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Multicriterion problem of allocation of resources in the heterogeneous distributed information processing systems

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Abstract. This study reviews the problem of allocation of resources in the heterogeneous distributed information processing systems, which may be formalized in the form of a multicriterion multi-index problem with the linear constraints of the transport type. The algorithms for solution of this problem suggest a search for the entire set of Pareto-optimal solutions. For some classes of hierarchical systems, it is possible to significantly speed up the procedure of verification of a system of linear algebraic inequalities for consistency due to the reducibility of them to the stream models or the application of other solution schemes (for strongly connected structures) that take into account the specifics of the hierarchies under consideration.

1. Introduction

A broad class of applied problems is associated with the distribution of limited resources in the information processing systems (IPS). The problems of master production scheduling, the problems of distribution of informational resources in the heterogeneous distributed information processing systems (HDIPS), the transport problems with intermediate points, the distribution of material and financial resources in the designing and manufacturing of some complex products, etc. may serve as the examples of such problems.

A resource utilization is the percentage of time within which a particular IPS resource is busy with the task, or otherwise, the percentage of a throughput capacity of the resource engaged with (depending on the type of the resource) computing or transmitting of the information. Otherwise, the "Resource utilization" parameter characterizes the load on the IPS [1].

A communication channel or a computational node represents a common resource, consequently their occupancy affects the response time of the entire system. It is important to determine whether a poor performance of IPS and the utilization rate of its individual components are interrelated.

It is recommended to observe the following rules to determine the maximum allowable utilization rate:

1. High resource utilization rate slows down the IPS only if this particular component represents the bottleneck of the system.

2. The maximum permissible resource utilization rate depends on the length of the computational route. The longer the route is, the lower the allowable utilization rate is.

The later the collision detected is, the greater the amount of overheads is and the greater the time spent on the task is. As the result, a response time of the system increases.

In the most general statement, the problem of allocation of resources in the HDIPS may be formulated as follows. There is the HDIPS, the elements of which may either collect, or transmit, or process the data. The elements of the system and the relationships between them are characterized by some resource constraints that determine the amount of resources that may circulate in the system. The problems of allocation of resources in the HDIPS involve finding such allowable amounts of resources at which the optimality criteria determining the system functioning efficiency reach their extrema. Availability of several optimality criteria in such problems makes them multicriterion problems. Hereinafter, a solution of the multicriterion problem shall mean an elaboration of the entire set of Pareto-optimal solutions [2-5].

2. Mathematical Model of the HDIPS

The study [6] suggests a model and a method for solving the resource allocation problem in the general statement (with the random system hierarchy), a correct functioning of which requires a determination of compatibility of the systems of linear constraints of the transport type.

The HDIPS is modeled with weighted connected directed graph $H = (V, A), A \subseteq V2$, free of any loops. On the set of graph vertices V, partition $V = V_s \cup V_p \cup V_c$ is marked. $F(i) = \{j \mid (i,j) \in A, j \in V\}$ denotes the set of the graph vertices immediately following vertex $i, i \in V$; $P(i) = \{j \mid (j,i) \in A, j \in V\}$ denotes the set of vertices immediately preceding vertex $j, j \in V$. The constructed model displays a multi-level hierarchical structure in which the resource is allocated from the sources (the set of vertices V_s), through the transmitting elements (the set of vertices V_p), to the users (the set of vertices V_c). It is assumed that $F(i) = \emptyset$ if $i \in V_c, P(j) = \emptyset$, if $j \in V_s$.

$x_i, i \in V$ denotes the amount of the resource corresponding to the i-th element (the amount of the "produced" resource for the source, the "transferred" resource for the transmitting element, and the "consumed" resource for the resource consumer). The values of x_i may be limited with both the maximum and the minimum values:

$$0 \leq A_i \leq x_i \leq B_i < \infty, i \in V. \quad (1)$$

y_{ij} denotes the amount of the resource transmitted through arc $(i,j), (i,j) \in A$. The throughput of each arc is determined with the values of C_{ij} and D_{ij} representing the bottom and the top boundaries of a segment of admissible values, $C_{ij} \geq 0, D_{ij} < \infty, (i,j) \in A$. Then the restrictions on the resource values transmitted through the arcs are determined with the system of constraints:

$$C_{ij} \leq y_{ij} \leq D_{ij}, (i,j) \in A. \quad (2)$$

Equilibrium criteria for the elements are:

$$\sum_{j \in P(i)} y_{ij} = x_i, i \in V \backslash V_s, \quad (3)$$

$$x_i = \sum_{j \in F(i)} y_{ij}, i \in V \backslash V_c. \quad (4)$$

The general problem of allocation of resources in the HDIPS is the determination of such values of $x_i, i \in V$, and $y_{ij}, (i,j) \in A$ for which constraints (1) - (4) are satisfied and some optimality criteria determining an efficiency of the system functioning reach the extrema.

3. Solution of the Multicriterion Optimal Resources Allocation Problem

Through introduction of the additional elements corresponding to the arcs of the system, it is possible to proceed to the problem with the constraints determined only for the elements of the system, the number of which increases from $|V|$ to $m = |V| + |A|$.

An element of the system shall be called a controlled element if the amount of the resource, it processes, transfers or receives, determines the efficiency of the system's functioning. It is assumed that the first n elements of the system are controlled, $n \leq m$.

In the most general statement, the problem may be formulated as follows. There are two Boolean matrices A and B, correspondingly, with sizes $m \times k$ and $n \times k$, real nonnegative vector $\vec{c}$ with size m and vector-valued function $\vec{F}(\vec{y})$ defined on the set of n-dimensional vectors in R^n with the values from $\{0, 1, \dots, p-1\}$. Introduced function $\vec{F}(\vec{y})$ determines space R^n on the set of the vertices of the n-dimensional p-ary cube. It is required to find vector $\vec{x}$ satisfying constraints $A\vec{x} \le \vec{c}$ taking into account minimized criteria $\vec{F}(B\vec{x})$. The resulting problem represents the n-criterion problem with a linear constraint of the constraints and the particular optimality criteria, the form of which depends on the form of function $\vec{F}(\vec{y})$. The system of constraint $A\vec{x} \le \vec{c}$ is equivalent to the system of linear algebraic inequalities of transport type $\sum_{j\in R(i)} x_j \le c_i, i = \overline{1,m}$, where $R(i)$ denotes the set of indices corresponding to the i-th row of the Boolean matrix $A, i = \overline{1,m}$, and vector $B\vec{x}$ has coordinates $\sum_{j\in G(i)} x_j, i = \overline{1,n}$, where $G(i)$ denotes the set of indices corresponding to the i-th row of the Boolean matrix B.

Function $\vec{F}(\vec{y})$ is determined as follows. For each component i, a set of nested segments $S_i^{t_i}, S_i^{t_i} \subseteq S_i^{t_i+1}, t_i = \overline{0, p-2}, p \ge 1, i = \overline{1,n}$ is considered. Then $F_i(\sum_{j\in G(i)} x_j) = t_i$ if $\sum_{j\in G(i)} x_j \in S_i^{t_i}$ but $\sum_{j\in G(i)} x_j \notin S_i^{t_i-1}$. After the completed transformations, the multicriterion problem of allocation of resources in the HDIPS is stated the following way: it is required to find a vector $\vec{x}$ satisfying constraints $\sum_{j\in R(i)} x_j \le c_i, i = \overline{1,m}$ taking into account minimized criteria $F_i(\sum_{j\in G(i)} x_j) = t_i, t_i \in \{0,1,\dots,p-1\}, i = \overline{1,n}$.

Resolution of the stated multicriterion problem may be achieved through application of various compromise schemes. The study [2] reviews the problem solution algorithm for at the lexicographic ordering of the optimality criteria, which may also be successfully applied to the problems in which a random complete linear order is given for the set of criteria for the problem. The study [7-8] presents the problem solution algorithm for the quadratic convolution of the partial optimality criteria. The study [9-10] reviews the problem of construction of the entire set of Pareto-optimal solutions (corners of a cube) for the multicriterion problem under consideration.

An efficient vertex of the n-dimensional p-ary cube is vertex $\vec{z}^0 = (z_1^0, z_2^0, \dots, z_n^0)$ for which $f(\vec{z}^0) = 1$, and for any vertex $\vec{z}^1 = (z_1^1, z_2^1, \dots, z_n^1), z_1^0 \ge z_1^1, i = \overline{1,n}$, for which there is at least one component $j, z_j^0 \ge z_j^1, f(\vec{z}^1) = 0$.

For each vertex of n-dimensional p-ary cube $\vec{z} = (z_1, z_2, \dots, z_n)$, a system of linear algebraic inequalities of transport type $S(\vec{z})\colon A\vec{x} \le \vec{c}, \vec{F}(\vec{x}) \le \vec{z}$ is assigned. For each vertex $\vec{z}$, this system always includes the initial constraints of the problem $A\vec{x} \le \vec{c}$ (this part of the inequalities remains unchanged and does not depend on the choice of the vertex). Additional conditions are assigned to the values of the criteria that should not exceed the boundaries specified by the vertex $\vec{z}$. This means that the solutions of the system of inequalities $S(\vec{z})$ are the admissible solutions of the multicriterion problem under consideration for which the additional conditions $F_i(\vec{x}) \le z_i, i = \overline{1,n}$ are satisfied.

Function $f(\vec{z})$, taking the value of 1 if system $S(\vec{z})$ is compatible, and 0 – otherwise, is marked on the set of the cube vertices. It is easy to show that function $f(\vec{z})$ is monotonous: if $\vec{z}^1 \le \vec{z}^2$ (componentwise), then $f(\vec{z}^1) \le f(\vec{z}^2)$.

4. Algorithm for Finding All Efficient Vertices of the Cube

In order to find the entire set of efficient vertices of the cube, an area of the search shall be restricted to the minimum possible value for each component of the vertex for which function $f(\vec{z})$ is equal to 1. The monotonicity of function $f(\vec{z})$ allows one to perform a binary search using the value of the current component of the cube under consideration, assuming the remaining values of the components equal to $p-1$. As a result, for each component there is boundary value $z_j^*, j, j = \overline{1,n}$ and vertex $\vec{z}^j = (p-1, p-1, \dots, z_j^*, \dots, p-1), j = \overline{1,n}$. Vertices $\vec{z}^j$ are placed into set M. Evidently, for all vertices that have the values of all components greater than or equal to the values of the corresponding components in $\vec{z}^j$; function f will also take the value of 1. Therefore, these vertices are excluded from the further search, since they are not efficient.

It is assumed that $\vec{z}^* = (z_1^*, z_2^*, \dots, z_n^*)$. In this case, the system of linear constraints corresponding to the vertex $\vec{z}^*$ may either be consistent or inconsistent. The total number of computations of the function $f(\vec{z})$ (verifications for the consistency of the systems of linear algebraic constraints of the transport type) for finding vertex $\vec{z}^*$ is of the order of $n \log_2 p$.

Theorem 1

If $f(\vec{z}^*) = 1$, then vertex $\vec{z}^*$ is the only efficient vertex of the cube.

Proof of the Theorem 1

The proof of the theorem is based on the fact that for any vertex of cube $\vec{z}^1$, any component of which is less than the corresponding component of vertex $\vec{z}^*$, there is $f(\vec{z}^1) = 0$. It is suggested to state vice versa, and for vertex $\vec{z}^1$, state $z_j^1 < z_j^*, f(\vec{z}^1) = 1$. Then for vertex $\vec{z}^j = \left(p-1, \dots, p-1, z_j^1, p-1, \dots, p-1\right)$, there is $f\left(\vec{z}^j\right) = 1$. It results in a contradiction, since z_j^* is the minimum value of the component j for which vertex $\left(p-1, p-1, \dots, z_j^*, \dots, p-1, \dots, p-1\right)$ corresponds to a joint system of the linear constraints. Consequently there is no effective cube vertex that is different from $\vec{z}^*$.

Corollary to Theorem 1

The proof of Theorem 1 means that all the vertices of the cube that correspond to joint systems of linear inequalities, and thus efficient vertices $\overline{z'} = (z_1', z_2', \dots, z_n')$, satisfy the condition $z_j' \geq z_j^*, j = \overline{1, n}$.

It is assumed that f $(\vec{z}^*)$ =0. All the vertices $\vec{z}, \vec{z} \neq \vec{z}^j$ for which $\vec{z} \geq \vec{z}^j$ (componentwise), where $\vec{z}^j = (p-1, p-1, \dots, p-1, z_j^*, p-1, \dots, p-1)$ are excluded from the consideration. Then $j = arg\ \min_{i=\overline{1,n}}(z_i^*)$ is found. The set of values of all components except the j-th is fixed equal to $z_1^*, z_2^*, \dots, z_{j-1}^*, z_{j+1}^*, \dots z_n^*$. For these fixed values, the binary search for the j-th component from z_j^* to $p-1$, finds minimum value z_j^0 for which $f\,(z_1^*, z_2^*, \dots, z_{j-1}^*, z_j^0, z_{j+1}^*, \dots, z_n^*) = f\,(\vec{z}^0) = 1$, or it verifies that there are no such values. If z_j^0 is found, then, proceeding from the property of monotonicity of function f, the vertices of the cube, all components of which do not exceed the component of the found $\vec{z}^0$, also have a value of the function f equal to 1. Therefore, they may be excluded from the further search. The vertex $\vec{z}^0$ is included in the set M. If the value of z_j^0 is not found, then the value of the first component is increased by one and the binary search among the remaining vertices of form $\vec{z}_j^1 = (z_1^* + 1, z_2^*, \dots, z_{j-1}^*, z_j^*, z_{j+1}^*, \dots, z_n^*)$ is performed again. Let us note that if the value z_j^0 was found at the previous step, then the search at the second step is performed from the values of z_j from z_j^* to z_j^0. Thus, the value of the first component increases until it reaches $p-1$. After that, it decreases to z_1^*, and the value of the second component of the vertex increases by one. Then again, the first component increases at the new fixed value of the second. After considering all possible options for changing the first and the second components, they are set equal to z_1^* and z_2^* respectively, and the third component increases by one. With this new value of the third component, it is necessary to search for all possible values of the first and the second one, etc. The algorithm is completed when all sets of values of the components other than the j-th one from $(z_1^*, z_2^*, \dots, z_{j-1}^*, z_j, z_{j+1}^*, \dots, z_n^*)$ to $(p-2, p-1, \dots, z_j, p-1, \dots, p-1)$ are considered.

Set M is called the set of boundary vertices or the vertices "suspicious of the efficiency".

Theorem 2

Set M of the boundary vertices contains the entire set of efficient vertices.

Proof of the Theorem 2

It is supposed that there is efficient vertex $\vec{z}^1 = (z_1^1, z_2^1, \dots, z_{j-1}^1, \vec{z}^1, z_{j+1}^1, \dots, z_n^1)$ that does not belong to set M that was not found by the procedure described. Then there is vertex $\vec{z}^m = (z_1^1, z_2^1, \dots, z_{j-1}^1, z_j^0, z_{j+1}^1, \dots, z_n^1)$ in set M, all components of which, except the j-th, coincide with the corresponding components of the vertex $\vec{z}^1$, and $\vec{z}_j^1 \neq \vec{z}_j^0$.

If $\vec{z}_j^1 > \vec{z}_j^0$, then it is evident that vertex $\vec{z}^1$ is not efficient, since vertex $\vec{z}^0$, which dominates it, is found.

If $\vec{z}_j^1 < \vec{z}_j^0$, then it means that at searching in the j-th direction, the vertex was found with the value other than the minimal value of the j-th criterion, for which the system remains joint, which is impossible. Consequently, all efficient vertices for the problem under consideration lie in the set M.

After the search for the j-th component from the found vertices of the set M, it is necessary to select the non-dominant ones. These are the efficient vertices of the original problem. Computational complexity of the proposed procedure:

$O((p-1-z_1^*) \times (p-1-z_2^*) \times \ldots \times \left(p-1-z_{j-1}^*\right) \times \log_2\left(p-1-z_j^*\right) \times \left(p-1-z_{j+1}^*\right) \ldots (p-1-z_n^*))$.

Example

There is a hierarchical system with source $\{1\}$, transferring elements $\{2, 3\}$ and consumers $\{3, 4, 5, 6, 7, 8\}$. It is supposed that $F\ (1) = \{2, 3\}, F\ (2) = \{4, 5, 6\}, F\ (3) = \{7, 8\}$. x_i denotes the quantity of the resource corresponding to the i-th element, $i = \overline{1,8}$. Restrictions for the elements of the system are:

$$\begin{array}{ll} \mathbf{24 \le x_1 \le 29} & \mathbf{3 \le x_5 \le 6} \\ \mathbf{12 \le x_2 \le 16} & \mathbf{2 \le x_6 \le 4} \\ \mathbf{12 \le x_3 \le 13} & \mathbf{3 \le x_7 \le 5} \\ \mathbf{5 \le x_4 \le 8} & \mathbf{7 \le x_8 \le 12}. \end{array}$$

It is assumed that y_{ij} is the amount of the resource that will be transferred through arc $(i, j), i, j = \overline{1,8}$. Equilibrium criteria for the elements of the system шы:

$$\begin{array}{ll} \mathbf{x_1 = y_{12} + y_{13}}, & \mathbf{x_5 = y_{25}}, \\ \mathbf{x_2 = y_{12} = y_{24} + y_{25} + y_{26}}, & \mathbf{x_6 = y_{26}}, \\ \mathbf{x_3 = y_{13} = y_{37} + y_{38}}, & \mathbf{x_7 = y_{37}}, \\ \mathbf{x_4 = y_{24}}, & \mathbf{x_8 = y_{38}}. \end{array}$$

It is supposed that the controlled elements are 1 and 2, and $S_1^0 = [29,29], S_1^1 = [28,29], S_1^2 = [24,29]$, and

$S_2^0 = [12,12], S_2^1 = [12,15], S_2^2 = [12,16] -$ the set of the segments corresponding to them.

A 2-dimensional 3-ary cube shall be considered (Figure 1).

For the example under consideration, $\vec{z}^* = (0,0)$ and $f(\vec{z}^*) = 0$.

Vertices $(2,1), (1,2)$ and $(2,2)$ are excluded from the consideration. Required set M contains vertices $(0,2), (1,1)$ and $(2,0)$. All of them are efficient vertices of the cube.

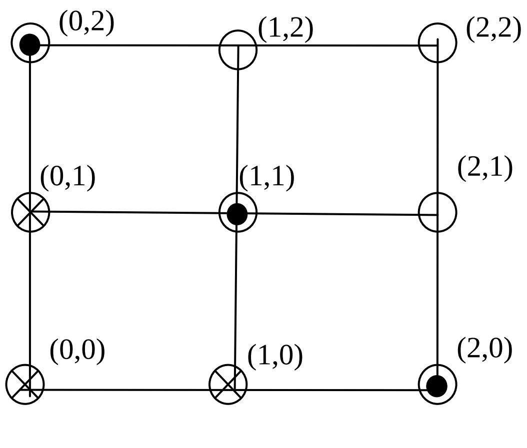

Figure 1. 2-dimensional 3-ary cube

The considered hierarchical system has a tree-like structure; therefore, the reduced boundaries procedure, which has a linear computational complexity, may be applied to determine the consistency of the systems of constraints.

5. Conclusions

This study reviews the problem of allocation of resources in the heterogeneous distributed information processing systems, which may be formalized in the form of a multicriterion multi-index problem with the linear constraints of transport type. The algorithms for solution of this problem suggest searching for the entire set of Pareto-optimal solutions. For some classes of hierarchical systems, it is possible to significantly speed up the procedure of verification of a system of linear algebraic inequalities for consistency due to the reducibility of them to the stream models or the application of other solution schemes (for strongly connected structures) that take into account the specifics of the hierarchies under consideration.

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