Mobile robot motion estimation using Hough transform

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Abstract. This paper proposes an algorithm for estimation of mobile robot motion. The geometry of surrounding space is described with range scans (samples of distance measurements) taken by the mobile robot's range sensors. A similar sample of space geometry in any arbitrary preceding moment of time or the environment map can be used as a reference. The suggested algorithm is invariant to isotropic scaling of samples or map that allows using samples measured in different units and maps made at different scales. The algorithm is based on Hough transform: it maps from measurement space to a straight-line parameters space. In the space of straight-line parameters, the problems of estimating rotation, scaling and translation are solved separately breaking down a problem of estimating mobile robot localization into three smaller independent problems. The specific feature of the algorithm presented is its robustness to noise and outliers inherited from Hough transform. The prototype of the system of mobile robot orientation is described.

1. Introduction

For simultaneous real time localization and mapping, two approaches are usually applied to estimate relative position of a mobile robot with measurements from the onboard sensors [1]:

- feature points-based method;
- samples mapping.

The feature points-based method is preferred in environments where distinguishable features can be identified. The positions of feature points are defined in the coordinate plane of a mobile robot and its localization is then defined from comparing feature points taken from two samples. This approach was an initiative way to address the SLAM problem at the beginning of its development [2]. It has an obvious drawback: errors that occur while comparing feature points causes incorrect estimation of robot localization.

On the contrary, samples mapping does not require comparing separate points. Samples are compared by minimizing a function that reflects the degree of geometrical mapping. Outliers in samples do not have a significant effect on the final result. The most popular algorithm is an Iterative Closest Point algorithm (ICP) [3, 4] which assumes iterative optimization of one sample`s position relative to another one`s, where the modifications of the Least Squares Method are used. The necessity to provide sufficiently accurate initial estimation of location and iterative nature that determine low speed response can be considered as drawbacks. An algorithm based on Hough transform was suggested by the group of authors [5-10]. This algorithm does not require the initial approximate solution and it is invariant to similarity transformations (rotation, translation, isotropic scaling). The algorithm described in their studies has several drawbacks: its translation invariance is limited to small translation values; defining the rotation period demands computations in the measurement space; there

is no description of the calculation method; the problem of scaling estimation is not considered. In one of the previous works [11], the authors have suggested the modification of an algorithm which solves the first two of the problems mentioned. This paper focuses on assessing the mobile robot motion and a relative scale of samples that is important for solving the problem of orientation with aprioristic maps and combinations of maps constructed by different robots.

2. Problem statement

The two-dimensional case of problem statement is considered. There are two samples of range scan taken by mobile robot sensors: reference sample M_1 and study sample M_2. The samples correspond to the same environment. These samples can be considered as:

- samples, gathered by one robot in two separate time points;
- a sample, gathered by the robot and a space map;
- samples, gathered by two robots.

Robot position at the moment of study sample capture defines relation between samples M_1 and M_2: study sample is rotated by α and translated by $(t_x, t_y)^T$. In case of various origin of the samples (a map or samples of different robots are used) it is also necessary to consider scaling coefficient s. Thus, to solve the problem of mobile robots localization in environment we need to calculate variables α, $(t_x, t_y)^T$, s, connecting M_1 and M_2.

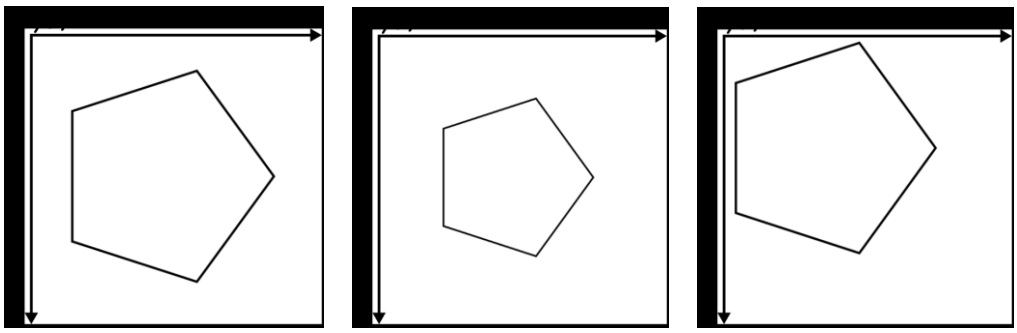
3. Concept of the algorithm

The algorithm for the estimation of rotation suggested in one of the authors' previous works [11] allows one to break down the formulated problem into three independent subproblems: estimation of rotation α, estimation of scaling s and estimation of translation $(t_x, t_y)^T$. In essence, the algorithm for the estimation of rotation α assumes that measurements of samples M_1 and M_2 are also transformed to Hough Space $[\theta, \rho]$ calculated by (1), the result of which are accumulators A_1 and A_2.

$$A(\theta,\rho) = \sum_{m \in M} f(\rho - (x_m \cos\theta + y_m \sin\theta)), \tag{1}$$

where $f(x) = \begin{cases} 1, x = 0 \\ 0, x \neq 0 \end{cases}$.

Further reasoning is illustrated by the example of samples processing shown in Figure 1.



a b c

Figure 1. Examples of samples: a) a reference sample b) a sample with scale, s = 1,33 c) a sample with translation $t_x = t_y = -50$.

Mutual accumulator offset along the θ axis corresponds to the amount of rotation α. The accumulator offset is defined by maximizing correlation function (2) of extrema distributions which are calculated by (3).

$$cor(D_1(\theta), D_1(\theta+\alpha)) \xrightarrow[\alpha]{} \max, \alpha = \overline{0,2\pi}, \tag{2}$$

$$D(A,\theta) = \max(A(\theta,\rho)). \tag{3}$$

This paper considers the problem of defining scaling coefficient s and the value of translation $(t_x, t_y)^T$ using Hough transform.

The algorithm for estimating scaling coefficient s is based on the concept that isotropic scaling an object in measurement space keeps straight lines parallel, changing the distance between them. Considering the process of scaling in Hough transform space it is possible to conclude that scaling changes the distance along axis ρ between the extrema corresponding to parallel straight lines in proportion to scaling coefficient. The phenomenon is shown in Figures 2a and 2b.

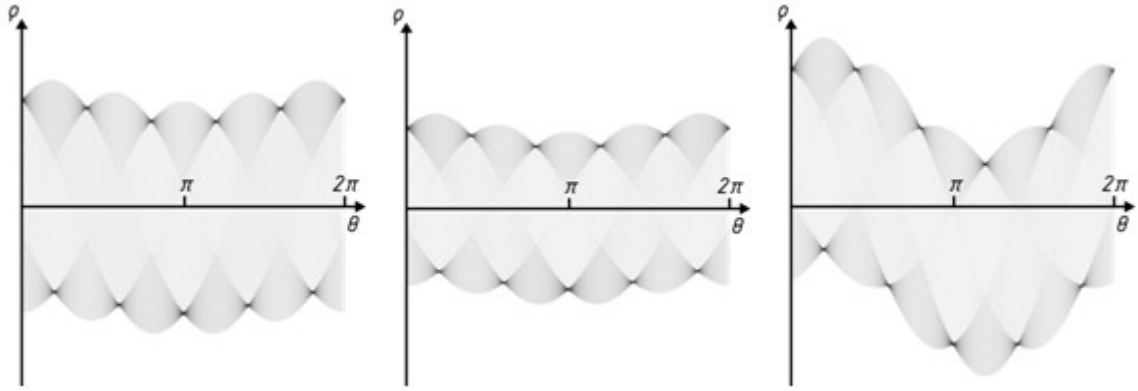


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Figure 2. Reference samples accumulators: a) a reference sample b) a sample with scale c) a sample with translation.

Robot translation causes a change in the distance between a robot and surrounding objects. Robot translation causes a change in the distance between a robot and surrounding objects. When one considers measurement space as a set of straight lines one can notice that there is no change in the distance between the mobile robot and the straight lines, parallel to translation vector, but the distance between the mobile robot and the straight lines, normal to translation vector, changes by an amount of translation. As to Hough transform space, the mobile robot translation defines corresponding extrema offset along the ρ axis, as can be seen in Figures 2a and 2c. The value of maximum offset of extrema along the ρ axis defines the value of the mobile robot translation. Value θ, where maximum extrema offset along axis ρ is achieved, defines the azimuth of translation.

Estimation of α is invariant to a mobile robot translation and samples' scale. Thus it is possible to compensate rotation irrespectively of scaling and translation. Compensation of rotation in Hough transform space is carried out by cyclic accumulator offset along the θ axis by amount α:

$$A_k(\theta,\rho) = A((\theta+\alpha) \bmod 2\pi, \rho). \tag{4}$$

4. Estimation of scaling

On the basis of the method for scaling estimation considered above, let us suggest an algorithm for the estimation of scaling coefficient s:

Step 1. Construct accumulators A_1 and A_2 of the Hough transform for study samples M_1 and M_2.

Step 2. Estimate rotation by (2) and compensate it in an accumulator space using equation (4).

Step 3. Estimate scaling coefficient s_i for each accumulator column (for each θ), see Figure 3.

Step 4. Calculate scaling coefficient s on the basis of the set of values s_i.

The scaling coefficient for the column can be calculated as follows:

1. as a ratio of range widths of the non-zero values in a column, calculated by (5);

2. as a ratio of spectral reflectance values in columns;
3. by maximizing correlation function of values in columns with scaling coefficient s_i and mutual translation:

$$s_i = \frac{w(A_1, i)}{w(A_2, i)}, \tag{5}$$

where $w(A, i)$ – range width of the non-zero values in the ith column of accumulator A.

It is obvious that the most precise result is provided with the last method. However it is extremely computationally expensive due to necessity of numerical optimization by two variables. Therefore its application is not reasonable. The most efficient is the first method since the range width can be calculated at a stage of accumulator creation by an elementary operation, that is why it is rather computationally cheap.

Let us suggest calculating required scaling coefficient s by choosing a median value of the set of scaling coefficients s_i by (6), (see Figure 3):

$$s = median(s_i). \tag{6}$$

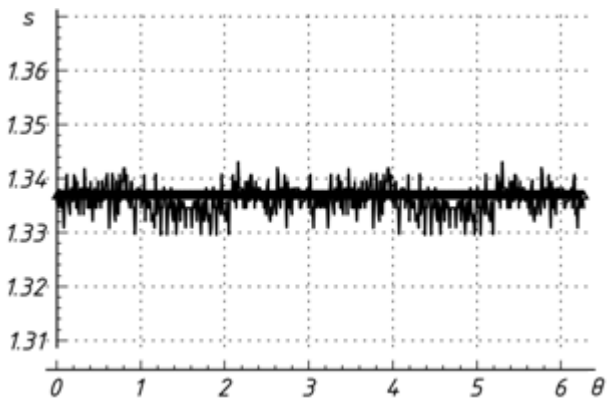


Figure 3. Scaling coefficients s_i and median value (bold line).

5. Estimation of translation

On the basis of the concept of translation estimation considered above, let us obtain an algorithm for estimation of translation $(t_x, t_y)^T$:

Step 1. Construct accumulators A_1 and A_2 of the Hough transform for study samples M_1 and M_2.

Step 2. Estimate rotation by (2) and compensate it in an accumulator space using equation (4).

Step 3. Estimate scaling by (5) and (6) and compensate it in an accumulator space.

Step 4. Estimate translation t_i for each accumulator column (for each θ), as maxima of the correlation function:

$$cor(A_1(i,0), A_2(i,t_i)) \xrightarrow[t_i]{} \max, \tag{7}$$

where $A(i, t_i)$ – i^{th} column of the accumulator, translated by t_i along axis ρ.

Step 5. Estimate amplitude t_A and phase offset of sine wave t_φ, described by a set of values t_i. In an ideal scenario, values t_i describe a sine wave with period 2π; however due to the existence of noise and outliers in source data, values t_i may also contain noise and outliers. It is recommended to make robust assessment t_A and t_φ with the discrete Fourier transform of the set of values. t_A and t_φ correspond to the amplitude and phase offset with frequency 2π.

Step 6. When considering t_A as an amount of mobile robot translation, and t_φ as a route, $(t_x, t_y)^T$ is calculated as follows:

$$t_x = \cos(t_\varphi) \cdot t_A; \quad t_y = \sin(t_\varphi) \cdot t_A, \tag{8}$$

Thus, when there are two samples of range-metric measurements, it ispossible to estimate rotation and translation of the mobile robot, producing calculations only in the Hough transform space.

6. Computational experiment

The ultimate goal of the research devoted to the construction of the algorithm for mobile robot angular and linear motion estimation is to develop the system of mobile robot orientation on the basis of range-metric data. There is a given set of samples of the range-metric data obtained by the systems of mobile robot onboard sensors $M_1, \ldots, M_N$. The task of the system is to generate a two-dimensional path $P_1, \ldots, P_N$, where $P_i = (x_i, y_i, \alpha_i)^T$ – robot position at i time: $(x, y)^T$ – coordinates in the world coordinate system, α – route (angular orientation). For simplicity it is considered that $P_0 = (0, 0, 0)^T$, i.e. the beginning of the coordinate system matches the mobile robot position at zero time.

For the computational experiment the authors have designed:

- the prototype of the system of mobile robot orientation;
- the imitative environment providing generation of the range-metric data samples on the basis of geometrical description of the environment and the mobile robot current position.

The experiment is made as follows: the experimenter interacts with the imitative environment by means of the graphic user interface, defining the next P_i of the mobile robot. The imitative environment transfers the generated sample to the system of the mobile robot orientation. The system of orientation estimates position $\hat{P}_i$ and adds it to the path constructed on the previous steps.

The purpose of the experiment is to provide qualitative estimation of the system operability that is carried out by expertise – the experimenter previews the compliance of the actual trajectory with the estimated one. The purpose of such approach to estimation of the prototype system operability is to define its operability in general, its feasibility and the ways of the system further development.

The prototype of the system functions on the basis of the trajectory generation algorithm, schematically represented in Figure 4.

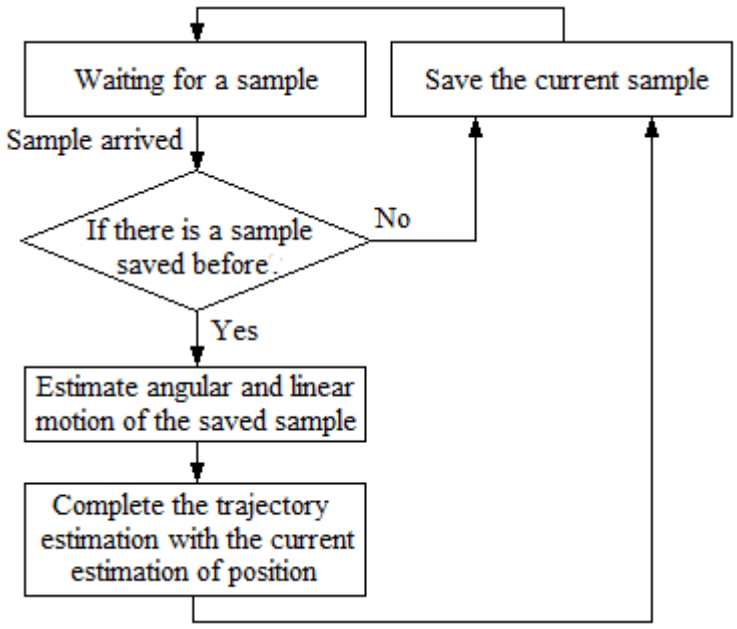
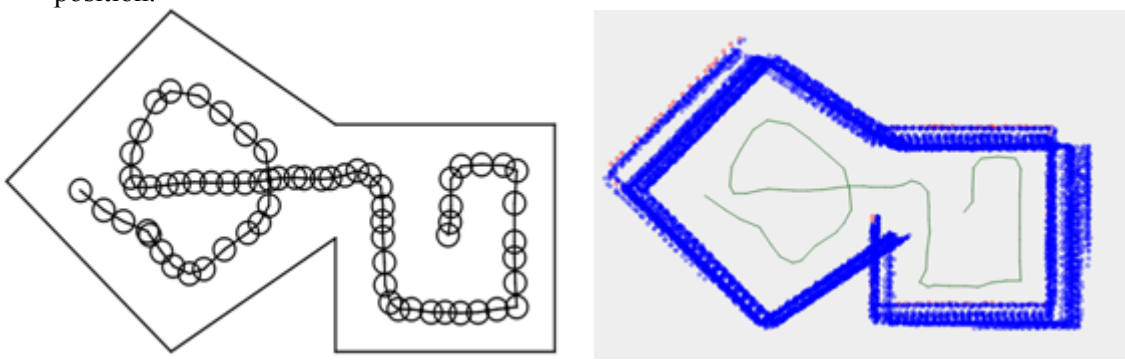


Figure 4. The algorithm of the prototype system operability.

In Figure 5 one can find the visualization of the experiment: the actual trajectory constructed in the imitative environment (Figure 5a) and the trajectory generated by the range-metric data (Figure 5b).

As a result of the experiment, the following conclusions were made:

- the algorithm suggested is efficient in general; it allows one to solve the problem of trajectory estimation;
- there has been observed error accumulation in estimation of the position caused by the fact that the current position is calculated as an offset relatively to the previous estimation of the position.



a b

Figure 5. Mobile robot trajectory: a) actual trajectory b) generated trajectory.

7. Conclusion

The problem of estimating the position of a mobile robot in surrounding environment is considered. The algorithm for linear motion estimation based on two-dimensional linear Hough Transform is described. Together with [11], published earlier, the algorithm described in this paper allows one to solve the problem of estimating the mobile robot trajectory on the basis of range-metric data.

The algorithm proposed inherits restrictions from the algorithm for estimation of angular orientation described earlier, being its derivation. In particular, the prevalence of straight lines should exist in the geometry of surrounding space (due to the choice of linear Hough Transform). Besides, it is not possible to define a mobile robot position in symmetric environments unambiguously.

The developed prototype of the system of mobile robot orientation with the algorithm for mobile robot position estimation is described. The experiment has proved the operability of the system. During the experimental sessions, the drawback of the approach to generation has been revealed – the estimation of the current position relatively to the previous position estimation with the algorithm suggested causes error accumulation.

Further consideration of issues concerning estimation of the current position relatively the map of the environment, generated from saved samples, is supposed to be in the authors' future research.

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