Siberia, Russia) in the summer of 2014 and 2015.

The measurements of currents were recorded using Acoustic Doppler Current Profilers 600 kHz and 1200 kHz at two points.

The measurements of temperature were recorded by termistor sensors distributing with depth at ten locations.

The first and second empirical orthogonal modes for temperature account for 70-90 % of the total energy. They were used to identify the periods of summer heating and the location of the thermocline.

The first mode for surface temperature accounts for about 96 % of the total energy and corresponds to surface temperature gradients.

The first mode for vertical velocities accounts for about 10 % of the total energy and the analysis of the corresponding modal coefficient makes it possible to determine the periods when water moves up or down vertically.

Keywords: empirical orthogonal analysis, velocity profile, temperature distribution

INTRODUCTION

The data of field measurements of temperature and velocity have the complex spatial-temporal structure so it is necessary to apply statistical methods to study these variables.

The empirical orthogonal functions (EOFs) method is among the most widely used statistical methods in oceanology [1-8]. This method is also known as principal component analysis [9].

The EOFs decomposition allows us to analyze temperature and velocity fields and to reduce the dimensions of these data.

The present paper extends our study of hydrophysical characteristics in the Shira lake by applying the EOF method [10].

In this study the vertical and horizontal EOFs of the Shira lake's thermal structure were analyzed.

The EOFs expansions for temperature and vertical velocity were compared for the same location.

MATERIALS AND METHODS

Long-term measurements of the temperature were conducted in the Shira lake by the Institute of Biophysics, Russian Academy of Sciences, Siberian Branch, during summer 2015.The temperature was measured with the 0.1 °C accuracy at ten locations indicated in Fig.1.

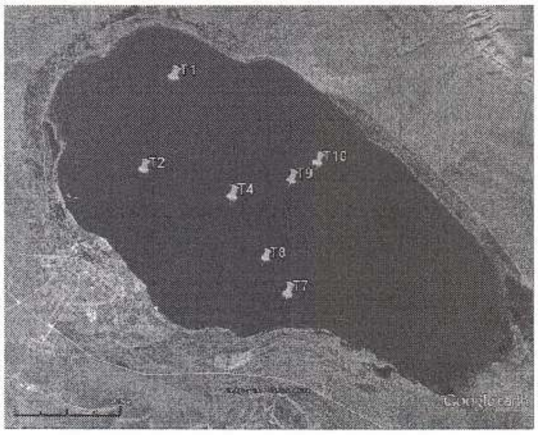

Fig. 1. Thermal sensors location.

The measurements of currents were performed at two points using Acoustic Doppler Current Profilers (ADCP) 600 kHz and 1200 kHz in summer of 2014 and 2015 [10].

The seasonal changes in temperature and vertical velocity were analyzed using the EOF method [1, 3] in three cases: the horizontal distribution of temperature, the distribution of temperature and vertical velocity measured at a variety of depths.

The measurements of temperature at a variety of depths were analyzed at location T1 (Fig. 1). The EOFs analysis was performed for two cases. In the first case, the data matrix was not processed. In the second case, the data matrix was constructed by calculating two numerical characteristics of the matrix, where $\{T_j\}$ is the average monthly temperature profile and similar signs $\{D_j\}$ is the variance vector. Then, the variables in the matrix were standardized so that the average temperature profile was subtracted and these values were divided by the standard deviation. Consequently, the data matrix with zero mathematical expectation and unit variance was constructed.

The covariance matrix $\mathrm{cov}(i, j)$ was formed and its eigenvalues $\{T_j\}$ and corresponding eigenvectors φ_j were found in accordance with [3]. These eigenvectors φ_j are the empirical orthogonal functions or modes. The eigenvectors are orthogonal to each other.

The percentage of each modes of the total energy was calculated as $\left(\lambda_j \Big/ \sum_{k=1}^{N} \lambda_k\right)$.

RESULTS

First, the distribution of temperature measured at a variety of depths was analyzed. The measurements were conducted in 28 points at various depths at location T1.Typical summer temperature profile is shown in Fig. 2a.

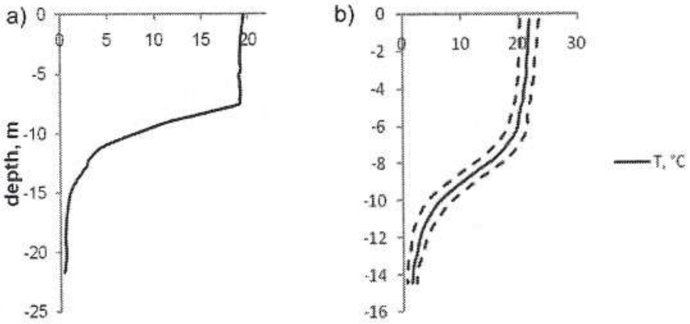

Fig. 2. Typical summer temperature profile in the Shira lake in summer 2015 (a); the average values and standard deviations of vertical temperature profile at location T1 (b).

The vertical structures of the temperature profile include the upper convective layer located at the depth of 0 to 7 meters and the seasonal thermocline located at the depth of 7 to 12 meters.

The values of standard deviations decreased monotonically with depth.

Fig. 3 shows the covariance matrix at location T1 in July 2015.

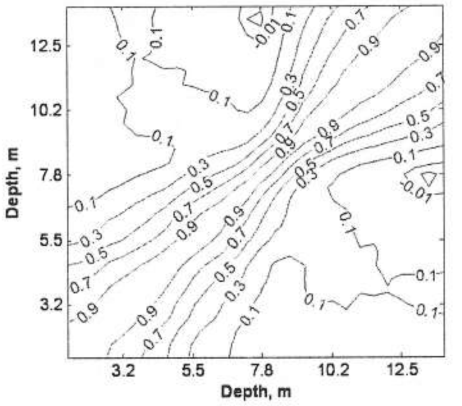

Fig. 3. The covariance matrix of distribution temperature with depth.

Fig. 3 demonstrates a strong linear relationship between EOFs of temperature in the upper layer at the depth of 0 to 4 meters. In the layer at the depth of 4 to 5.5 meters, a mildly linear relationship is observed, the relationship is very weak below 5.5 meters.

The formal decomposition in complete orthonormal basis functions is found using the EOFs method. In order to associate the obtained modes with corresponding physical processes an a priory hydrological information is needed, so the possibility of a physical interpretation is discussed later.

The EOF method was applied to standardized and non-standardized data. The values of the accumulated part of energy in these cases are very similar (Fig. 4a, b). These modes almost coincide, excluding first mode (Fig. 4 c).

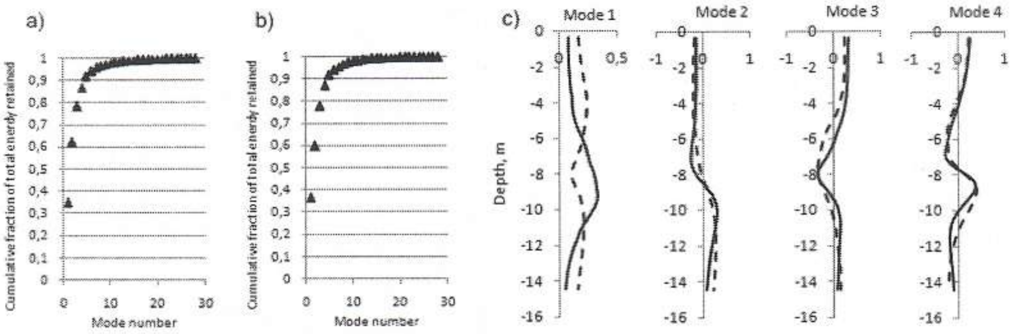

Fig. 4. The accumulated part of energy: a) standardization data; b) non-standardization data; c) the first modes for standardization data (dotted line) and non-standardization data (solid line).

The satisfactory approximation of data measurements was found using the first four modes in the case of standardization data and eight modes in the case of non-standardization data (Fig. 5).

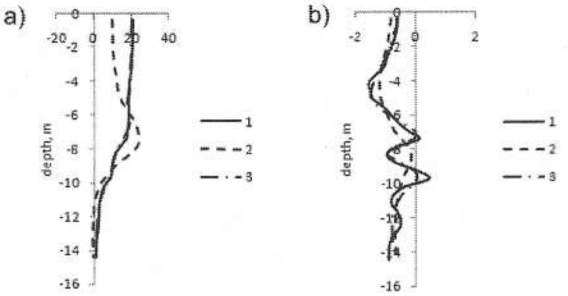

Fig. 5. a) non-standardization data, solid line 1 is data of measurements on 1 July 2015, dotted line 2 is graph for the case of using the first two modes, dash-dotted line 3 is graph for the case of using the first four terms; b) standardization data, solid line 1 is data of measurements on 1 July 2015, dotted line 2 is graph for the case of using the first six modes, dash-dotted line 3 is graph for the case of using the first eight modes.

Further analysis of horizontal temperature distribution in the lake was also carried out.

Three cases were studied. In the first case, the data of temperature measurements was analyzed for seven locations in near-surface layer. In the second case the layer was at the depth of 5 meters, in the third case the layer was at the depth of 11 meters.

A value of temperature in near-surface layer is a value at the first measured depth. If measurements at the depths of 5 meters or 11 meters were absent, then these values were found by applying Lagrange polynomial of the third degree.

Covariance matrices of horizontal temperature distribution in near-surface layer, at the depths of 5 and 11 meters are shown in Fig. 6.

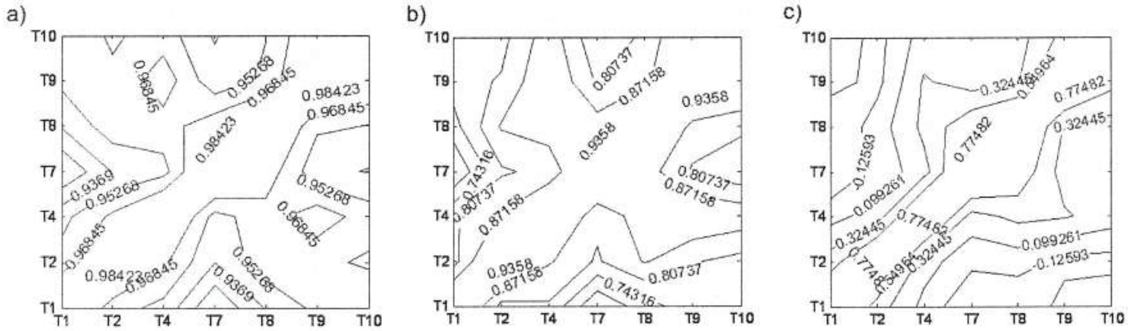

Fig. 6. The covariance matrix of horizontal temperature distribution: a) in near-surface layer; b) at the depth 5 m; c) at the depth of 11 m.

The comparison of the covariance matrix in Fig. 6 and covariance matrix of distribution temperature with depth in Fig. 4 shows that covariance matrix in Fig. 6 demonstrates a strong linear relationship between all values of the EOFs in near-surface layer and at the depth of 5 meters. The relationship is weaker at the depth of 11 meters.

Fig. 7 shows the part of the total energy for the EOFs of temperature at seven locations in near-surface layer, at the depths of 5 and 11 meters.

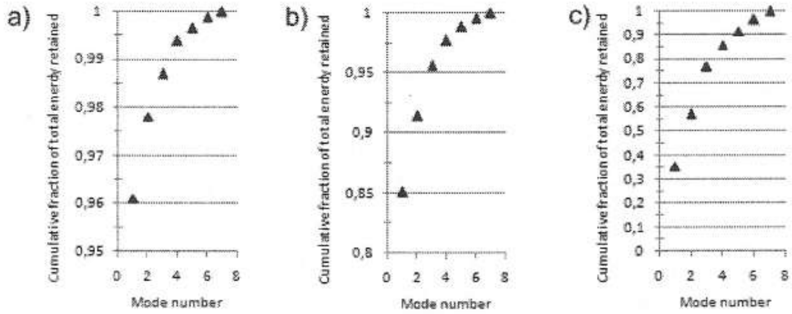

Fig.7. The part of the total energy a) in near-surface layer; b) at the depth 5 m; c) at the depth of 11 m.

The part of the total energy for first mode at the depth 11 meters as expected is significantly less than in near-surface layer and at the depth of 5 meters.

Fig. 8 shows the principal component (first mode) of the near-surface temperature. The difference of temperature in near-surface layer demonstrates warmer areas at the surface of the lake.

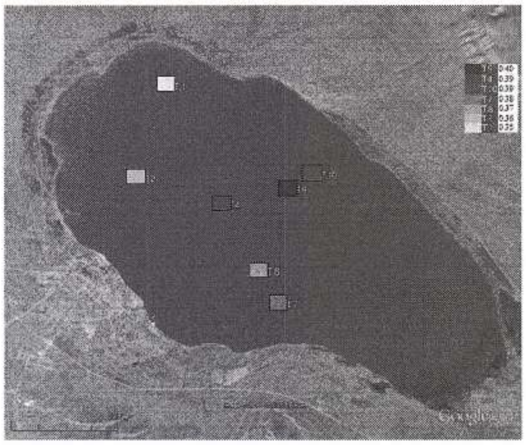

Fig. 8. The principal component of near-surface temperature.

The temperature and vertical velocity were analyzed at the location T1.

The analysis of accumulated energy shows that first mode for vertical velocity at this location is only 10 % of the total energy (Fig. 9a).

The first mode for vertical velocities accounts for about 10 % of the total energy and the analysis of the corresponding modal coefficient makes it possible to determine the periods when water moves up or down vertically.

The first modes for the vertical velocity and temperature are positive (Fig. 9b), the corresponding modal coefficient for temperature is positive and for vertical velocity is negative (Fig. 10). Consequently, this process determines water moves down.

The second and the fourth modes are mostly agreed by signs, but their modal coefficients have opposite signs. The third modes are opposite signs, but signs of modal coefficients are the same.

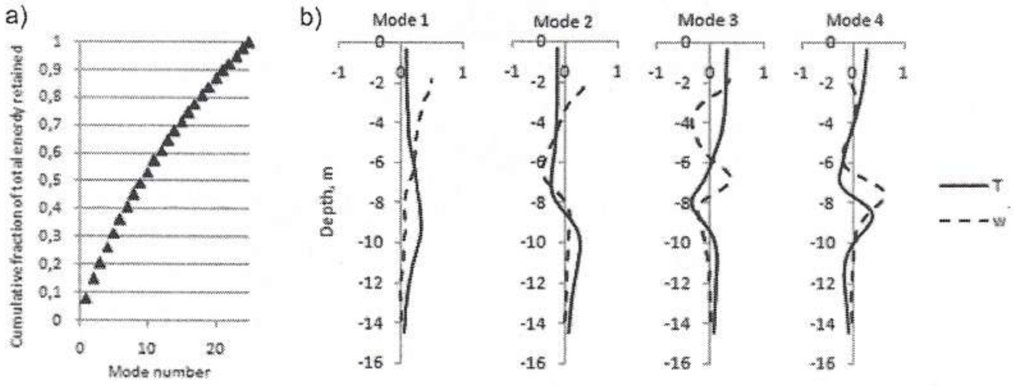

Fig. 9.The first four temperature (T) and vertical velocity modes (w) at location T1.

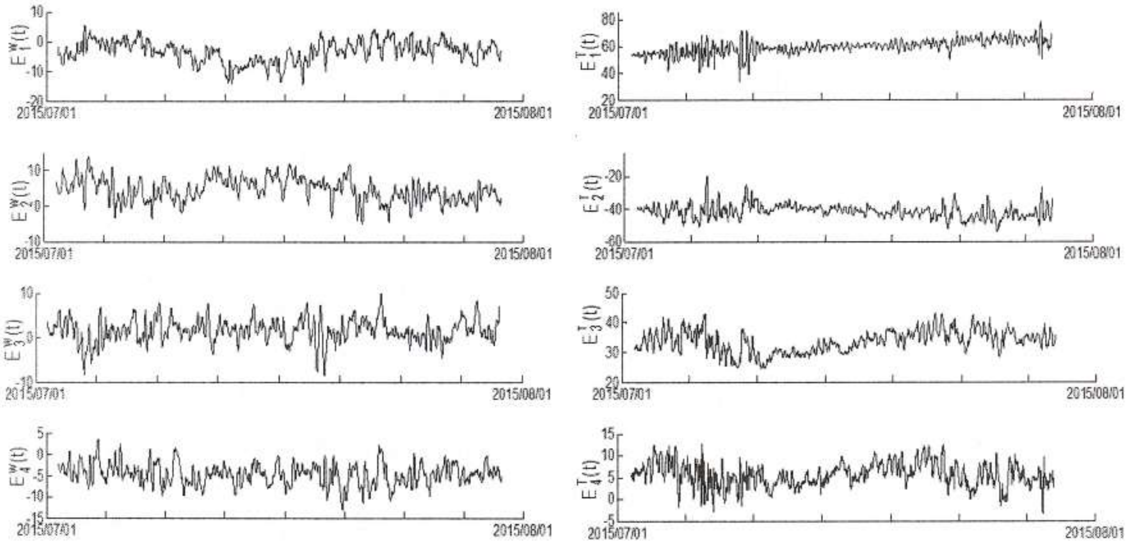

Fig. 10. Modal coefficients for the first four empirical eigenfunctions.

CONCLUSION

The following results were obtained by applying the empirical orthogonal functions method for the analysis of the temperature's data in the Shira lake: temperature is changing significantly with depth, but for its reconstruction using the EOFs method the first three modes is enough.

Temperature in near-surface layer in the horizontal direction changes insignificantly, so even the first mode gives us good impression of the temperature variation. This gives us the possibility to determine warmer area at the surface lake in summer.

Vertical velocity is most difficult for analysis. Its first mode accounts for 10 % of the total energy.

The relationships between the empirical orthogonal functions expansion for temperature and vertical velocity at the same location was analyzed.

Both first modes are positive, but the corresponding modal coefficients for temperature increased linear. Its demonstrates the summer heating process.

At the same time and location first modal coefficient for vertical velocity is negative. These results corresponds the summer heating process and emphasizes the relationships between heating process and wind-induced mixing with moving of liquid from the lake surface to the bottom.

REFERENCES

[1] Lorenz E.N., Empirical orthogonal functions and statistical weather prediction. Sci. Rep. No. 1, Statistical Forecasting Project, M.I.T., Cambridge, MA, 1956, p 48.

[2] Hartmann D. L. Pacific sea surface temperature and the winter of 2014 //Geophysical Research Letters. vol. 42/issue 6, 2015, p. 1894-1902.

[3] Kundu P.K., Allen J.S., Smith R.L., Modal decomposition of the velocity field near the Oregon Coast, Journal of Physical Oceanography, USA, vol. 5/issue 4, pp. 683-704, 1975;

[4] Kundu P. K., Blanton J. O., Janopaul M. M. Analysis of current observations on the Georgia shelf //Journal of Physical Oceanography, 1981, vol. 11/issue 8, pp. 1139-1149.

[5] Belokopytov V.N., Bagaev A.V. Statistical analysis of the thermohaline fields of the Black Sea according to the data of the buoy-profilers ARGO, Ekologicheskaya bezpeka priberezhnoï za ulfovoe zona ta komplektne vikoristanna resursiv shelf, 2012, no. 26 (2), pp. 128-142.

[6] Fujii Y., Kamachi M. Three dimensional analysis of temperature and salinity in the equatorial Pacific using a variational method with vertical coupled temperature salinity empirical orthogonal function modes //Journal of Geophysical Research: Oceans. – 2003, vol. 108/issue C9.

[7] Kuzin V. I., Lobanov A. S. Analysis of variations in the surface temperature of tropical and Northern Pacific Ocean //Izvestiya, Atmospheric and Oceanic Physics. – 2016, vol. 52/issue 5, pp. 550-559.

[8] Alishouse J. C., Crone L. J., Fleming H. E., Van Cleef F. L., Wark D. Q. A discussion of empirical orthogonal functions and their application to vertical temperature profiles //Tellus,1967, vol. 19/issue 3, pp. 477-482.

[9] Gorban A. N., Kegl B., Wunsch D., Zinovyev A. Y. (Eds.), Principal Manifolds for Data Visualisation and Dimension Reduction, Series: Lecture Notes in Computational Science and Engineering 58, Berlin, 2008, p 340.

[10] Volodko O., Kompaniets L., Gavrilova L., Analysis of the velocity profile in Lake Shira in summer using principal component analysis. International Multidisciplinary Scientific GeoConference SGEM, 2017, vol. 17/issue 31, pp. 831–838.