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Analysis of an Exact Solution of Problem of the Evaporative Convection (Review). Part I. Plane Case

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Development of theory describing the convection under conditions of "liquid–gas" phase transition, is caused by the active experimental study of the convective phenomena accompanied by evaporation/condensation at interphase. Results of the analytical and numerical investigation of new nonstandard problems of heat and mass transfer in domains with free surfaces or interfaces allow one to evaluate the adequacy of new mathematical models and to derive new characteristic criteria. The obtained fundamental knowledge on physical mechanisms of the studied processes provides the basis of modification and improvement of the fluidic technologies using the evaporating liquids and gas-vapor mixtures as working media. In the paper the analysis of the exact solution of the convection equations, which gives a possibility to model the two-layer convective fluid flows with evaporation, is presented.

Keywords: evaporative convection, exact solution, two-layer flows, thermocapillary interface.

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Introduction

For many years the problems of gravitational and thermocapillary convection of fluids under conditions of terrestrial gravitation and in the weak force fields are very actual. These problems are studied theoretically, numerically and experimentally [1–8]. In this broad area of

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hydrodynamics the convective motions in the domains with interfaces and free boundaries arouse particular interest. If the system of the Oberbeck–Boussinesq equations (or the Navier–Stokes equations in the Oberbeck–Boussinesq approximation) is used for description of the convection of the fluids then the mathematical model is called the classical one. The Oberbeck–Boussinesq system, that describes the convective flows, is a rather complicated not only for its nonlinearity and high order but also because it is not related to any classical type (see, for example, [9]).

Among all results of theoretical investigations of the convective processes with respect to evaporation on the basis of the mathematical models in the complete statements, such results are emphasized, which are related to finding of the exact solutions of the governing equations. Sometimes an obtaining of a solution of special type allows one to get to a problem of a smaller dimension. The exact solutions allow one to model effectively and rapidly the real fluid flows including the flows under conditions of phase transfer in the domains with free boundaries and interfaces. The importance of the exact solutions is that their use gives a possibility to specify on the qualitative level the physical factors defining the basic flow mechanisms. In the case of a multiparameter problem the exact solution, as a solution of special type, allows one to study degrees and character of influence of particular physical factors and their mutual combinations, to clarify a mathematical model in order to describe the investigated flows properly. Let us note that the systematic analysis of the exact solutions of the convection equations, including determination of the solution form, pertains to problems, which are solved using the group analytical methods of the differential equations [4, 10–12].

In the paper we present analysis of exact solution of the classical convection equations that describes the two-layer flows with evaporation on the thermocapillary interface in the two-dimensional case. The constructed exact solution is the analog of the Ostroumov–Birikh solution of the system of the Oberbeck–Boussinesq differential equations. The obtained velocity and temperature profiles enabled one to study the impact of the boundary conditions and the characteristics of interaction of the different effects and mechanisms of the evaporative convection in the gravitational field and in weightlessness.

1. Modeling of flows with mass transfer on the interfaces on the basis of the exact solutions

The convection of fluids in a horizontal layer with free boundary is studied theoretically. In a common statement the problem is to investigate the combined influence of the gravitational and thermocapillary mechanisms on the dynamics and heat exchange in a liquid and also to find the effects caused by action of a laminar gas flux moving over liquid layer and thermal load imposed on the external boundaries of the flow domain.

For the first time the problems, connected with construction of the exact solutions for description of the convective flows under action of the arbitrary oriented temperature gradient, have been solved in [13]. In the cited monograph [13] the solutions of the Oberbeck–Boussinesq equations have been studied, when the temperature could linearly depend on one of the spatial coordinates. In [14] the exact solutions describing the dynamics of the fluid flows with the constant tangential stresses on the free surface have been presented. It turns out, that the exact solution of the Oberbeck–Boussinesq convection equations obtained in [15] is applicable for description of a wide class of the stationary convective fluid flows in an infinite strip having non-deformable free boundary and being under action of a constant longitudinal temperature gradient. It was generalized to the cases of the thermocapillary convection in a two-layer system

with the mass transfer through the interface [16], of the flows with evaporation/condensation on the interphase boundary [17] and of the three-dimensional flows [18, 19] (also see [20–22]).

The idea about an applicability of the exact solutions of the Ostroumov – Birikh type to modeling of the joint liquid – gas flows and evaporation processes at interface occurred due to the analysis of the experimental results [23]. Note that in [24, 25] the liquid evaporation through the interface was not taken into account explicitly, but its modeling was carried out with the help of an appropriate temperature condition at interface. In the condition the agreement of the gas flux direction with a choice of the thermal boundary regime and first of all with the value of the longitudinal temperature gradient corresponded qualitatively to the physical experiment [23, 26–29]. In [22, 24, 25] the isothermal gas flow was modeled with the help of the Poiseuille solution of the Navier – Stokes equations. Upon that the requirement of the closed flow is not imposed as in the experiments. The principal opportunity to control the thermocapillary convection with help of gas flux was also demonstrated. Velocity can possess both the positive and negative values on the interface. The latter case corresponds to regimes of the reverse flows. The parameter relations ensuring the realization of each of the two regimes and fulfillment of the flow closure condition were found. That solution is one more confirmation of the physical plausibility of the Birikh solution analogs and their possible interpretation. In [30] the solution was applied for description of the two – layer thermocapillary fluid flows with the interface in a channel with fixed walls in the frame of the Oberbeck – Boussinesq approximation. The exact expressions of the unknown functions were found under additional condition of zero total flow rate of the liquid mass. A generalization of the Ostroumov–Birikh solution for a problem of the thermodiffusion, presented in [8], was studied in [31] with regard to motion of binary mixtures in cases of different boundary thermal regimes.

The stationary two-layer gas-liquid flows were studied in [17, 32] in the case, when the liquid evaporation at the interface should be taken into account. As the mathematical model the Navier – Stokes equations in the Oberbeck – Boussinesq approximation are used. In the upper layer, that is a mixture of the gas and liquid vapor, the diffusion process occurs and the effect of diffusive thermal conductivity is taken into consideration. In the later papers [33, 34] the investigations were performed with simultaneous consideration of the effects of thermodiffusion and diffusive thermal conductivity (of the Soret and Dufour effects).

1.1. Problem statement and exact solution form

In [17, 32–38] it is supposed that a liquid and gas-vapor mixture fill the infinite layers of the thickness l and h , $\Omega_1 = \{(x, y) : -\infty < x < \infty, -l < y < 0\}$, $\Omega_2 = \{(x, y) : -\infty < x < \infty, 0 < y < h\}$, respectively. The coordinate system is chosen so that the gravitational force vector is $\mathbf{g} = (0, -g)$.

As the mathematical model the Navier – Stokes equations in the Oberbeck – Boussinesq approximation are used:

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p'}{\partial x} + \nu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right), \quad (1.1)$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho} \frac{\partial p'}{\partial y} + \nu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + g(\beta T + \underline{\gamma C}), \quad (1.2)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1.3)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \chi \left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \delta \left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} \right) \right], \quad (1.4)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \left[\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} + \alpha \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \right]. \quad (1.5)$$

The underlined terms and equation (1.5) are taken into account by modeling flows of the vapor-gas mixture in the upper layer¹. Equation (1.5) describes a vapor transfer (vapor diffusion) as a passive admixture. This equation is a consequence of the Fick's laws and more common Maxwell–Stefan equation, which describes a diffusion in the multicomponent systems. Actually the Fick's laws found a strict experimental confirmation in [39] by the study of the solutions of low concentrations (see, for instance, [40]). It is also valid for interdiffusion of various gases [41]. In the system (1.1)–(1.5) u , v are the projections of the velocity vector on the Ox and Oy axes, respectively, p' is the modified pressure (deviation from the hydrostatic one), T is the temperature, C is the vapor concentration. Physical characteristics of the media ρ , ν , χ , D , β , γ (density, coefficients of kinematic viscosity, thermal diffusivity, vapor diffusion in the gas, heat and concentration expansion, respectively) are assumed to be given. The coefficients δ and α characterize the Dufour and Soret effects, respectively, which are considered taking into account the fraction concentration values of the evaporated substance.

The exact solutions of the equations (1.1)–(1.5) (a generalization of the Ostroumov–Birikh solution) take the form

$$\begin{aligned} u_j &= u_j(y), \quad v_j = 0, \quad p'_j = p'_j(x, y), \\ T_j &= (a_1^j + a_2^j y)x + \vartheta_j(y), \quad C = (b_1 + b_2 y)x + \varphi(y). \end{aligned} \quad (1.6)$$

Here (and everywhere below) the indices $j = 1$, $j = 2$ are used to identify the medium characteristics in the lower and upper layers (evaporating liquid and gas–vapor mixture, respectively). One can see that only the longitudinal component of the velocity is not equal to zero and depends on the transverse coordinate. The temperature distributions in the layers and vapor concentration have the linear terms with respect to the longitudinal coordinate.

1.2. Boundary conditions

The kinematic and dynamic conditions are fulfilled on the thermocapillary interface Γ defined here by the equation $y = 0$. The interface remains non-deformed and plane. The first condition is fulfilled identically due to equality $v = 0$. The dynamic conditions have the form

$$\rho_1 \nu_1 u_{1y} = \rho_2 \nu_2 u_{2y} - \sigma_T \left. \frac{\partial T}{\partial x} \right|_{y=0}, \quad p_1 = p_2. \quad (1.7)$$

The first relation (projection on the tangential vector to the interface $y = 0$) is written under the assumption that the surface tension coefficient linearly depends on temperature ($\sigma = \sigma_0 - \sigma_T(T - T_0)$, where σ_0 is the representative value of the surface tension, $\sigma_T > 0$ for the majority of liquids).

¹Problem statement about two-layer flows with evaporation and with respect to the Dufour effect in the vapor-gas medium and subsequent construction of an exact solution in the two-dimensional case [32] were a result of the fruitful discussions with Professor M. Hennenberg (M. Hennenberg, Microgravity Research Center, Brussels, Belgium).

The condition for heat fluxes with respect to the mass transfer is the following:

$$k_1 \frac{\partial T_1}{\partial y} - k_2 \frac{\partial T_2}{\partial y} - \delta k_2 \frac{\partial C}{\partial y} \Big|_{y=0} = -LM. \quad (1.8)$$

Here L is the latent heat of evaporation, M is the mass flow rate of the evaporating liquid, k_1, k_2 are the coefficients of thermal conductivity. Value of M is determined from the mass balance equation on the interface:

$$M = -D\rho_2 \left(\frac{\partial C}{\partial y} + \alpha \frac{\partial T}{\partial y} \right) \Big|_{y=0}. \quad (1.9)$$

The saturated vapor concentration can be found with the help of the relation

$$C|_{y=0} = C_* \left[1 + \varepsilon \left(T_2|_{y=0} - T_0 \right) \right]. \quad (1.10)$$

Here $\varepsilon = L\mu/(RT_0^2)$, μ is the molar mass of the evaporating liquid, R is the universal gas constant, C_* is the saturated vapor concentration at $T_2 = T_0$ (in [16] the value T_0 is considered to be equal to 20 °C). The relation (1.10) is a result of the Clapeyron – Clausius equation [42] for the saturation vapor pressure $P = P_0 \exp[L\mu(T - T_0)/(RT_0T)]$ and of the Mendeleev – Clapeyron for the ideal gas $\rho_v RT = \mu P$. Here (P_0, T_0) is some initial state, $\rho_v = C\rho_2$. Let us assume that the non-dimensional parameter εT_* (T_* is characteristic value of the temperature drop) is quite small and linearize the relation $C = \tilde{C}_* \exp[-L\mu/(RT)]/T$, where $\tilde{C}_* = \text{const}$. Then, the linear dependence of the vapor concentration at interface (1.10) takes place for the moderate temperature drops. Note that the similar approach, when the Clapeyron – Clausius equation is used for formulation of the condition at interface, was referenced in [16] (see also [43]).

In the presented statement of the convection problem under phase transfer conditions the evaporation effects (heat consumption on vaporization) are taken into consideration only in the heat balance condition (1.8) on the interface. At the same time only the diffusive mass flux is supposed (1.9); the convective mass transport is not taken into account (see also [16]). If we use the exact condition of the vapor mass balance (1.9), then the mass flow rate value M will be calculated according to one of the formulae:

$$M = -\frac{D\rho_2}{1-C} \frac{\partial C}{\partial n}, \quad M = -\frac{D\rho_2}{1-C} \left(\frac{\partial C}{\partial n} + \alpha \frac{\partial T}{\partial n} \right).$$

The first relation is used in the absence of the Soret effects in the gas – vapor layer, the second one is employed in cases when the thermodiffusion effect is taken into account. Negative values of M correspond to situation when the condensation occurs in the system, positive values of M relate to the evaporation process. Under conditions of smallness of the parameter εT_* , that is ensured by moderate values of temperature and temperature drops in the system, the interface value of C can be set equal to C_* . In this case the coefficient D in (1.9) should be called the modified diffusion coefficient equal to the diffusion coefficient divided by $(1 - C_*)$. The influence of the Soret and Dufour effects on the evaporation rate is studied in [44, 45].

Fulfillment of the continuity conditions of velocity and temperature is required additionally

$$u_1|_{y=0} = u_2|_{y=0}, \quad T_1|_{y=0} = T_2|_{y=0}. \quad (1.11)$$

It is noted, if the condition of temperature continuity is to be valid on the interface then $a_1^1 = a_1^2 = A$ and the temperature distribution takes the form

$$T_i = (A + a_2^j y)x + \vartheta_i(y), \quad i = 1, 2. \quad (1.12)$$

The temperature is distributed linearly with respect to the longitudinal coordinate and the no-slip conditions should be satisfied on the rigid impermeable channel boundaries:

$$T_1|_{y=-l} = A_1x + \vartheta^-, \quad T_2|_{y=h} = A_2x + \vartheta^+, \quad u_1|_{y=-l} = 0, \quad u_2|_{y=h} = 0. \quad (1.13)$$

Two types of conditions for vapor concentration on the upper rigid boundary are studied. The equality to zero of the vapor flux

$$\left(\frac{\partial C}{\partial y} + \alpha \frac{\partial T}{\partial y} \right) \Big|_{y=h} = 0 \quad (1.14)$$

or the condition of a complete absorption (zero vapor concentration)

$$C|_{y=h} = 0 \quad (1.15)$$

can be set. The condition (1.14) takes into account the impact of the Soret effect. Relative to the Soret effect it should be noted that its consideration in (1.14) is necessary only in the limited range of values of the problem parameters. The question of taking into consideration of the Soret effect in the boundary condition (1.14) requires particular additional analysis. In the condition (1.14) the temperature effects can be neglected under certain conditions with an error, that does not exceed 1%. The fulfillment of the following equality can be demanded:

$$\frac{\partial C}{\partial y} \Big|_{y=h} = 0. \quad (1.16)$$

In [17, 33–38] the solution of the stationary problem was obtained at given gas flow rate

$$Q = \int_0^h \rho_2 u_2(y) dy. \quad (1.17)$$

The exact solutions (1.6) gives an opportunity to analyze the presented mathematical model in the view of revealing the significant factors, which influence on the characteristics of the two-layer flows with evaporation, and to evaluate adequacy of the model in the sense of physical plausibility of the obtained results. It turns out, that the Soret effect enables one to consider the interface as a thermocapillary surface also in the case, when the upper rigid boundary of the channel has a property of vapor absorption (see condition (1.15)). Without considering the Soret effect in the case of using condition (1.15) it is necessary to put the longitudinal temperature gradient A equal to zero on the interface. If thermodiffusion effect is taking into account, one should predetermine alone value of temperature gradients among A , A_1 , A_2 and the other two values can be calculated with help of specific relations. If condition (1.14) is used and the Soret effect is taken into account it is possible to describe two-layer flow with given value of thermal gradient being the same for channel walls and interface ($A = A_1 = A_2$). In general case, value of one of the three temperature gradients (e.g. A) is defined with help of given the other two gradients (A_1 , A_2). Then, coefficients b_1 , b_2 are calculated thus defining the longitudinal gradient of vapor concentration. The analysis of possible flow regimes with the Dufour effect depending on taking into account the Soret effect is presented in Tab. 1. These results allow one to specify ways to control the flows by means of boundary conditions.

Thus, the Soret effect leads to significant alteration of qualitative and quantitative pattern of the flow. Also, it enables one to change mechanism of controlling the flow structure with help of boundary thermal load on the channel walls under agreement of values of some parameters. Since α is the regular parameter of the problem, the limiting transition of solution (1.6) constructed taking into account the Soret effect to solution obtained without taking into consideration the thermodiffusion effect takes place at $\alpha \rightarrow 0$ [35].

Table 1. Analytical results of investigation of the Soret effect influence on the fluid flow features through the temperature regime and vapor concentration on boundaries

	Flow 1: Condition (1.15) and taking into account the Soret effect ($\alpha \neq 0$)	Flow 2: Condition (1.15) without considering the Soret effect ($\alpha = 0$)	Flow 3: Condition (1.14) and taking into account the Soret effect ($\alpha \neq 0$)	Flow 4: Condition (1.14) without considering the Soret effect ($\alpha = 0$)
A	calculated: $A = \frac{\alpha A_1}{\alpha + \frac{l}{h} \frac{k_2}{k_1} C_* \varepsilon (\alpha \delta - 1)}$	$A = 0$ ($A = \frac{b_1}{C_* \varepsilon}$)	given (A, A_1, A_2 can be given and equal)	given
A_1	given	given	given (A, A_1, A_2 can be given and equal)	given
A_2	calculated: $A_2 = \frac{A_1(\alpha + C_* \varepsilon)}{\alpha + \frac{l}{h} \frac{k_2}{k_1} C_* \varepsilon (\alpha \delta - 1)}$	calculated: $A_2 = -A_1 \frac{h k_1}{l k_2}$ (consequence of heat transfer condition)	calculated: $A_2 = A + (A - A_1) \frac{h k_1}{l k_2} \frac{1}{1 - \alpha \delta}$ (A, A_1, A_2 can be given and equal)	calculated: $A_2 = A + (A - A_1) \frac{h k_1}{l k_2}$
b_1	$b_1 = C_* \varepsilon A$ ($b_1 = -b_2 h$)	$b_1 = 0$ ($b_1 = -b_2 h$)	$b_1 = C_* \varepsilon A$	$b_1 = C_* \varepsilon A$
b_2	$b_2 = -\frac{C_* \varepsilon A}{h}$	$b_2 = 0$ ($b_2 = -\alpha \frac{A_2 - A}{h}$; consequence of mass balance condition)	$b_2 = -\alpha \frac{A_2 - A}{h}$	$b_2 = 0$

2. Classification of flow regimes

In [35] a classification of the flow types which can be described by the exact solution (1.6) was proposed. Three classes of the flows are specified depending on dominant forces: purely thermocapillary, mixed and Poiseuille's flows. In general case, when different thermal load is applied at channel boundaries ($A_1 \neq A_2$), the classification can be expanded by supplemental particular subsets (some of them are not distinguished at $A_1 = A_2 = A$):

- purely thermocapillary flow; it is characterized by completely reverse flow in the liquid phase; basic mechanism is the thermocapillary effect (Fig. 1);
- mixed flow; basic mechanisms are the interacting shear stresses, thermocapillary effect, temperature gradients, viscous forces; three subsets are specified:
 - the first type mixed flow (mixed I); it is characterized by the velocity "delamination" near the interface and occurrence of zones with the reverse flow near the interface; basic mechanisms are oppositely directed the shear stresses and the thermocapillary effect (Fig. 2(a–c));
 - the second type mixed flow (mixed II); it is characterized by the velocity "delamination" near the interface with the positive longitudinal component of the vector velocity; basic mechanisms are the shear stresses and the thermocapillary effect which act unidirectionally (Fig. 2(d–f));
 - the third type mixed flow (mixed III); it is characterized by the velocity field structure close to the Couette's distribution within one of the phases (Fig. 2(g–i)) or simultaneously in both media;

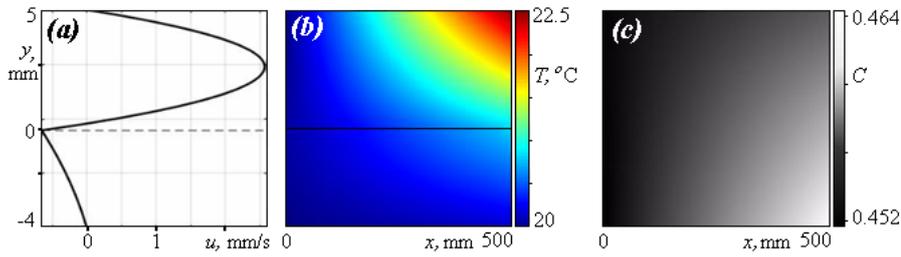


Fig. 1. Distributions of the velocity (a), temperature (b) and concentration (c) for the purely thermocapillary flows: $g = g_0 \cdot 10^{-2}$, $A_1 = 0.4$ K/m, $A_2 = 5$ K/m, $A = 1.4899$ K/m, $M = 1.5983 \cdot 10^{-7}$ kg/(m²·s)

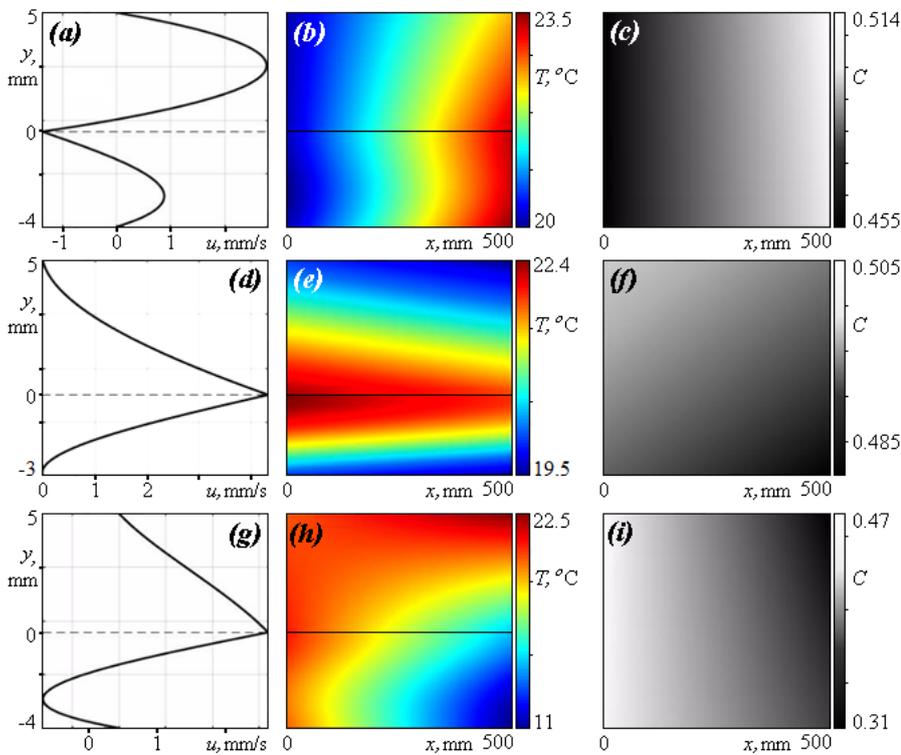


Fig. 2. Distributions of the velocity (a, d, g), temperature (b, e, h) and concentration (c, f, i) for the mixed flows: (a–c) – $A_1 = 7.4$ K/m, $A_2 = 5$ K/m, $A = 6.8313$ K/m, $M = 1.1955 \cdot 10^{-6}$ kg/(m²·s); (d–f) – $l = 3$ mm, $A_1 = A_2 = A = -11.5$ K/m, $M = -1.9276 \cdot 10^{-5}$ kg/(m²·s); (g–i) – $A_1 = -18.4$ K/m, $A_2 = 5$ K/m, $A = -12.8556$ K/m, $M = -2.433 \cdot 10^{-6}$ kg/(m²·s)

- Poiseuille's flow; it is characterized by the velocity fields close to parabolic one in both fluids; three subsets are specified:
 - purely Poiseuille's flow (Poiseuille's flow I); it is characterized by the positive values of the longitudinal component of the velocity in each phase; basic mechanisms are the pressure gradients in both fluids (Fig. 3(a–c));
 - the first type conditionally Poiseuille's flow (Poiseuille's flow II); it is characterized by formation of zones with the return motion near the channel walls in one of the layers;

- basic mechanisms are the pressure gradients and viscous forces (Fig. 3(d–f));
- the second type conditionally Poiseuille’s flow (Poiseuille’s flow III); it is characterized by formation of stagnant zones in the liquid (liquid is at rest) and gas velocity profile close to parabolic one; basic mechanisms are the thermocapillary effect and shear stresses (Fig. 3(g–i)).

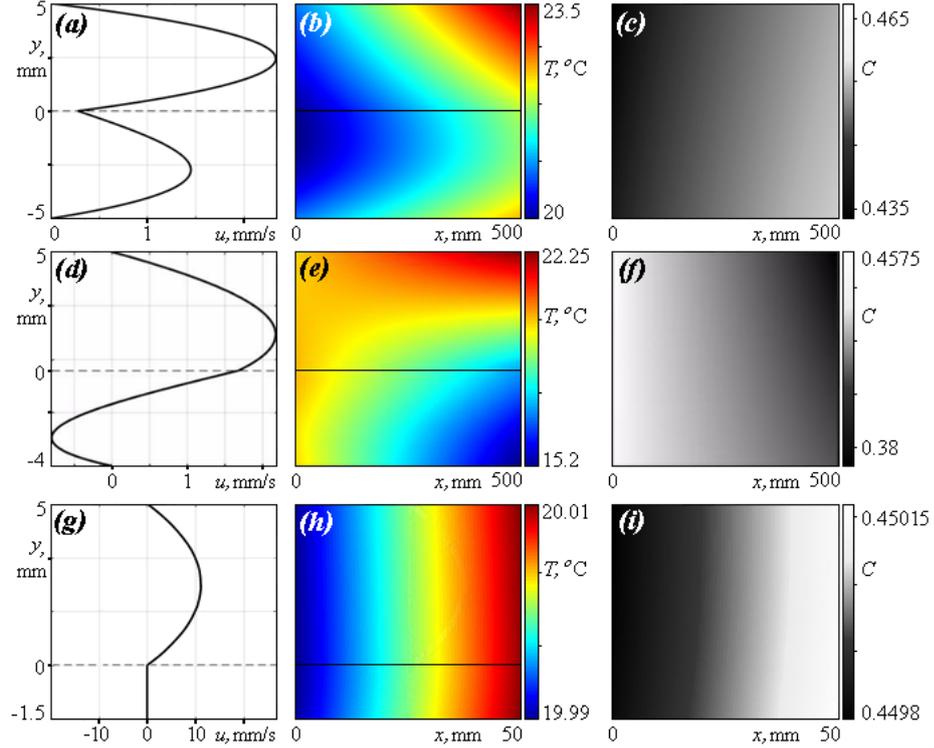


Fig. 3. Distributions of the velocity (a, d, g), temperature (b, e, h) and concentration (c, f, i) for the Poiseuille type flows: (a–c) – $l = 5$ mm, $A_1 = 3$ K/m, $A_2 = 5$ K/m, $A = 3.5592$ K/m, $M = 5.6295 \cdot 10^{-7}$ kg/(m²·s); (d–f) – $A_1 = -9.8$ K/m, $A_2 = 5$ K/m, $A = 6.2933$ K/m, $M = -1.276 \cdot 10^{-6}$ kg/(m²·s); (g–i) – $\alpha = 0$, $l = 1.5$ mm, $A_1 = A_2 = 0.4$ K/m, $M = 6.7046 \cdot 10^{-8}$ kg/(m²·s)

Typical distributions of the basic characteristic of the flows presented in Fig. 1–3 are obtained for "HFE-7100 – nitrogen" system at $Q = 9.6 \cdot 10^{-6}$ kg/(m·s), $\alpha = 5 \cdot 10^{-3}$, $l = 4$ mm, $h = 5$ mm, $g = g_0 = 9.81$ m/s² (if not specified otherwise). Values of the longitudinal temperature gradients and mass flow rate are given in figure captions for each configuration. Values of the physical parameters of the substances are given in Tab. 2.

Complete study of the exact solution properties involves investigation of the solution stability characteristics and analysis of the perturbation spectrum. In [36] the linearized equations for the amplitudes of the normal disturbances of the basic solution, long-wave asymptotics of the eigenvalues and eigenfunctions were obtained. In [37, 38] the spectrum of the characteristic perturbations of the velocity, temperature and concentration was calculated. Dependence of the type and structure of the perturbations on the system geometry, disturbance wave-length and intensity of external actions (temperature gradient on the channel walls and flow rate of the

Table 2. Values of the physical parameters for the system "HFE-7100 – nitrogen"

Parameter \ Medium	HFE-7100	Nitrogen
Density ρ , kg/m ³	$1.5 \cdot 10^3$	1.2
Kinematic viscosity ν , m ² /s	$0.38 \cdot 10^{-6}$	$0.15 \cdot 10^{-4}$
Coefficient of temperature expansion β , K ⁻¹	$1.8 \cdot 10^{-3}$	$3.67 \cdot 10^{-3}$
Coefficient of thermal conductivity k , W/(m·K)	0.07	0.02717
Coefficient of thermal diffusivity χ , m ² /s	$0.4 \cdot 10^{-7}$	$0.3 \cdot 10^{-4}$
Temperature coefficient of surface tension σ_T , N/(m·K)	$1.14 \cdot 10^{-4}$	
Latent heat of evaporation L , (W·s)/kg	$1.11 \cdot 10^5$	
Molar mass of evaporating liquid μ , kg/mol	0.25	
Coefficient of diffusion D , m ² /s		$0.7 \cdot 10^{-5}$
Coefficient of concentration expansion γ		-0.5
Saturated vapor concentration C_*		0.45
Dufour coefficient δ , K		10^{-5}

working media) was investigated. It was found that the perturbations can lead to formation of the vortex, thermocapillary and hybrid structures corresponding to different mechanisms of instability.

Case of the different thermal load on the channel walls ($A_1 \neq A_2$) does not allow using the normal mode method [35]. The problem reduces to the solving space-time one in "stream function – vorticity" terms. Influence of the intensity and character of the thermal load (heating/cooling), gas flow rate and amplitude of the initial perturbations on a type of arising instabilities was studied. Stability of the basic flow is ensured only under quite small thermal gradients and gas flow rates. Instability can appear by generation of monotonic and oscillatory regimes. The first is characterized by formation of the vortex and thermocapillary structures. In another regime "pulsatory" vortexes can arise.

Further study of the solution (1.6) involves obtaining the critical characteristic of stability in the space of similarity criteria, taking into account interface deformation and investigation of influence of the thermophysical properties of working media.

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Анализ точного решения задачи испарительной конвекции (обзор). Часть I. Плоский случай

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Развитие теории, описывающей конвекцию в условиях фазового перехода "жидкость – газ", во многом вызвано активным экспериментальным изучением конвективных явлений, сопровождающихся испарением/конденсацией на межфазных границах. Результаты аналитического и численного исследования новых нестандартных задач тепло- и массопереноса в областях со свободными границами или поверхностями раздела позволяют оценить адекватность новых математических моделей и вывести новые определяющие критерии. Полученные фундаментальные знания о физических механизмах изучаемых процессов служат основой модификации и совершенствования жидкостных технологий, использующих в качестве рабочих сред испаряющиеся жидкости и парогазовые смеси. В настоящей работе представлен анализ точного решения уравнений конвекции, позволяющего моделировать двухслойные конвективные течения с испарением.

Ключевые слова: испарительная конвекция, точное решение, двухслойные течения, термокапиллярная граница раздела.