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Model Problems for Two Nonlinear Equations that Type Depends on the Solution

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Model problems for two nonlinear second-order partial differential equations that type depends on the solution are considered in this article. One of the equations can be called a nonlinear analog of the Lavrent'ev-Bitsadze equation.

Keywords: type of equation, elliptic and hyperbolic equations, Lavrentev-Bitsadze's equation, Tricomi problem.

Introduction

Two nonlinear equations that can be a model for statement and research of boundary value problems for nonlinear equations that type depends on the solution are considered in this article:

$$\operatorname{sign} U \frac{\partial^2 U(x, y)}{\partial x^2} + \frac{\partial^2 U(x, y)}{\partial y^2} = 0, \quad (1)$$

$$\left(\frac{\partial^2 U(x, y)}{\partial x^2} \right)^2 - \left(\frac{\partial^2 U(x, y)}{\partial y^2} \right)^2 = \left(\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} \right) \left(\frac{\partial^2 U}{\partial x^2} - \frac{\partial^2 U}{\partial y^2} \right) = 0. \quad (2)$$

The equation (1) changes type if the solution switches sign or equals zero in considered domain. When $U > 0$, we have Laplace's equation (elliptic type). When $U < 0$ we have a wave equation (hyperbolic type). If solution $U(x, y)$ equals zero in $B \subset D$ set, then there is a parabolic degeneracy in set B . D is the domain where specific solution $U(x, y)$ is considered.

The equation (1) may be considered as a nonlinear analogue of Lavrent'ev-Bitsadze equation [1]

$$\operatorname{sign} y \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} = 0, \quad (3)$$

which is one of the canonical equations in the theory of mixed-type equations.

The type of equation (2) depends on the multiplier that makes solution equal zero.

Let's consider the matter of statement of the boundary value problems for equations (1),(2). Let D be a domain where the solution is sought. It should be noted that solving might be implemented along with the domain search. The solution appertains to $C^1(D) \cap C^2(D_i)$, $D_i \subseteq D$. The solution saves equation's type in domain D_i : elliptic or hyperbolic type.

The solution of the problem that does not change the type of equation in the whole domain D will be called *trivial*. If in the whole area D the solution satisfies the wave equation, then it is the *trivial hyperbolic solution*. If in the whole area D it satisfies the Laplace's equation –

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trivial elliptical one. The trivial solutions of the equation (1) do not equal zero in domain D . The solution that satisfies Laplace's equation in one part of the domain and wave the equation in another one will be called a *changing type of the equation* solution.

When setting boundary conditions for the equation (2) the possible or prescribed sign of the solution at the bound should be taken into account, since the sign of solutions there will be determined by the continuity in the neighborhood and the type of equation as well, hence the form of possible boundary conditions that are typical for given type.

So the components that define the boundary value problem (the area where the solution is sought, the boundary conditions, changeable type of the equation, solution smoothness) must be strictly consistent.

If the boundary value problem is stated, for example if the domains with boundary conditions are specified, then it could be considered as a problem with unknown line. When the line is crossed, the solution changes equation's type.

The different approach exists. The curve is specified. The domain that contains this curve is defined in that way: the curve is passed by the solution, the type of equation changes and given boundary conditions are satisfied.

1. Model problems

Let us consider some of model problems for (1), (2).

The Dirichlet problem. In the bounded domain D with bound Γ it is required to find the solution that satisfies

$$U|_{\Gamma} = \varphi(s). \quad (4)$$

The Dirichlet problem for equation (1).

In case of $\varphi(s) \geq 0$ the harmonic function satisfying the boundary condition (4) is a trivial elliptic solution of the problem, the function is positive in D because of the maximum principle. There are no trivial solutions if boundary function $\varphi(s)$ changes sign.

The Dirichlet problem for equation (2). A harmonic function that satisfies boundary condition (5) is a trivial elliptic solution.

The model Dirichlet problem. D is a circle of radius R ($r^2 = x^2 + y^2 \leq R^2$), $\varphi(s) = H$.

The model Dirichlet problem for equation (1).

Let $\varphi(s) = H > 0$. The solution $U = H > 0$ is a trivial elliptic one. The function

$$U(x, y) = \begin{cases} \frac{H}{2 \ln \frac{a}{R}} \left(1 - \frac{r^2}{a^2} \right), & \text{if } 0 \leq r \leq a, \\ H \frac{\ln \frac{a}{r}}{\ln \frac{a}{R}}, & \text{if } a \leq r \leq R \end{cases} \quad (5)$$

for any fixed a ($0 < a < R$) is continuously differentiable in a circle $r < R$, negative inside the circle $r < a$ satisfying the wave equation, positive and harmonic in the ring $a < r < R$. When $r = R$, $U = H$. When $r = a$, $U = 0$.

Let $\varphi(s) = H < 0$. The function

$$U(x, y) = C(r^2 - R^2) + H$$

is a trivial hyperbolic solution of the Dirichlet problem, $C > \frac{H}{R^2}$ is an arbitrary constant.

Thus, the model Dirichlet problem for an equation (1) when $H > 0$ has one trivial elliptic solution and an infinite number of solutions that changes equation's type. There is an infinite number of trivial hyperbolic solutions when $H < 0$.

Model Dirichlet problem for equation (2). There is an infinite number of solutions of this problem:

$$U(x, y) = \begin{cases} C(r^2 - a^2) + 2Ca^2 \ln \frac{a}{R} + H, & \text{if } 0 \leq r \leq a, \\ 2Ca^2 \ln \frac{r}{R} + H, & \text{if } a \leq r \leq R, \end{cases} \quad (6)$$

that changes equation's type and an infinite number of hyperbolic trivial solutions.

$$U(x, y) = C(r^2 - R^2) + H,$$

for any $C, a, a \in (0, R)$.

Neumann problem. In the bounded domain D with bound Γ it is required to find solution satisfying

$$\left. \frac{\partial U}{\partial n} \right|_{\Gamma} = \varphi(s). \quad (7)$$

Let $\int_{\Gamma} \varphi(s) ds = 0$ and $V(x, y)$ is an arbitrary harmonic function in D that satisfies the boundary condition (7).

There is the infinite number of trivial elliptic solutions of the Neumann problem for equations (1), (2):

$$U = V(x, y) + C, \quad C > \max_D |V(x, y)|$$

for the equation (1),

$$U = V(x, y) + C,$$

for the equation (2), C is an arbitrary constant.

Model Neumann problem. D is a circle of radius R and $\varphi(s) = K$.

Model Neumann problem for the equation (1).

There is an infinite number of solutions when $K > 0$

$$U(x, y) = \begin{cases} \frac{KR}{2} \left(\frac{r^2}{a^2} - 1 \right), & \text{if } 0 \leq r \leq a, \\ KR \ln \frac{r}{a}, & \text{if } a \leq r \leq R, \end{cases}$$

that changes equation's type for any a ($0 < a < R$). When the inequalities are implemented

$$C < -\frac{KR}{2}, \text{ if } K \geq 0; \quad C < 0, \text{ if } K < 0$$

there is the infinite number of trivial hyperbolic solutions

$$U = \frac{Kr^2}{2R} + C. \quad (8)$$

Model Neumann problem for the equation (2). The problem has the infinite number of solutions

$$U(x, y) = \begin{cases} \frac{KR}{2a^2} (r^2 - a^2) + KR \ln \frac{a}{R} + C, & \text{if } 0 \leq r \leq a, \\ KR \ln \frac{r}{R} + C, & \text{if } a \leq r \leq R, \end{cases}$$

that changes equation's type and an infinite number of trivial hyperbolic solutions

$$U = \frac{Kr^2}{2R} + C$$

for any $C, a, a \in (0, R)$.

The considered Dirichlet and Neuman model problems for the equations (1) and (2), thanks to the arbitrary parameter a , have the infinite amount of solutions that changes the type of the equation. When Dirichlet and Neumann conditions for the equation (1) are combined (what leads to a Cauchy problem), the value of a is uniquely determined.

Model case of Cauchy problem. D is a circle of the radius R , $U|_{r=R} = H$, $\left. \frac{\partial U}{\partial r} \right|_{r=R} = K$.

Model case of Cauchy problem for the equation (1).

Let $H > 0$, $K > 0$. Satisfying Neumann condition in solution (5) of Dirichlet problem $\left. \frac{\partial U}{\partial r} \right|_{r=R} = K$, the value a is determined: $a = Re^{-\frac{H}{KR}}$.

We have one solution of the model Cauchy problem that changes the type of the equation.

Satisfying the Dirichlet condition in trivial hyperbolic solutions (8) of the model Neumann, we obtain a single trivial hyperbolic solution of the Cauchy model problem:

$$U = \frac{K}{2R}(r^2 - R^2) + H, \quad H \leq 0, \text{ or } H - \frac{KR}{2} < 0,$$

where K of any sign.

Model case of Cauchy problem for equation (2). The problem has an infinite number of solutions

$$U(x, y) = \begin{cases} \frac{KR}{2a^2}(r^2 - a^2) + KR \ln \frac{a}{R} + H, & \text{if } 0 \leq r \leq a, \\ KR \ln \frac{r}{R} + H, & \text{if } a \leq r \leq R, \end{cases}$$

that changes equation's type and an infinite number of trivial hyperbolic solutions

$$U = \frac{Kr^2}{2R} + H - \frac{KR}{2}$$

for any $a, a \in (0, R)$.

Further problems are considered only for the equation (1).

Model Dirichlet problem for the upper half-plane. Domain D is the upper half-plane $y > 0$, $U|_{y=0} = 0$. The problem has an infinite number of solutions that changes the type of equation:

$$U(x, y) = \begin{cases} Ce^x \sin y, & \text{if } 0 \leq y \leq \pi, \\ Ce^x \operatorname{sh}(\pi - y), & \text{if } y \geq \pi, \end{cases}$$

for any $C > 0$. When $C = 0$ we obtain the zero solution .

Solution for the whole plane. The function

$$U(x, y) = \begin{cases} C(r^2 - a^2), & \text{if } 0 \leq r \leq a, \\ 2Ca^2 \ln \frac{r}{a}, & \text{if } a \leq r < +\infty, \end{cases}$$

for arbitrary $a > 0$, $C > 0$ is a solution of the equation (1) that changes the type of the equation on the whole plane.

Goursat problem. In the domain $D : y + x > 0, y - x + 1 > 0$ it is required to find a solution of the equation (1) satisfying the conditions

$$U|_{y=-x} = f_1(x), \quad x \leq \frac{1}{2}; \quad U|_{y=x-1} = f_2(x), \quad x \geq \frac{1}{2}; \quad f_1\left(\frac{1}{2}\right) = f_2\left(\frac{1}{2}\right). \quad (9)$$

The lines $y = -x$, $y = x - 1$ are the characteristics of the wave equation. The function

$$U = f_1\left(\frac{x-y}{2}\right) + f_2\left(\frac{x+y+1}{2}\right) - f_1\left(\frac{1}{2}\right) \quad (10)$$

is the solution of the wave equation that satisfies (9).

If $(x, y) \in D$, then $\frac{x-y}{2} \leq \frac{1}{2}$, $\frac{x+y+1}{2} \geq \frac{1}{2}$. For example requiring

$$f_1(x) - f_1\left(\frac{1}{2}\right) \leq 0, \quad f_2(x) \leq 0,$$

we obtain that the function (10) is negative in D , thus it is a trivial hyperbolic solution of the Goursat problem.

Tricomi problem. Let us consider the classical formulation of the Tricomi problem for the Lavrent'ev-Bitsadze equation (3). Let D be a domain bounded by the line σ with endpoints $A(-1, 0)$, $B(1, 0)$, when $y > 0$ and the characteristics are AC $y = -x - 1$, BC $y = x - 1$ when $y < 0$. D is the typical domain to the Tricomi problem. It is required to determine the function U with the following properties:

- 1) $U(x, y)$ is the solution of the equation (1) in D when $y \neq 0$;
- 2) $U(x, y)$ is continuous in the enclosed domain D ;
- 3) the partial derivatives $\frac{\partial U}{\partial x}$, $\frac{\partial U}{\partial y}$ are continuous within the region D and near the points A , B it could turn into infinity order less than one;
- 4) on the line σ and on the characteristics AC $U(x, y)$ takes given values

$$U|_{\sigma} = \varphi(s), \quad (11)$$

$$U|_{AC} = f(x). \quad (12)$$

In A.V. Bitsadze's research [2] the existence and uniqueness of the solution of the Tricomi problem (reviewed above) is proved and if the bound σ is a semicircle, then the solution is obtained explicitly.

Let D_1 , D_2 be the parts of the domain D where $y > 0$ and $y < 0$ are respectively.

We show there is a solution of the equation (1) that changes its type and satisfies only one condition

$$U|_{\sigma} = \varphi(s) \geq 0, \quad \varphi(-1, 0) = \varphi(1, 0) = 0, \quad (13)$$

which is positive in D_1 , negative in D_2 and the boundary condition (12) on the AC characteristics is not required.

Let $U(x, y)$ be a harmonic function in D_1 satisfying (13) and the condition

$$U(x, 0) = 0, \quad -1 \leq x \leq 1. \quad (14)$$

Because of the maximum principle and the boundary conditions (13), (14) the harmonic function $U(x, y)$ is positive in D_1 . Since when $y = 0$, it has the lowest value, then by the Hopf lemma for harmonic functions $\frac{\partial U}{\partial n}\Big|_{y=0} < 0$ for all $x \in (-1, 1)$. The normal n is external to D_1 . Hence

$$\nu(x) = \frac{\partial U}{\partial y}\Big|_{y=0} > 0, \quad -1 < x < 1.$$

The function

$$U(x, y) = \frac{1}{2} \int_{x-y}^{x+y} \nu(x) dx \quad (15)$$

is the solution of the wave equation in D_2 and

$$U(x, 0) = 0, \quad \left. \frac{\partial U}{\partial y} \right|_{y=0} = \nu(x), \quad -1 < x < 1.$$

Since $\nu(x) > 0$ and in D_2 $x + y < x - y$, then the solution given by (15) is negative in D_2 . Considering in (15) $x + y = -1$ we find the value of the solution on the AC characteristic:

$$f(x) = U|_{AC} = -\frac{1}{2} \int_{-1}^{2x+1} \nu(x) dx < 0, \quad x \in (-1, 0], \quad U(-1, 0) = 0.$$

Thus, setting only one boundary condition (13) we have a solution of the equation (1) that changes its type in the domain that is typical to the Tricomi problem.

Note that searching of the solution was begun from satisfying the boundary condition (13), finding a solution in D_1 ($y > 0$) at first and then in D_2 ($y < 0$). In other words, moving **"top-down"**.

If σ is a semicircle $x^2 + y^2 = 1, y \geq 0$, then the solution can be written explicitly. In D_1 :

$$U(x, y) = \frac{1}{\pi} \int_{\sigma} \varphi \frac{\partial G}{\partial n} ds,$$

$$G(\bar{x}, \bar{\xi}) = \ln \frac{|\bar{\xi}^* - \bar{x}| |\bar{\xi}^- - \bar{x}|}{|\bar{\xi}^+ - \bar{x}| |\bar{\xi} - \bar{x}|}, \quad \bar{x} = (x, y), \quad \bar{\xi} = (\xi, \eta), \quad \bar{\xi}^* = \frac{1}{|\bar{\xi}|} \bar{\xi}, \quad \bar{\xi}^- = (\xi, -\eta), \quad \bar{\xi}^+ = \frac{1}{|\bar{\xi}|} \bar{\xi}^-.$$

$G(\bar{x}, \bar{\xi})$ is a Green's function of the Dirichlet problem for the domain D_1 .

After the direct calculation:

$$\nu(x) = \frac{\partial U(x, 0)}{\partial y} = \frac{4(1-x^2)}{\pi} \int_0^\pi \varphi(\cos, \sin \theta) \frac{\sin \theta}{(1-2x \cos \theta + x^2)^2} d\theta.$$

The solution of the problem in D_2 is defined by (15).

Let us find the solution of the equation (1) that is positive in D_1 and negative in D_2 , but when moving **"bottom-up"** satisfying the boundary condition (12) on the characteristics of AC firstly.

The solution in D_2 in the form

$$U(x, y) = f_1(x + y) + f_2(x - y).$$

is sought. Satisfying the condition (12) on the characteristics of $AC(y = -x - 1)$ and the condition (14) we obtain

$$U(x, y) = f\left(\frac{x - y - 1}{2}\right) - f\left(\frac{x + y - 1}{2}\right). \tag{16}$$

In the domain $D_2 : \frac{x - y - 1}{2} > \frac{x + y - 1}{2}$. The solution is negative in D_2 , if the function $f(x)$ decreases on $(-1, 0)$. Taking into account that $f(-1) = 0$, we obtain the condition of negativity in D_2 for the solution (16):

$$f(-1) = 0, \text{ function } f(x) \text{ decreasing, } f(x) < 0, \quad -1 \leq x \leq 0.$$

From (16):

$$\nu(x) = \frac{\partial U(x, 0)}{\partial y} = -f'\left(\frac{x - 1}{2}\right). \tag{17}$$

For the existence of $\nu(x) = \frac{\partial U(x,0)}{\partial y}$ it is required the existence of $f'(x)$. Since $f(x)$ is decreasing then $f'(x) < 0$. Then according to (17) $\nu(x) = \frac{\partial U(x,0)}{\partial y} > 0$, $-1 < x < 1$.

To find a solution in D_1 with the condition (14) we come to the problem of finding a harmonic function in D_1 satisfying $U|_{\sigma} = \varphi(s) \geq 0$, $U(x,0) = 0$, $\frac{\partial U(x,0)}{\partial y} = -f' \left(\frac{x-1}{2} \right) > 0$.

The obtained problem is overdetermined if the domain D_1 or boundary function $\varphi(s)$ are previously defined even without requiring non-negativity of $\varphi(s)$. If a harmonic function exists in any neighborhood of $y > 0$ that is the solution of the Cauchy problem

$$U(x,0) = 0, \quad \frac{\partial U(x,0)}{\partial y} = -f' \left(\frac{x-1}{2} \right), \quad -1 < x < 1, \quad (18)$$

then the function $\nu(x) = -f' \left(\frac{x-1}{2} \right)$ is analytical by x on the interval $(-1,1)$. It follows that the function given on the characteristic AC must be analytical.

By Kowalewski theorem in a certain neighborhood $y > 0$, $-1 < x < 1$ there a harmonic function exists satisfying the analytical conditions of the Cauchy problem (18). From the conditions

$$U(x,0) = 0, \quad \frac{\partial U(x,0)}{\partial y} = -f' \left(\frac{x-1}{2} \right) > 0, \quad -1 < x < 1$$

the existence of a neighborhood of $y > 0$ that is adjacent to the entire interval $(-1,1)$ follows where this harmonic function is positive. This neighborhood or any other one that contained in it that is also adjacent to the entire interval $(-1,1)$, defines D_1 when formulating the Tricomi problem, if setting the boundary value only on the characteristics of AC :

$$U|_{AC} = f(x), \quad f(-1) = 0, \quad f'(x) < 0, \quad \text{function } f(x) \text{ is analytical.} \quad (19)$$

We obtain an explicit formula of the solution of the Cauchy problem (18) for the Laplace equation. Turning to the complex plane $z = x + iy$, $\bar{z} = x - iy$, Laplace equation can be written as $\frac{\partial^2 U}{\partial z \partial \bar{z}} = 0$. Taking $U = f_1(z) + f_2(\bar{z})$, after satisfying the conditions (18) of the Cauchy problem we obtain

$$U(x,y) = i \left(f \left(\frac{z-1}{2} \right) - f \left(\frac{\bar{z}-1}{2} \right) \right) \quad (20)$$

The area of harmonicity is defined by the possibility of an analytical extension of the function $f \left(\frac{x-1}{2} \right)$ from the interval $(-1,1)$.

Thus, we come to the two constructions (area + boundary conditions) of, perhaps, a lot of formulations of boundary value problems for the equation (1).

Let D be a domain that is typical to the Tricomi problem.

The Tricomi problem 1. The boundary condition (13) is set only on σ that is the upper bound of D_1 .

The Tricomi problem 2. Only the one condition (12) is set on the characteristics of AC . The domain D_1 is defined during the solution process.

Consider few examples of the Tricomi problem 2.

Example. $f(x) = -(1+x)^3$. According to (16), (20)

$$U(x,y) = \begin{cases} \frac{y}{4}(3(x+1)^2 - y^2), & \text{if } (x,y) \in D_1, \\ \frac{y}{4}(3(x+1)^2 + y^2), & \text{if } (x,y) \in D_2. \end{cases}$$

D_1 is an arbitrary domain in the Tricomi problem. The points in D_1 satisfy $\sqrt{3}(x+1) - y \geq 0$. In this case $U(x, y) > 0$ in D_1 .

Example. Let the boundary function $f(x)$ set on $[-1, 0]$ be represented by series $f(x) = \sum_{n=0}^{\infty} a_n \left(x + \frac{1}{2}\right)^n$.

According to (16) in D_2 : $U = \sum_{n=0}^{\infty} \frac{a_n}{2^n} ((x-y)^n - (x+y)^n)$.

According to (20) in D_1 :

$$U = \iota \sum_{n=0}^{\infty} \frac{a_n}{2^n} (z^n - \bar{z}^n) = - \sum_{n=0}^{\infty} \frac{a_n}{2^{n-1}} r^n \sin(n\varphi). \quad (21)$$

Domain D_1 is determined by researching of the sign of the function (21).

Remark 1. It should be noted that in all considered problems there remains an open-ended question whether all solutions are found, even if in the problem an infinite number of solutions is obtained.

Remark 2. The equations (1), (3) as well as Laplace's and wave equations are the special cases of the equation (2). So, all boundary value problems are typical to Laplace's, wave, Lavrent'ev-Bitsadze equations and considered above for the equation (1) are included in possible statements of boundary value problems for the equation (2).

Remark 3. The examples of the formulations of Tricomi 1 and Tricomi 2 boundary value problems give an opportunity to suppose that two components that define the boundary problem (domain and boundary conditions), may be produced according to the method of finding a solution.

Remark 4. The Tricomi problem 1 shows a particular impact of the domain's geometry to the formulation of boundary value problems for the equation (1). Let D be the domain that is typical to the Tricomi problem. When formulating the Dirichlet problem for Lavrent'ev-Bitsadze equation we come to the classical Tricomi problem, where the boundary condition is not specified for one of the characteristics. For the equation (1) with the additional non-negativity condition of the function set on the bound of σ , the boundary conditions might not be set on two characteristics.

2. Other equations

A natural generalization of the equation (2) is

$$\prod_{i=1}^n (L_i(u) - f_i(u)) = 0, \quad (22)$$

where, for example, $L_i(u)$ are linear second-order differential operators. We seek a continuously differentiable in D function and disjoint areas D_i , $\bigcap_{i=1}^n D_i = D$, where $L_i(u) = f_i(u)$. On the bound of D and bounds of D_i the additional conditions are specified.

If $L_i(u) = \Delta u$, then the equation (22) describes the general problem of splicing the vortex and potential flows of an perfect fluid [3]. Here $u(x, y)$ is a stream function and $\omega_i = f_i(u)$ is a vorticity in D_i .

For example, if D is a bounded domain and a problem is

$$\Delta U(x, y)(\Delta U(x, y) - \omega) = 0, \quad \omega > 0, U|_{\Gamma} = \varphi(s) \geq 0, U|_{\Gamma_1} = 0,$$

where Γ_1 is the bound of $D_1 \subset D$ where $\Delta U(x, y) = \omega$, then it is the Goldshtik's problem that describes detached flows by Lavrentev's scheme [4].

The considered equations could be model in gas dynamics and hydromechanics problems, heat, wave and other processes. Solutions of these problems have to satisfy different equations depending on process settings and solution properties.

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Модельные задачи для двух нелинейных уравнений, тип которых зависит от решения

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В статье рассматриваются модельные задачи для двух нелинейных уравнений с частными производными второго порядка, тип которых зависит от решения. Одно из уравнений можно назвать нелинейным аналогом уравнения Лаврентьева–Бицадзе.

Ключевые слова: тип уравнения, уравнения эллиптического, гиперболического типа, уравнение Лаврентьева–Бицадзе, задача Трикоми.