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Construction and Analysis of Exact Solution of Oberbeck–Boussinesque Equations

Irina V. Stepanova*
Institute of Computational Modelling, SB RAS
Akademgorodok, 50/44, Krasnoyarsk, 660036
Institute of Mathematics and Computer Science
Siberian Federal University
Svobodny, 79, Krasnoyarsk, 660041
Russia

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A unidirectional stationary binary mixture motion in a horizontal channel is under study. New exact solution of the Oberbeck–Boussinesque equations is constructed for description of the mentioned flow. The obtained solution is applied for investigation of the separation process in mixture of water and isopropanol located between two differently heated rigid walls.

Keywords: Oberbeck–Boussinesque equations, exact solution, binary mixture flow.
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The Oberbeck–Boussinesque equations are often used in mathematical modelling of convection in fluids [1]. Application of these equations for description of mixture motion is possible if an equation for mass transfer is added on momentum and heat transfer equations. The obtained system of equations is nonlinear and has high order. That is why construction of exact solutions of the system is difficult and important problem.

Analysis of compatibility of the Oberbeck–Boussinesque equations for unidirectional motion allows to conclude that if the temperature function is a polynomial then it has degree not higher than three. For the linear function of temperature some exact solutions of the system under study are described in papers [2, 3]. They are constructed in cases of linear with respect to horizontal coordinate distribution of temperature or heat flux on the rigid walls. Work [4] is devoted to study of group properties of the two dimensional Oberbeck–Boussinesque equations and their invariant solutions. Not only solutions with linear temperature function are discussed there but more complex dependencies of all unknown functions on spatial coordinates are obtained. Quadratic law for temperature and concentration distributions is used in [5] for solution of joint motion of binary mixture and homogeneous liquid with common interface. The field of velocities in that paper corresponds to Himenz-type velocity. Different statements of boundary value problems are suggested in [6] for the parabolic dependence of the parameters of state on the horizontal coordinate.

The present paper continues the study started in [6]. It is devoted to exhausted description of constructing exact solution of the Oberbeck–Boussinesque equations for unidirectional motions in case of quadratic dependence of temperature and concentration with respect to horizontal coordinate. The functions of velocity, temperature, concentration and pressure are obtained. This solution is applied for description of mixture motion between two rigid walls. The parabolic

*stepiv@icm.krasn.ru

temperature law is posed on the lower wall, the upper wall is supposed to be thermal isolated. The algorithm of finding all constants of integration is proposed. Calculations are carried out for the binary mixture of water and isopropanol. Distributions of velocity, temperature and concentration are shown in figures and analyzed.

1. Governing equations

We consider stationary equations of unidirectional binary mixture motion (velocity vector is $\mathbf{u} = (u(y), 0, 0)$) [1]

$$\begin{gathered} \nu u_{yy} = \frac{1}{\rho_0} p^*_x, \quad g(\beta_1\theta + \beta_2 c) = \frac{1}{\rho_0} p^*_y, \\ u\theta_x = \chi(\theta_{xx} + \theta_{yy}), \quad uc_x = D(c_{xx} + c_{yy}) + D^\theta(\theta_{xx} + \theta_{yy}), \end{gathered} \tag{1}$$

where $u = u(y)$ is the horizontal velocity, $\theta = \theta(x, y)$ is the temperature, $c = c(x, y)$ is the light component concentration, $\mathbf{g} = (0, -g, 0)$ is the gravity acceleration vector, $p^* = p - g\rho_0 y$ is the modified pressure. The constant $\nu > 0$, $\chi > 0$, $D > 0$, D^θ are the kinematic viscosity, thermal diffusivity, diffusion and thermal diffusion respectively. We deal with normal thermal diffusion at $D^\theta < 0$ when the light component tends to more heated region. If $D^\theta > 0$ the light component tends to less heated region. This thermal diffusion is named abnormal.

Equations (1) describe the binary liquid motion in the Oberbeck–Boussinesq approximation. It means that the equation of state has the form $\rho = \rho_0(1 - \beta_1\theta - \beta_2 c)$, where β_1, β_2 are the thermal and concentration extension coefficients, ρ_0 is the average mixture density.

Let L be the scale of length while $\triangle T$ be the characteristic temperature difference. Then we use the dimensionless variables in the form

$$\hat{x} = \frac{x}{L}, \quad \hat{y} = \frac{y}{L}, \quad \hat{u} = \frac{\nu}{g\beta_1 \Delta T L^2}\, u, \quad \hat{p^*} = \frac{1}{\rho_0 g\beta_1 \Delta T L}\, p^*, \quad \hat{\theta} = \frac{1}{\Delta T}\,\theta, \quad \hat{c} = \frac{\beta_2}{\beta_1 \Delta T}\, c.$$

In these variables equations (1) take the form

$$\begin{gathered} u_{yy} = p_x, \quad \theta + c = p_y, \\ \mathrm{Gr}\, u\, \theta_x = \frac{1}{\mathrm{Pr}}\,(\theta_{xx} + \theta_{yy}), \quad \mathrm{Gr}\, u\, c_x = \frac{1}{\mathrm{Sc}}\,[c_{xx} + c_{yy} - \psi(\theta_{xx} + \theta_{yy})], \end{gathered} \tag{2}$$

where $\mathrm{Gr} = g\beta_1\Delta T L^3/\nu^2$ is the Grashof number, $\mathrm{Pr} = \nu/\chi$ is the Prandtl number, $\mathrm{Sc} = \nu/D$ is the Schmidt number, $\psi = -\beta_2 D^\theta/(\beta_1 D)$ is the separation ratio. The symbols "hat" and "asterisk" are omitted.

2. Solution of the governing equations

We differentiate the first equation from (2) with respect to y and the second one with respect to x. Then we compare the obtained expressions and have

$$u''' = (\theta + c)_x. \tag{3}$$

Here and below the prime denotes a derivative with respect to y. The left hand side of (3) does not depend on x. Then the right hand side can be represented in the form

$$\theta + c = \alpha(y)x + \beta(y), \tag{4}$$

the functions α and β are arbitrary. Substituting $c(x,y) = \alpha(y)x + \beta(y) - \theta(x,y)$ (see (4)) into the last equation in (2) we obtain

$$\theta_x = \frac{\alpha'' x + \beta''}{\mathrm{GrSc}\Psi\, u} - \frac{\alpha}{\Psi}, \tag{5}$$

where $\Psi = \mathrm{Pr}(1+\psi)/\mathrm{Sc} - 1$ is supposed to be not zero. From equation (5) we can derive a formula for θ and derivatives of θ with respect to x and y. These derivatives should be used for substitution into the third equation of (2). After the substitution we split the resulting expression with respect to powers of x and have three equations

$$\left(\frac{\alpha''}{u}\right)'' = 0, \quad \left(\frac{\beta''}{u} - \mathrm{ScGr}\,\alpha\right)'' = \mathrm{PrGr}\alpha'', \quad \frac{\alpha''}{u} + \mathrm{Sc}\gamma'' = \mathrm{PrGr}(\beta'' - \mathrm{ScGr}\,\alpha). \tag{6}$$

From equations (6) we conclude that

$$\begin{gathered} \alpha'' = (a_0 y + a_1)u, \quad \beta'' = (\mathrm{Gr}(\mathrm{Sc}+\mathrm{Pr})\,\alpha - b_0 y - b_1)u, \\ \gamma'' = \frac{\mathrm{PrGr}}{\mathrm{Sc}}\left(\alpha \mathrm{GrPr} - b_0 y - b_1\right)u - \frac{a_0 y + a_1}{\mathrm{Sc}}, \end{gathered} \tag{7}$$

where a_0, a_1, b_0, b_1 are arbitrary constants connecting with integration of two first equations in (6). Using the expression for α in (7), equations (3) and (4) we derive the equation for velocity

$$u^{(V)} - (a_0 y + a_1)u = 0. \tag{8}$$

If $a_0 = a_1 \equiv 0$ we have the Birikh-type solution for the velocity. It was constructed in [2] for a homogeneous fluid and in [7] for description of joint motion of a liquid and a gas with common interface. This solution is not considered here. In this paper we study the case $a_0 \equiv 0$ and $a_1 \neq 0$. Then we have the following solution of equation (8)

$$u(y) = C_1 e^{\lambda y} + e^{\lambda\mu_1 y}(C_2 \cos\lambda\mu_2 y + C_3 \sin\lambda\mu_2 y) + e^{\lambda\mu_3 y}(C_4 \cos\lambda\mu_4 y + C_5 \sin\lambda\mu_4 y). \tag{9}$$

Here C_i, $i = 1,\ldots,5$, are constant. The other constants in (9) are

$$\lambda = \sqrt[5]{a_1}, \quad \mu_1 = \frac{\sqrt{5}-1}{4}, \quad \mu_2 = \frac{\sqrt{10+2\sqrt{5}}}{4}, \quad \mu_3 = -\frac{\sqrt{5}+1}{4}, \quad \mu_4 = \frac{\sqrt{10-2\sqrt{5}}}{4}.$$

It is interesting to note that μ_i, $i = 1,\ldots,4$, have some properties we used in calculations further

$$\mu_1 + \mu_3 = -0.5, \quad \mu_1^2 + \mu_2^2 = 1, \quad \mu_3^2 + \mu_4^2 = 1, \quad \mu_1^2 - \mu_2^2 = \mu_3, \quad \mu_3^2 - \mu_4^2 = \mu_1,$$

$$\begin{gathered} 1 + \mu_1\mu_3 + \mu_2\mu_4 = 1 + \mu_1, \quad 1 + \mu_1\mu_3 - \mu_2\mu_4 = 1 + \mu_3, \\ (1+\mu_1)^2 + \mu_2^2 = 2(1+\mu_1), \quad (1+\mu_3)^2 + \mu_4^2 = 2(1+\mu_3). \end{gathered}$$

From equation (4) and expression (3) the function $\alpha = u'''$, i. e.

$$\alpha = P_1 e^{\lambda y} + e^{\lambda\mu_1 y}(P_2 \cos\lambda\mu_2 y + P_3 \sin\lambda\mu_2 y) + e^{\lambda\mu_3 y}(P_4 \cos\lambda\mu_4 y + P_5 \sin\lambda\mu_4 y) \tag{10}$$

with notation in the following form

$$\begin{gathered} P_1 = AC_1, \quad P_2 = A(C_2\mu_3 - 2C_3\mu_1\mu_2), \quad P_3 = A(C_3\mu_3 + 2C_2\mu_1\mu_2), \\ P_4 = A(C_4\mu_1 - 2C_5\mu_3\mu_4), \quad P_5 = A(C_5\mu_1 + 2C_4\mu_3\mu_4), \quad A = \frac{a_1}{\lambda^2}. \end{gathered}$$

Further we can find the functions β and γ from the second and the third equations in (7)

$$\beta = \mathrm{Gr}(\mathrm{Sc}+\mathrm{Pr})M - N + C_6 y + C_7, \quad \gamma = \frac{\mathrm{Gr}^2\mathrm{Pr}^2}{\mathrm{Sc}}M - \frac{\mathrm{GrPr}}{\mathrm{Sc}}N - \frac{a_1 y^2}{2\mathrm{Sc}} + C_8 y + C_9. \tag{11}$$

Here C_i, $i = 6, \dots, 9$, are constants, the other notation is written below.

$$M = t_1 e^{2\lambda y} + \frac{e^{\lambda(1+\mu_1)y}}{2\lambda(1+\mu_1)}\Big(t_2\cos\lambda\mu_2 y + t_3\sin\lambda\mu_2 y\Big) + \frac{e^{\lambda(1+\mu_3)y}}{2\lambda(1+\mu_3)}\Big(t_4\cos\lambda\mu_4 y + t_5\sin\lambda\mu_4 y\Big) +$$
$$+\frac{e^{\lambda(\mu_1+\mu_3)y}}{2\lambda}\Big(t_6\cos\lambda m^- y + t_7\sin\lambda m^- y + t_8\cos\lambda m^+ y + t_9\sin\lambda m^+ y\Big) +$$
$$+\frac{e^{2\lambda\mu_1 y}}{2\lambda}\Big(t_{10} + t_{11}\cos 2\lambda\mu_2 y + t_{12}\sin 2\lambda\mu_2 y\Big) + \frac{e^{2\lambda\mu_3 y}}{2\lambda}\Big(t_{13} + t_{14}\cos 2\lambda\mu_4 y + t_{15}\sin 2\lambda\mu_4 y\Big),$$
$$N = (h_1 y + h_2)e^{\lambda y} + e^{\lambda\mu_1 y}((h_3 y + h_4)\cos\lambda\mu_2 y + (h_5 y + h_6)\sin\lambda\mu_2 y) +$$
$$+e^{\lambda\mu_3 y}((h_7 y + h_8)\cos\lambda\mu_4 y + (h_9 y + h_{10})\sin\lambda\mu_4 y).$$

The constants t_j, $j = 1, \dots, 15$, and h_k, $k = 1, \dots, 10$, depend on constants C_i, $i = 1, \dots, 5$, and a_1, b_0, b_1 nonlinearly. In spite of cumbersomeness we list them here.

$$t_1 = \frac{\bar{A}C_1^2}{2\lambda}, \quad t_2 = \frac{\bar{A}C_1(C_2(2\mu_3-1) - 2C_3\mu_2)}{2(1+\mu_1)}, \quad t_3 = \frac{\bar{A}C_1(C_3(2\mu_3-1) + 2C_2\mu_2)}{2(1+\mu_1)},$$
$$t_4 = \frac{\bar{A}C_1(C_4(2\mu_1-1) - 2C_5\mu_4)}{2(1+\mu_3)}, \quad t_5 = \frac{\bar{A}C_1(C_5(2\mu_1-1) + 2C_4\mu_4)}{2(1+\mu_3)},$$
$$t_6 = \frac{\bar{A}(C_2(C_4 + 2\bar{\mu}C_5) + C_3(C_5 - 2\bar{\mu}C_4))}{4(1+\mu_3)^2}, \quad t_7 = \frac{\bar{A}(C_3(C_4 + 2\bar{\mu}C_5) - C_2(C_5 - 2\bar{\mu}C_4))}{4(1+\mu_3)^2},$$
$$t_8 = \frac{\bar{A}(C_2(C_4 - 2\tilde{\mu}C_5) - C_3(C_5 + 2\tilde{\mu}C_4))}{4(1+\mu_1)^2}, \quad t_9 = \frac{\bar{A}(C_2(C_5 + 2\tilde{\mu}C_4) + C_3(C_4 - 2\tilde{\mu}C_5))}{4(1+\mu_1)^2},$$
$$t_{10} = \frac{\bar{A}\mu_3(C_2^2 + C_3^2)}{2\mu_1^2}, \quad t_{11} = \frac{\bar{A}((C_2^2 - C_3^2)\mu_1 + 2C_2C_3\mu_2)}{2}, \quad t_{12} = \frac{\bar{A}(2C_2C_3\mu_1 - (C_2^2 - C_3^2)\mu_2)}{2},$$
$$t_{13} = \frac{\bar{A}\mu_1(C_4^2 + C_5^2)}{2\mu_3^2}, \quad t_{14} = \frac{\bar{A}((C_4^2 - C_5^2)\mu_3 + 2C_4C_5\mu_4)}{2}, \quad t_{15} = \frac{\bar{A}(2C_4C_5\mu_3 - (C_4^2 - C_5^2)\mu_4)}{2},$$

$$h_1 = \frac{b_0 C_1}{\lambda^2}, \quad h_2 = -\frac{\bar{B}C_1}{\lambda^3}, \quad h_3 = \frac{b_0(C_2\mu_3 - 2C_3\mu_1\mu_2)}{\lambda^2}, \quad h_4 = -\frac{C_2\bar{B}\mu_3 - 2C_3 B\mu_1\mu_2}{\lambda^3},$$
$$h_5 = \frac{b_0(C_3\mu_3 + 2C_2\mu_1\mu_2)}{\lambda^2}, \quad h_6 = -\frac{C_3\bar{B}\mu_3 - 2C_2 B\mu_1\mu_2}{\lambda^3}, \quad h_7 = \frac{b_0(C_4\mu_1 - 2C_5\mu_3\mu_4)}{\lambda^2},$$
$$h_8 = -\frac{C_4\bar{B}\mu_1 + 2C_5 B\mu_3\mu_4}{\lambda^3}, \quad h_9 = \frac{b_0(C_5\mu_1 + 2C_4\mu_3\mu_4)}{\lambda^2}, \quad h_{10} = -\frac{C_5\bar{B}\mu_1 - 2C_4 B\mu_3\mu_4}{\lambda^3}.$$

We use the notation in the formulas for t_j and h_k

$$B = 2b_0 + \lambda b_1, \quad \bar{B} = 2b_0 - \lambda b_1, \quad \bar{A} = \frac{a_1}{2\lambda^3}, \quad \bar{\mu} = \mu_2 - \mu_4, \quad \tilde{\mu} = \mu_2 + \mu_4.$$

After determination of the functions α, β and γ we can write the functions of temperature and concentration. They have the form

$$\theta(x,y) = \frac{1}{\mathrm{Gr}\Psi}\left(\frac{a_1}{2\mathrm{Sc}}x^2 + \frac{\alpha(y)\mathrm{GrPr} - b_0 y - b_1}{\mathrm{Sc}}x + \gamma(y)\right), \quad c(x,y) = \alpha(y)x + \beta(y) - \theta(x,y). \tag{12}$$

Formulas (9) and (12) are the exact solution of equations (2). The modified pressure p can be calculated by integrating

$$p(x,y) = x\int \alpha(y)\,dy + \int \beta(y)\,dy + p_0$$

with arbitrary constant p_0. The solution obtained can be used for description of a binary mixture motion in layers extended enough in the horizontal direction. In the next section we apply solution (9), (12) for study of separation process of binary mixture in layer between two differently heated rigid walls.

3. Motion in a horizontal channel with thermal isolated upper wall

We consider a motion of binary mixture between two rigid walls. Boundary conditions on the lower wall are written as follows. The first of them is a quadratic distribution of the temperature with respect to x

$$\theta|_{y=0} = \theta_0 x^2 + \theta_1 x + \theta_2, \tag{13}$$

where θ_i, $i = 0, 1, 2$, are given dimensionless constants. This form of temperature is connected with presentation of the temperature in formula (12). The second condition is the absence of mass flow through the lower wall $y = 0$

$$(\theta' - \psi c')|_{y=0} = 0, \tag{14}$$

and the last one is no-slip condition

$$u(0) = 0. \tag{15}$$

Conditions (13) and (14) are splitted with respect to powers of x into five equations

$$a_1 = 2\mathrm{ScGrPr}\Psi\theta_0, \quad b_1 = \mathrm{Gr}(\alpha(0)\mathrm{Pr} - \mathrm{Sc}\Psi\theta_1), \tag{16}$$

$$\gamma(0) = \mathrm{Gr}\Psi\theta_2, \quad \beta'(0) - \frac{1+\psi}{\mathrm{Gr}\Psi}\gamma'(0) = 0, \quad \alpha'(0)\left(1 - \frac{(1+\psi)\mathrm{Pr}}{\mathrm{Sc}\Psi}\right) + \frac{(1+\psi)b_0}{\mathrm{Gr}\Psi\mathrm{Sc}} = 0. \tag{17}$$

The upper wall is supposed to be thermal isolated. That is why the conditions for temperature and concentration are

$$\theta'|_{y=1} = 0, \quad c'|_{y=1} = 0, \tag{18}$$

and no-slip condition is

$$u(1) = 0. \tag{19}$$

Splitting conditions (18) with respect to powers of x we have four equations

$$\gamma'(1) = 0, \quad \alpha'(1)\mathrm{GrPr} - b_0 = 0, \quad \beta'(1) = 0, \quad \alpha'(1)\left(1 - \frac{\mathrm{Pr}}{\Psi\mathrm{Sc}}\right) + \frac{b_0}{\mathrm{Gr}\Psi\mathrm{Sc}} = 0. \tag{20}$$

From the second and fourth equations in (20) we conclude that $b_0 = 0$ and $\alpha'(1) = 0$, then $\alpha'(0) = 0$ from the last equation in (17) because $\mathrm{Sc} \neq 0$. Thereby, we have eight conditions for nine constants C_i and once more boundary condition should be added for the problem closure. We use the integral condition for concentration in section $x = 0$

$$\int_0^1 (\mathrm{Gr}\Psi\beta + \gamma)\,dy = \mathrm{Gr}\Psi C_0, \tag{21}$$

where C_0 is the average concentration.

We suggest the algorithm of finding constants in functions u, β and γ. Firstly, we express C_i, $i = 2, 3, 4, 5$, through C_1 from equations (15), (19) and $\alpha'(0) = \alpha'(1) = 0$. These expressions are linear with respect to C_1. Secondly, we determine C_i, $i = 6, 7, 8, 9$, from the third equation in (20), condition (21) and the first equations from (20) and (17) respectively. At last, the second equation from (17) has not used yet. After substitution of all values of constants expressed by C_1 into the second equation from (17) we have the quadratic equation for C_1. We should use a such initial data for which the roots of the obtained equation remain real. After C_1 is found we can reconstruct all constants C_i, $i = 2, \dots, 9$. It is obviously the couple of such constant sets is obtained in general case.

Further we demonstrate the solution of the problem under study for the mixture of water-isopropanol (30 %/70 %). The physical parameters of this mixture are given in [8]: Pr = 398.403, Sc = 112040.8 and $\psi = -0.1144$. The Grashof number depends on temperature difference $\triangle T$ and layer width L. We use values $\theta_1 = 0.01$, $\theta_2 = 0.1$, $\triangle T = 2\,K$ and analyze the behaviour of the velocity in dependence on θ_0 and L. There are distributions of velocity with respect to the thermal load θ_0 in Fig. 1. The decrease of thermal load θ_0 leads to the essential velocity decrease. The layer width L has influence on motion as well as θ_0. Constriction of the distance between lower and upper walls generates intensification of the flow. It can be seen in Fig. 2. For all considered cases we observe direct and inverse zones of flow, the velocity changes a sign near the center of the layer.

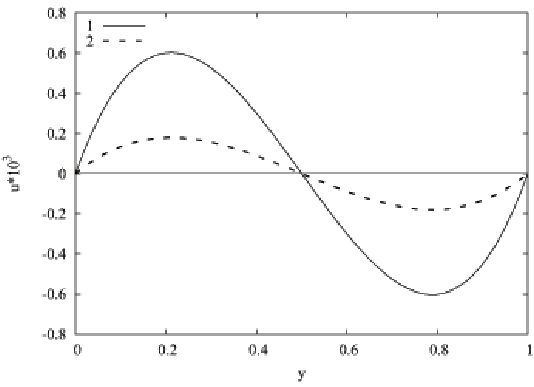

Fig. 1. Dependence of velocity on θ_0: 1 — $\theta_0 = -0.0001$, 2 — $\theta_0 = -0.00001$, $L = 0.001\,m$

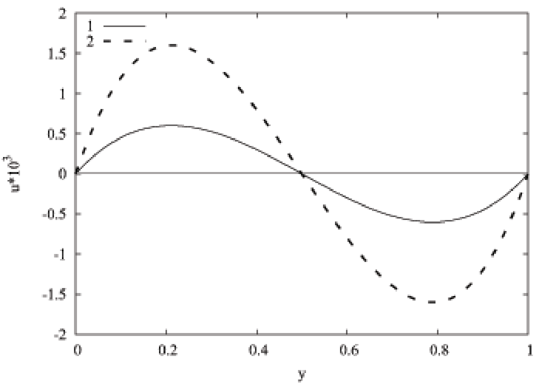

Fig. 2. Dependence of velocity on L: 1 — $L = 0.001\,m$, 2 — $L = 0.0005\,m$, $\theta_0 = -0.0001$

Temperature $\theta(x, y)$ and concentration $c(x, y)$ are shown in Fig. 3. Both of them are close to a flat surface in spite of that the functions θ and c in (12) are essentially nonlinear. The concentration for the parameters used changes from its average value $C_0 = 0.7$ essentially. It should be noted that we deal with abnormal thermal diffusion because $\psi < 0$. It means the light component tends to less heated wall. It is shown in Fig. 3 the concentration of the isopropanol is bigger near $y = 1$, where smaller temperature is. Figs. 1, 2 and 3 are constructed for positive root C_1. For the negative root C_1 we have mirror symmetry for the velocity profiles.

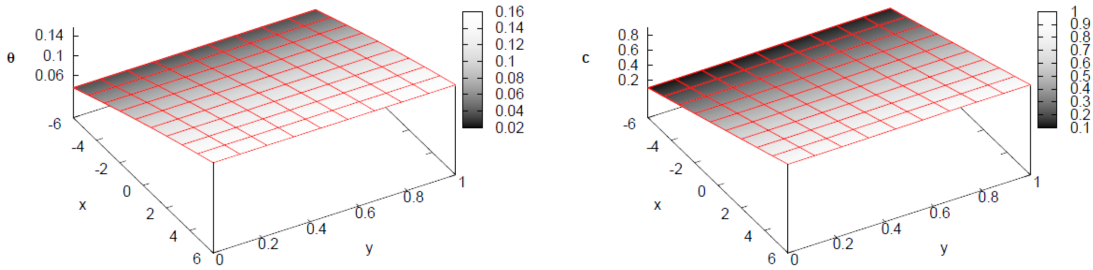

Fig. 3. Temperature (left) and concentration (right) for parameters $L = 0.001\,m$, $\triangle T = 2\,K$, $\theta_0 = -0.0001$

Conclusion

The main feature of the paper is the construction of new exact solution of the Oberbeck–Boussinesque equations. The equations are used for mathematical modelling of liquid mixture motion under the action of buoyancy force linearly depending on temperature and concentration. The constructed solution is significantly different from known Birikh-type solution widely used in physical applications. It should be reminded that the Birikh-type solution presents the linear dependence of temperature and concentration on the horizontal coordinate. The solution obtained in the present paper is essentially nonlinear with respect both spatial variables. For the mixture water-isopropanol the velocity profile changes a sing, regions with direct and inverse motion occur. Increasing of thermal load on the lower wall leads to intensification of mixing, the constriction of layer width generates more intensive flow as well. The parameters used for the construction of temperature and concentration dependence are such that plots are similar to a flat surface. The separation effect displays strongly for this binary mixture, changes of concentration are significant. It is fair to note that nonlinearity of temperature and concentration fields is not be pronounced tangibly for this mixture. It is verisimilar that there are binary mixtures for which the structure of the obtained solution is more complicated and it can be applied for description of effects which are not caught by earlier known solutions.

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Построение и анализ точного решения уравнений Обербека–Буссинеска

Ирина В. Степанова
Институт вычислительного моделирования СО РАН
Академгородок, 50/44, Красноярск, 660036
Институт математики и фундаментальной информатики
Сибирский федеральный университет
Свободный, 79, Красноярск, 660041
Россия

Построено новое точное решение уравнений Обербека–Буссинеска для описания стационарного однонаправленного течения бинарной смеси в горизонтальном канале. Полученное решение использовано для изучения режима разделения бинарной смеси на компоненты. Проанализировано влияние геометрии течения и тепловой нагрузки на стенке на скорость смеси вода-изопроапанол.

Ключевые слова: уравнения Обербека–Буссинеска, точное решение, течение бинарной смеси.