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Variability of Circular Depolarization Ratio
in Radar Sensing of the Medium Filled
with Hydrometeors

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Circularly polarized waves are most sensitive to the influence of the propagation medium factors such as differential phase shift and differential attenuation. The reason of that is a feature of the wave polarization transformation. Such circumstances require consideration during designing radars working with circularly polarized waves. The aim of the study is to develop of approach for estimating the effect of the wave polarization transformation on a value of one of the most informative parameters based on the use of Jones vector component functional dependence on a polarization ellipse orientation angle and ellipticity angle. The task of this work is obtaining the analytical equation for the determination of that polarimetric radar informative parameter. As a result, the solution for the case of wave backscatter by the propagation medium was obtained when the medium basis orientation differs from the radar basis orientation. The estimation of the circular depolarization ratio providing the determination of regions with its raised value was obtained.

Keywords: circular polarization, differential attenuation, differential phase shift, polarization radar.

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Изменчивость кругового деполяризационного отношения при дистанционном зондировании среды, заполненной гидрометеорами

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Сигналы с круговой поляризацией в силу специфики трансформации поляризационной структуры сигнала наиболее сильно подвержены воздействию дифференциальных факторов (дифференциального фазового сдвига и дифференциального ослабления) среды распространения. Указанные обстоятельства требуют обязательного учета при проектировании радиолокационных систем, использующих данный вид поляризации. Целью данной работы является разработка подхода для оценки влияния трансформации поляризационной структуры на величину одного из наиболее характерных используемых в радиолокации информативных параметров, основанного на использовании функциональной зависимости компонент вектора Джонса от угла эллиптичности и угла наклона эллипса поляризации. Задача данной работы – получение расчетных соотношений для определения указанного выше информативного параметра поляризационной радиолокации. Получены расчетные соотношения для случая обратного рассеяния средой, ориентация поляризационного базиса которой отлична от ориентации измерительного базиса. Выявлены оценки указанного выше информативного параметра, позволяющие определить области с его повышенным значением.

Ключевые слова: круговая поляризация, дифференциальное ослабление, дифференциальный фазовый сдвиг, поляризационная радиолокация.

Introduction

The prerequisite to the use for displaying and interpreting the measurements results of the circular depolarization ratio (CDR) [1] is pronounced dependence of the circularly polarized received signals amplitude on the shape of the water droplets. The ratio, as it shown in [2−4], is independent of the orientation of the hydrometeors and is less subject to the influence of third party noises. The higher intensity of the rain (in other words, the more water droplets shape differs from the spherical shape), the more differences of a signal received with right and left circular polarizations [5] in the case when the probe signal polarization has one of the pointed above circular polarization states.

Task statement

The character of the differences pointed above is the leading cause of the variability of the circular depolarization ratio CDR which determines in decibels (dB) on the base of amplitude measurements when the wave with the right circular polarization is transmitted, and the wave with the right circular polarization is received (e. g. $\dot{E}_{RR}$), or when the wave with the right circular polarization is transmitted, and the wave with the left circular polarization is received (e. g. $\dot{E}_{RL}$)

$$CDR(z) \ = \ 20 \cdot lg\big(\big|\dot{E}_{RR}\big|/\big|\dot{E}_{RL}\big|\big), \tag{1}$$

where $\dot{E}_{RR}$ is the amplitude of the signal transmitted and received with the right circular polarization (the first letter of the index means polarization of the transmitted signal, the second letter of the index means polarization of the received signal); $\dot{E}_{RL}$ is the amplitude of the signal transmitted with the right circular polarization and received with the left circular polarization.

The differential attenuation $\Delta\alpha$ [dB/km] and the differential phase shift $\Delta\Phi$ [deg/km] concern to the main factors determining the transformation of the signal polarization structure in the process of its propagation in the medium filled with hydrometeors. These circumstances dictate the need for searching paths to evaluate the importance of the effect of these factors on the obtained measurement results and to determine the character of the variability of the measured parameter CDR.

The task of this work is obtaining the analytical solution for the evaluation of the circular depolarization ratio on the base of Jones vector component functional dependence on a polarization ellipse orientation angle β and ellipticity angle α.

Method of determining the circular depolarization ratio

According to the results of [6–8], the Jones vector of the radar signal in the eigen basis of the propagation medium can be represented using trigonometric functions of angles α and β as

$$\dot{E}^{OUT}(z) \;=\; \begin{vmatrix} \cos\beta(z)\cos\alpha(z) + j\sin\beta(z)\sin\alpha(z) \\ -\sin\beta(z)\cos\alpha(z) + j\cos\beta(z)\sin\alpha(z) \end{vmatrix}, \tag{2}$$

where z is the length of the signal propagation path in a medium filled with hydrometeors. The ellipticity angle α and the orientation angle β of the polarization ellipse of the circular polarized signal propagating in the medium defines by results of [9] as

$$\alpha(z) \;=\; 0.5\arcsin\left(\frac{2\cdot10^{0.05\Delta\alpha z}\sin(\Delta\Phi z+0.5\pi)}{1+10^{0.1\Delta\alpha z}}\right), \tag{3}$$

$$\beta(z) \;=\; 0.5\,\mathrm{arctg}\left(\frac{2\cdot10^{0.05\Delta\alpha z}\sin(\Delta\Phi z+0.5\pi)}{1-10^{0.1\Delta\alpha z}}\right) + 0.5\pi n, \tag{4}$$

where n = 1, 2, 3…

The complex amplitude of the received circularly polarized signal can then be determined from the following operator sequence:

$$\dot{E}^{IN}(z) \;=\; \left[\dot{T}\right] \cdot [R(\beta_0)]^{-1} \cdot \dot{E}^{OUT}(z), \tag{5}$$

where $\left[\dot{T}\right]$ is the operator of the polarization converter of the received signal; $[R(\beta_0)]^{-1}$ – the operator of the transition from the medium basis to the measuring basis; β_0 – the angle of orientation of the measuring basis relative to the medium basis.

For the case of receiving the signal with the right circular polarization $\left[\dot{T}\right]$ defines as

$$\left[\dot{T}\right] \;=\; \frac{1}{\sqrt{2}}e^{j\frac{\pi}{4}}\begin{vmatrix} 1 & -j \\ 0 & 0 \end{vmatrix}. \tag{6}$$

For the case of receiving the signal with the left circular polarization $\left[\dot{T}\right]$ defines as

$$\left[\dot{T}\right] \;=\; \frac{1}{\sqrt{2}}e^{j\frac{\pi}{4}}\begin{vmatrix} 1 & j \\ 0 & 0 \end{vmatrix}. \tag{7}$$

The received right circularly polarized signal can then be found by performing multiplication of the matrixes as:

$$\dot{E}_{RR}^{IN}(z) = \frac{1}{\sqrt{2}} e^{j\frac{\pi}{4}} \begin{vmatrix} 1 & -j \\ 0 & 0 \end{vmatrix} \cdot \begin{vmatrix} \cos\beta_0 & -\sin\beta_0 \\ \sin\beta_0 & \cos\beta_0 \end{vmatrix} \cdot \begin{vmatrix} \cos\beta(z)\cos\alpha(z) + j\sin\beta(z)\sin\alpha(z) \\ -\sin\beta(z)\cos\alpha(z) + j\cos\beta(z)\sin\alpha(z) \end{vmatrix} =$$

$$= \frac{1}{\sqrt{2}} e^{j\frac{\pi}{4}} [\cos\beta(z)\cos\alpha(z) + \cos\beta(z)\sin\alpha(z) + j(\cos\alpha(z)\sin\beta(z) + \sin\alpha(z)\sin\beta(z))] \cdot \tag{8}$$

$$\cdot (\cos\beta_0 - j\sin\beta_0).$$

After the simple manipulations, we finally obtain:

$$\dot{E}_{RR}^{IN}(z) = \frac{1}{\sqrt{2}} e^{j\left(\frac{\pi}{4} - \beta_0 + \beta(z)\right)} (\cos\alpha(z) + \sin\alpha(z)). \tag{9}$$

As can be seen from (9), the expression for the amplitude of the signal received with right circular polarization is

$$\left|\dot{E}_{RR}^{IN}(z)\right| = \frac{1}{\sqrt{2}} (\cos\alpha(z) + \sin\alpha(z)). \tag{10}$$

The phase of the signal received with right circular polarization is

$$\varphi_{RR}(z) = \frac{\pi}{4} - \beta_0 + \beta(z). \tag{11}$$

In the case of receiving a signal with left circular polarization, we have

$$\dot{E}_{RL}^{IN}(z) = \frac{1}{\sqrt{2}} e^{-j\frac{\pi}{4}} \begin{vmatrix} 1 & j \\ 0 & 0 \end{vmatrix} \cdot \begin{vmatrix} \cos\beta_0 & -\sin\beta_0 \\ \sin\beta_0 & \cos\beta_0 \end{vmatrix} \cdot$$

$$\cdot \begin{vmatrix} \cos\beta(z)\cos\alpha(z) + j\sin\beta(z)\sin\alpha(z) \\ -\sin\beta(z)\cos\alpha(z) + j\cos\beta(z)\sin\alpha(z) \end{vmatrix}. \tag{12}$$

By performing the transformation similar to the previous one, we obtain the expression for the received signal with left circular polarization:

$$\dot{E}_{RL}^{IN}(z) = \frac{1}{\sqrt{2}} e^{-j\left(\frac{\pi}{4} + \beta_0 + \beta(z)\right)} (\cos\alpha(z) - \sin\alpha(z)). \tag{13}$$

Then the amplitude of the signal defined in (13) will be

$$\left|\dot{E}_{RL}^{IN}(z)\right| = \frac{1}{\sqrt{2}} (\cos\alpha(z) - \sin\alpha(z)). \tag{14}$$

The phase of the signal received with left circular polarization is

$$\varphi_{RR}(z) = -\left(\frac{\pi}{4} + \beta_0 + \beta(z)\right). \tag{15}$$

To obtain desired expression for the circular depolarization ratio, let us define its value at the output of the logarithmic receiver as

$$CDR(\alpha(z)) = 20 \cdot \lg(\left|\dot{E}_{RR}(z)\right| / \left|\dot{E}_{RL}(z)\right|). \tag{16}$$

After the substituting (10) and (14) expression (16) can be written as

$$CDR(\alpha(z)) = 20 \cdot \lg\left(\frac{\cos\alpha(z) - \sin\alpha(z)}{\cos\alpha(z) + \sin\alpha(z)}\right). \tag{17}$$

Results analysis

Fig. 1-3 show the plots of the CDR as a function of the observation distance z calculated according to (17) for several values of the rain intensity. Differential attenuation $\Delta\alpha$ and differential phase shift
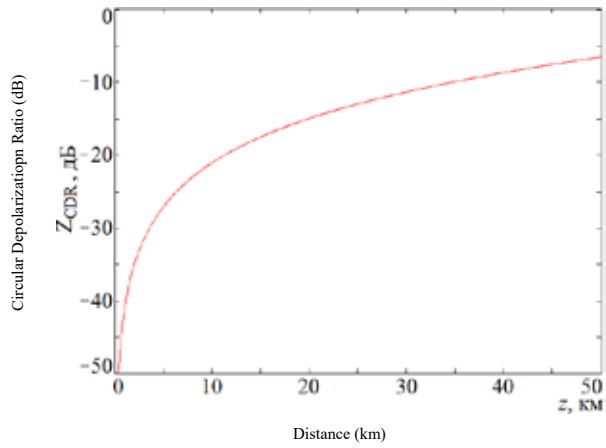
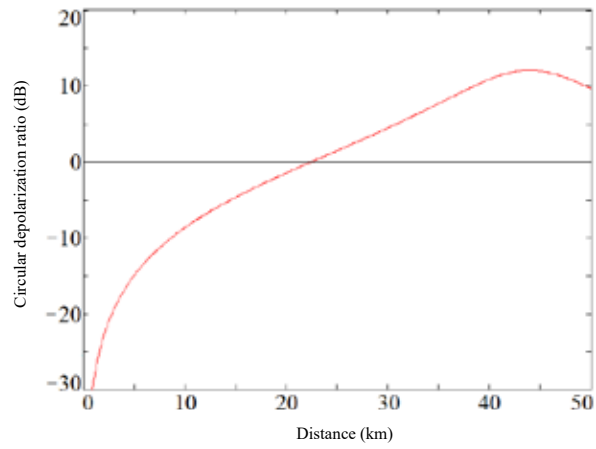Fig. 1. CDR as a function of observation distance z for the rain intensity R = 12.5 mm/h

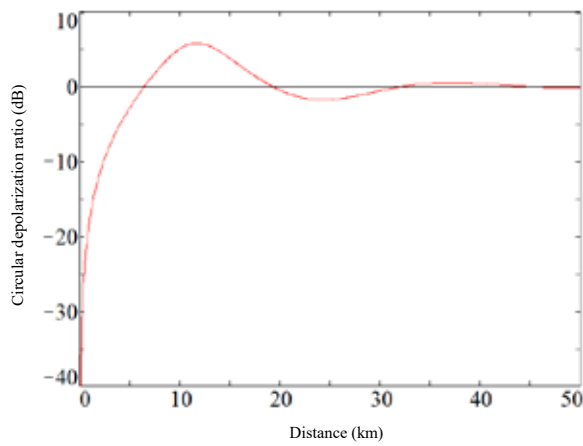Fig. 2. CDR as a function of observation distance z for the rain intensity R = 50 mm/h

Fig. 3. CDR as a function of observation distance z for the rain intensity R = 150 mm/h

ΔΦ values measured for the three-centimeter distance signals have used in the calculations. For the different values of the rain intensity R, these values were [9]: (a) $\Delta\alpha = 0.02$ dB/km, $\Delta\Phi = 1$ deg/km for R = 12.5 mm/h; (b) $\Delta\alpha = 0.1$ dB/km, $\Delta\Phi = 4$ deg/km for R = 50 mm/h; (c) $\Delta\alpha = 0.8$ dB/km, $\Delta\Phi = 14$ deg/km for R = 150 mm/h.

The analysis of the calculated curves of the CDR (see Fig. 1-3) at short distances z (for the front boundary of the meteorological formation) shows a substantial difference in the ratio value which at z = 5 km is about 25 dB for R = 12.5 mm/h, 15 dB for R = 50 mm/h, 40 dB for R = 150 mm/h respectively.

In this case, starting with the precipitates intensity of 50 mm/h, areas of positive values of the CDR evaluation appear. For R = 50 mm/h, the maximum of the CDR in the positive region is 12.5 dB, and for R = 150 mm/h is 6…7 dB. In the last two cases, CDR equals zero at a distance of 22.5 km (for R = 50 mm/h) and has four zero values at the distances of 6.45; 19.3; 32.15 and 45 km (for R = 150 mm/h).

Conclusion

The analytical solution for the evaluation of the circular depolarization ratio CDR on the base of Jones vector component functional dependence on a polarization ellipse orientation angle β(z) and ellipticity angle α(z) was obtained.

As it seen from (17), the characteristic feature of CDR is its dependence only on the variability of the ellipticity angle α(z). In this case, the ratio is independent of the orientation angle β(z) changing.

The magnitude of the circular depolarization ratio CDR as it follows from (3) and (17) depends on differential characteristics of propagation medium – differential attenuation Δα and differential phase shift ΔΦ.

The variability of the ratio has a pronounced character and substantial differences (up to 50 dB) at the short distances (z is about 12 km) from the front boundary of the meteorological formation.

Established CDR sign change is an undoubted indication of high-intensity precipitates presence in the sensed meteorological formation.

The procedure of high-intensity precipitation meteorological formations determining can be performed with the use of the algorithm (see above) for the CDR evaluating based on transmission and reception of signals with circular polarization ($\left|\dot{E}_{RR}\right|$ and $\left|\dot{E}_{RL}\right|$). In this case, the appearance of the positive ratio can indicate the meteorological formation with a high rain intensity.

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