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Applications of Operational Approach to Evaluation of Projects of Economic Systems Development

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The paper proposes a technique of obtaining the investment project optimal cost for the development of economic systems. The considered systems are described by multi-step multicriteria linear programming problems with discounting coefficients in the objective functions. The proposed technique is based on the operator which is equivalent to the z -transform for finite time interval. Application of the proposed technique allows one to classify the projects as unsuitable or potentially effective during the preliminary analysis stage.

Keywords: development of economic systems, investment project, the multi-step multicriteria linear programming problem, operational approach.

Introduction

In today's economy it is difficult to choose the best variant of the productive economic system (ES) development without dynamic optimization models because a wide range of products is put out. An economic system is characterized by the variety of different indicators — a number of these products, demand for them, productivity of the fixed assets involved in the production process, etc. Moreover, it is necessary to evaluate the efficiency of ECs taking into account interests of some individuals e.g., consumers, producers, tax authorities, etc., that is, many criteria are included. The study of such models numerically is complicated because we deal with multiparametric and large scale economic models. It justifies the use of analytical methods to evaluate the economic systems effectiveness. This paper proposes an approach to theoretically analyze the investment project (IP) of the ES development. The project is described by a multicriterion multistep problem of linear programming (MMPLP) with discounted coefficients in objective criteria and it allows one to take into account the decrease in the value of cash flow over time. The approach is based on the use of operator that is similar to the z -transform for the finite planning horizon. The approach allows a decision maker (DM) to classify an investment project as effective if its net present value is not less than the value claimed by the investor (or another participant of IP) as the result of the practical implementation of the project.

1. Formulation of the problem of the economic system development

Let us show the application of the approach on the example of the problem given in [1]. A company has its own initial capital and plans to produce several types of products. The products sales volume does not exceed the demand for them. Technical and economic characteristics of the basic production assets (BPA) participating in production process are given. These

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characteristics are assets value, assets service life, productivity of BPA unit and production cost of each product. It is necessary to determine the volume of investment allocated by the investor (tax centre — TC) and by the company for implementation of the project. One needs to determine both total investment and investments for each unit of BPA separately. One also needs to find an optimal output of each type of product subject to the condition that the total discounted cash flows of the participants of IP are maximized. Here the optimality is understood in the Pareto sense. Let us call the formulated problem as the problem of the " company-TC" economic system development or the problem A. The considered problem can be treated as an IP evaluation problem for the development of the specified economic system. If, in addition, it is required that at the end of the project the state of the specified system is coincident with the given state then we have modified problem. We call this problem as the problem with the fixed final state or the problem A'. The problem A' can be treated as a problem of optimal planning of real investments to determine an order of the total value of all BPA gained and to determine the output of each type of product, total residual value of products, the volume of financial resources and accumulated investment expenditures at the end of the project.

The specified approach to the IP efficiency analysis of economic system development is illustrated at a microeconomic level, i.e. at the company level. However it can be applied to dynamic models of meso- or macroeconomic systems that are described by the mentioned above type of problems [2, 3].

2. Mathematical models

Let us accept the following basic assumptions: 1) taxes form the most part of the company expenses and they are a value added tax (VAT), a tax on profit (TP), a property tax (PT), amount of insurance premiums (AIP) and deductions to the Fund of remuneration of labour (FRL); 2) a company has sufficient reserves of raw materials; 3) a period T of the IP duration is less than service life T_k of each type of BPA unit: $T < T_k$ ($k = 1, \dots, n$); 4) each BPA unit is spent to produce only one type of product (a principle of net branches). Taking into account all the assumptions, the formulated above problem A can be written as the following MMPLP:

$$\begin{aligned}
 x_k(t+1) &= x_k(t) + u_k(t) \quad (k = 1, \dots, n; t = 0, \dots, T-1), \\
 x_{n+1}(t+1) &= x_{n+1}(t) + \sum_{k=1}^n u_k(t) \quad (t = 0), \\
 x_{n+1}(t+1) &= -\sum_{k=1}^n x_k(t)/T_k + x_{n+1}(t) + \sum_{k=1}^n u_k(t) \quad (t = 1, \dots, T-1), \\
 x_{n+2}(t+1) &= -\alpha_2 x_{n+1}(t) + x_{n+2}(t) - \sum_{k=1}^n u_k(t) + u_{2n+1}(t) + u_{2n+2}(t) \quad (t = 0), \\
 x_{n+2}(t+1) &= \alpha_3 \sum_{k=1}^n \frac{x_k(t)}{T_k} - \theta x_{n+1}(t) + x_{n+2}(t) - \\
 &\quad - \sum_{k=1}^n u_k(t) + \gamma \sum_{k=1}^n u_{n+k}(t) + u_{2n+1}(t) \quad (t = 1, \dots, T^1 - 1), \\
 x_{n+2}(t+1) &= \alpha_3 \sum_{k=1}^n \frac{x_k(t)}{T_k} - \theta x_{n+1}(t) + x_{n+2}(t) - \sum_{k=1}^n u_k(t) + \gamma \sum_{k=1}^n u_{n+k}(t) \\
 &\quad (t = T^1, \dots, T-1); \\
 x_{n+3}(t+1) &= x_{n+3}(t) \quad (t = T^1, \dots, T-1); \\
 x_{n+3}(t+1) &= x_{n+3}(t) + u_{2n+1}(t) \quad (t = 0, \dots, T^1 - 1), \\
 x_k(0) &= 0 \quad (k = 1, \dots, n+3);
 \end{aligned} \tag{1}$$

$$\begin{aligned}
 & x_{n+2}(t) \geq 0 \quad (t = 1, \dots, T); \\
 & -\sum_{k=1}^n \frac{x_k(t)}{T_k} - \alpha_2 x_{n+1}(t) + (1 - \beta) \sum_{k=1}^n u_{n+k}(t) \geq 0 \quad (t = 1, \dots, T - 1); \\
 & 0 \leq u_{n+k}(t) \leq q_k(t + 1), \quad u_{n+k}(t) \leq \delta_k x_k(t) \quad (k = 1, \dots, n; t = 1, \dots, T - 1), \\
 & x_{n+3}(T^1) \leq I_0, \quad u_{2n+2}(0) \leq K_0, \\
 & u_k(t) \geq 0 \quad (k = 1, \dots, n; t = 0, \dots, T - 1), u_{2n+1}(t) \geq 0 \quad (t = 0, \dots, T^1 - 1), u_{2n+2}(0) \geq 0; \\
 & J = \{J_1, J_2\} \rightarrow \max,
 \end{aligned}$$

where

$$\begin{aligned}
 J_1 &= -\sum_{t=0}^{T^1-1} \frac{u_{2n+1}(t)}{(1+r)^t} - u_{2n+2}(0) + \sum_{t=T^2}^{T-1} \frac{\left[\alpha_3 \sum_{k=1}^n \frac{x_k(t)}{T_k} - \theta x_{n+1}(t) + \gamma \sum_{k=1}^n u_{n+k}(t) \right]}{(1+r)^t} + \frac{\delta x_{n+1}(T)}{(1+r)^{T-1}}, \\
 J_2 &= \sum_{t=T^2}^{T-1} \frac{\left[-\alpha_3 \sum_{k=1}^n \frac{x_k(t)}{T_k} + \theta x_{n+1}(t) + \rho \sum_{k=1}^n u_{n+k}(t) \right]}{(1+r)^t}
 \end{aligned}$$

are discounted amounts of the company own funds and TC, respectively. Here the control variables $u_k(t)$ ($t = 0, \dots, T - 1$), $u_{n+k}(t)$ ($k = 1, \dots, n; t = 1, \dots, T - 1$) and $u_{2n+1}(t)$ ($t = 0, \dots, T^1 - 1$) are values of the received BPA, proceeds from the realization of the k -th type of product, external and internal investments, respectively; phase variables $x_k(t), x_{n+1}(t), x_{n+2}(t)$ and $x_{n+3}(t)$, ($k = 1, \dots, n; t = 0, \dots, T$) are accumulated values of all BPA of the k -th type, the total residual value of all the BPA, current amount of cash of the company and the accumulated amount of foreign investments at the moment t , respectively; $q_k(t + 1)$ ($t = 1, \dots, T - 1$), V_k , T_k , c_k , and P_k – forecasted demand for the moment $t+1$, productivity, service life, a unit cost of BPA and the unit production cost of the of the k -th type, ($k = 1, \dots, n$), respectively; I_0 , K_0 are sums of external and internal investments, given for the duration of IP; α_1 , α_2 , α_3 and α_4 are VAT rate, PT, TP and AIP, respectively (VAT is included in the price of the products, so we assume that $\alpha_1 = 0$); β is a share of sales devoted to FRL; T^1 and T ($1 \leq T^1 < T$) are the moment of foreign investment completion and duration of the IP, respectively; $\theta = (1 - \alpha_3)\alpha_2$, $\delta_k = P_k V_k / c_k$ ($k = 1, \dots, n$), $\gamma = (1 - \alpha_3)(1 - \beta)$, $\rho = (1 - \beta)\alpha_3 + \alpha_4\beta$; r is the rate of the IP profitability. Parameter δ ($0 \leq \delta \leq 1$) is a share of residual value of all BPA with respect to its balance value for the moment $t=T$. Generally, this parameter is determined by expertise evaluation. Thus, the problem A' differs from the model A in the additional terminal condition $x_k(T) = x_k^T$ ($k = 1, \dots, n + 3$), where $x_k^T = \text{const}$ are known terminal values of the corresponding phase variables.

Without the loss of generality, one can assume that

$$x_k(T) = 0 \quad (k = 1, \dots, n + 3), \quad (2)$$

because the change of variables $\bar{x}_k(t) = x_k(t)$ ($t = 0, \dots, T - 1$); $\bar{x}_k(T) = x_k(T) - x_k^T$ ($k = 1, \dots, n + 3$) results in condition (2) for variables $\bar{x}_k(t)$.

Let us note that according to [4], MMPLP (1) and its modification (1), (2) are equivalent to one-criterion problems with the same restrictions and with the conditions that the criteria convolutions $J(\mu) = \mu J_1 + (1 - \mu) J_2 \rightarrow \max$ ($\mu \in (0; 1)$) and $J' = \mu J'_1 + (1 - \mu) J'_2 \rightarrow \max$ ($\mu \in (0; 1)$). As this takes place the following inequality holds

$$J'(\mu) \leq J(\mu) \quad (\mu \in (0; 1)). \quad (3)$$

By applying operator $Z_T(x(t)) = X(z, T) = \sum_{t=0}^{T-1} x(t)z^{-t}$ to MMPLP A' (i.e. to problem (1), (2)) at $z = 1 + r$, where r is discounted rate of IP and taking into account the feature $Z_T(x(t +$

1)) = $z[X(z, T) + x(T)z^{-T} - x(0)]$ we obtain a static problem of linear programming (PLP) that depends on parameters μ and z :

$$\begin{aligned}
 zX_k &= X_k + U_k \quad (k = 1, \dots, n), \quad zX_{n+1} = -\sum_{k=1}^n \frac{X_k}{T_k} + X_{n+1} + \sum_{k=1}^n U_k, \\
 zX_{n+2} &= \alpha_3 \sum_{k=1}^n \frac{X_k}{T_k} - \theta X_{n+1} + X_{n+2} - \sum_{k=1}^n U_k + \gamma \sum_{k=1}^n U_{n+k} + U_{2n+1} + u_{2n+2}(0), \\
 zX_{n+3} &= X_{n+3} + U_{2n+1}; \\
 zX_{n+2} &\geq 0, \quad -\sum_{k=1}^n \frac{X_k}{T_k} - \alpha_2 X_{n+1} + (1 - \beta) \sum_{k=1}^n U_{n+k} \geq 0, \quad U_{n+k} \leq Q_k, \\
 U_{n+k} &\leq \delta_k X_k \quad (k = 1, \dots, n), \\
 U_{2n+1} &\leq I_0, \quad u_{2n+2}(0) \leq K_0; \quad U_k(z) \geq 0 \quad (k = 1, \dots, 2n + 1); \quad u_{2n+2}(0) \geq 0; \\
 \bar{J}'(\mu) &= \mu \bar{J}'_1 + (1 - \mu) \bar{J}'_2 \rightarrow \max \quad (\mu \in (0; 1)).
 \end{aligned} \tag{4}$$

Here

$$\begin{aligned}
 \bar{J}'_1 &= -U_{2n+1} - u_{2n+2}(0) + \alpha_3 \sum_{k=1}^n \frac{X_k}{T_k} - \theta X_{n+1} + \gamma \sum_{k=1}^n U_{n+k}, \\
 \bar{J}'_2 &= -\alpha_3 \sum_{k=1}^n \frac{X_k}{T_k} + \theta X_{n+1} + \rho \sum_{k=1}^n U_{n+k}, \\
 U_j(z, T) &= \sum_{t=0}^{T-1} u_j(t)z^{-t}, \quad U_{n+j}(z, T) = \sum_{t=1}^{T-1} u_{n+j}(t)z^{-t} \quad (j = 1, \dots, n); \\
 U_{2n+1}(z, T^1) &= \sum_{t=0}^{T^1-1} u_{2n+1}(t)z^{-t}; \quad X_k(z, T) = \sum_{t=0}^{T-1} x_k(t)z^{-t} \quad (k = 1, \dots, n + 3)
 \end{aligned}$$

are sets of the corresponding dynamic variables and the summation is over all t for which these variables are defined. This explains, without the loss of generality, the existence in problem (4) both aggregative variables U_j ($j = 1, \dots, 2n + 1$) and dynamic variables $u_{2n+2}(0)$; z, T, T^1 are omitted for short; $Q_k(z, T) \stackrel{def}{=} \sum_{t=1}^{T-1} q_k(t + 1)z^{-t}$ ($k = 1, \dots, n$) is the aggregate demand for all types of manufactured products. Note that the last problem is equivalent to double-criterion PLP based on the same restrictions and condition $\bar{J}' = \{\bar{J}'_1, \bar{J}'_2\} \rightarrow \max$. We call this model $Z_T A'$ (an aggregative model of A'). The ultimate version of the model $Z_T A'$ obtained for the case $T \rightarrow +\infty$ will be called the ZA' model (or the ZA model since models of A and A' are equivalent when $T \rightarrow +\infty$).

3. Theoretical analysis of dynamic and aggregative models

If we construct convolutions $J'(\mu), \bar{J}'(\mu)$ for problems (1), (2) and (4) then the following inequality is true:

$$J'(\mu) \leq \bar{J}'(\mu) \quad (\mu \in (0; 1)). \tag{5}$$

Let us denote by $*$ the optimal value of criteria convolution. For problems A and A' theorem 1 holds. This theorem is necessary for the justification of further results (see consequence 1 and 2) on a finite time interval [2].

Theorem 1. *The optimal values of convolutions $J^*(\mu), J^{*'}(\mu)$ in the projects described by the models A and A' are non-decreasing functions of the parameter T at the constant values of the other parameters and $\mu \in [0; 1]$.*

The following lemma is true. It is used for the further analysis of problem A' (see theorems 2 and 3).

Lemma 1. *Conditions $\bar{\beta}_k = \theta \left[1 - \frac{1}{rT_k} \right] + \frac{1 - \alpha_3}{T_k} > 0$ ($k = 1, \dots, n$), where $\theta = (1 - \alpha_3)\alpha_2$, are equivalent to the following relations:*

$$\begin{cases} r \geq \alpha_2 \\ \begin{cases} r < \alpha_2 \\ T_k > \frac{1}{r} - \frac{1}{\alpha_2} \end{cases} \end{cases} \quad (k = 1, \dots, n), \quad (6)$$

Proof. Necessity. Let $\bar{\beta}_k > 0$. Taking into account the expression for θ and the inequality $1 - \alpha_3 > 0$ (it follows from economical sense of a tax on profit rate α_3), rewrite the condition $\bar{\beta}_k > 0$ in the following form $\alpha_2 \left[1 - \frac{1}{rT_k} \right] + \frac{1}{T_k} > 0$, or in the form

$$\alpha_2 + \frac{1}{T_k} \left(1 - \frac{\alpha_2}{r} \right) > 0. \quad (7)$$

If in the last relation $1 - \frac{\alpha_2}{r} \geq 0$ then (7) is obviously correct because a property tax rate $\alpha_2 > 0$ and $T_k > 0$, where T_k is the service life of BPA unit of the k -th type. If $1 - \frac{\alpha_2}{r} < 0$ then (7) could be rewritten as $\frac{1}{T_k} \left(\frac{\alpha_2}{r} - 1 \right) < \alpha_2$ whence it follows that $T_k > \frac{1}{r} - \frac{1}{\alpha_2}$.

Sufficiency. Let one of conditions (6) is true. If $r \geq \alpha_2$, i.e. $1 - \frac{\alpha_2}{r} \geq 0$, then $\frac{1}{T_k} \left(1 - \frac{\alpha_2}{r} \right) \geq 0$, whence (7) follows and it means that $\bar{\beta}_k > 0$. In that case when $T_k > \frac{1}{r} - \frac{1}{\alpha_2}$, i.e. $T_k > \frac{\alpha_2 - r}{\alpha_2 r}$, by multiplying the last equation by $\frac{\alpha_2}{T_k} > 0$ we obtain $\alpha_2 > \frac{\alpha_2 - r}{T_k r}$ or $\alpha_2 > \frac{1}{T_k} \left(\frac{\alpha_2}{r} - 1 \right)$, whence we get (7) and it means that $\bar{\beta}_k > 0$. Moreover if $r \geq \alpha_2$ then the inequality $T_k > \frac{1}{r} - \frac{1}{\alpha_2}$ is uninformative because of economic condition $T_k > 0$. In this case it would appear natural that $r < \alpha_2$, i.e. we obtain condition (7). \square

Taking into account PLP (4), condition (5) and lemma 1 we will formulate the following theorem [1] that is used in the numerical example given below.

Теорема 2. *If conditions (6) are true then the solution of the problem A' exists on the finite time interval and the following equality holds*

$$J^{*'}(\mu) \leq \Gamma'_{T,0}(\mu) \quad (\mu \in (0; 1)), \quad (8)$$

where

$$\Gamma'_{T,0}(\mu) = \begin{cases} \left\{ \mu\gamma + (1 - \mu)\rho - \frac{(1 - 2\mu)\gamma}{r} \sum_{\bar{\gamma}_j < 0} \frac{\bar{\gamma}_j}{\bar{\beta}_j} \right\} \sum_{k=1}^n Q_k & (\mu \in (0; 1/2]) \\ \left\{ \mu\gamma + (1 - \mu)\rho - \frac{(1 - 2\mu)\gamma}{r} \sum_{\bar{\gamma}_j > 0} \frac{\bar{\gamma}_j}{\bar{\beta}_j} \right\} \sum_{k=1}^n Q_k & (\mu \in (1/2; 1)), \end{cases} \quad (9)$$

$$\bar{\beta}_k = \theta \left[1 - \frac{1}{rT_k} \right] + \frac{1 - \alpha_3}{T_k}, \bar{\gamma}_k = \left[\frac{\alpha_3}{T_k} - \theta \right] r + \frac{\theta}{T_k} \quad (k = 1, \dots, n).$$

Let us consider the double-criterion problem A' and take the convolution of criteria $J'(\mu) = \mu J'_1 + (1 - \mu)J'_2$ as the average value of the total funds of the producer and the tax centre (obtained at the end of IP) with weights μ and $(1 - \mu)$, respectively. Then theorem 2 has the following meaning. The optimal value of IP in the model of problem A' is no greater than the value given by formula (9). The similar estimates of the optimal value of the project can be obtained with the analysis of other restrictions in problem (4).

Taking the limit $T \rightarrow +\infty$ in (9) and taking into account 3) and the fact that condition (6) is equivalent to $\lim_{T \rightarrow +\infty} \bar{\beta}_k = \theta > 0$ ($k = 1, \dots, n$) we obtain the following consequence of theorem 2 [1].

Corollary 1. *If the demand for the product is finite:*

$$\max_{t=1, \dots} q_k(t+1) < +\infty \quad (k = 1, \dots, n) \tag{10}$$

then the problems A and A' at $T \rightarrow +\infty$ are solvable and the following inequality holds

$$J^*(\mu) \leq \Gamma_0(\mu, z) \quad (\mu \in (0; 1)), \tag{11}$$

where

$$\Gamma_0(\mu, z) = \begin{cases} [\mu\gamma + (1 - \mu)\rho - (1 - 2\mu)\gamma n] \sum_{k=1}^n Q_k(z) & (\mu \in (0; 1/2]) \\ [\mu\gamma + (1 - \mu)\rho] \sum_{k=1}^n Q_k(z) & (\mu \in (1/2; 1)), \end{cases} \tag{12}$$

$$Q_k(z) = \sum_{t=1}^{\infty} q_k(t+1)z^{-t} \quad (k = 1, \dots, n).$$

The analysis of the problem ZA also leads to consequence 1. Moreover, the given statement has the economic sense which is similar to theorem 2: the optimal value of IP in model A or model A' at $T \rightarrow +\infty$ does not exceed the estimate given in (12).

After studying problem (4) the following theorem which is similar to theorem 2 can be proved.

Theorem 3. *If conditions (6) are fulfilled then there is a solution of the problem A' on a finite time interval and the following inequality holds*

$$J^*(\mu) \leq \Gamma'_{T,1}(\mu) \quad (\mu \in (0; 1)), \tag{13}$$

where

$$\Gamma'_{T,1}(\mu) = \begin{cases} \left\{ -\frac{(1 - 2\mu)\gamma\bar{\gamma}_{min}}{r\bar{\beta}_{min}} + \mu\gamma + (1 - \mu)\rho \right\} \sum_{k=1}^n Q_k & (\mu \in (0; 1/2]) \\ \left\{ -\frac{(1 - 2\mu)\gamma\bar{\gamma}_{max}}{r\bar{\beta}_{min}} + \mu\gamma + (1 - \mu)\rho \right\} \sum_{k=1}^n Q_k & (\mu \in (1/2; 1)), \end{cases} \tag{14}$$

$$\bar{\beta}_{min} = \min_{k=1, \dots, n} \bar{\beta}_k, \quad \bar{\gamma}_{min} = \min_{k:\bar{\gamma}_k < 0} \bar{\gamma}_k, \quad \bar{\gamma}_{max} = \max_{k:\bar{\gamma}_k > 0} \bar{\gamma}_k.$$

At $T \rightarrow +\infty$ consequence 2 [2] follows from theorem 3 for the unbounded planning horizon.

Proof. Expressing X_k from equations of model (4) we obtain the equivalent problem

$$\begin{aligned} U_{n+k} &\leq Q_k, \quad U_{n+k} \leq \delta_k U_k / r \quad (k = 1, \dots, n), \\ -\sum_{k=1}^n \bar{\alpha}_k U_k - r \left[\gamma \sum_{k=1}^n U_{n+k} + U_{2n+1} + u_{2n+2}(0) \right] &\leq 0, \\ \sum_{k=1}^n \bar{\beta}_k U_k - \gamma r \sum_{k=1}^n U_{n+k} &\leq 0, \quad U_{2n+1} \leq I_0, \quad u_{2n+2}(0) \leq K_0; \end{aligned}$$

$$U_k \geq 0 \quad (k = 1, \dots, 2n + 1); \quad u_{2n+2}(0) \geq 0, \quad (15)$$

$$\begin{aligned} \bar{J}'(\mu) = & -\frac{(1-2\mu)}{r^2} \sum_{k=1}^n \bar{\gamma}_k U_k + [\mu\gamma + (1-\mu)\rho] \sum_{k=1}^n U_{n+k} - \\ & -\mu [U_{2n+1} + u_{2n+2}(0)] \rightarrow \max(z > 1; \mu \in (0; 1)), \end{aligned} \quad (16)$$

where $\bar{\alpha}_k = \frac{\alpha_3}{T_k} + \theta \left(\frac{1}{rT_k} - 1 \right) - r \quad (k = 1, \dots, n)$.

Suppose that conditions (6) are satisfied. Let us show that a set \bar{D}' of admissible variable values of problem (15), (16) is a non-empty compactum. The boundedness of variables $U_{n+k} \quad (k = 1, \dots, n)$, U_{2n+1} and $u_{2n+2}(0)$ follows from the seventh (at $k = n + 1, \dots, 2n + 1$) and also from the first, fifth and sixth inequalities (15). We also have $\sum_{j=1}^n \bar{\beta}_j U_j \leq \gamma r \sum_{k=1}^n Q_k$ from the first and fourth conditions (15). Then due to conditions (6) and the seventh inequality (15) for $k = 1, \dots, n$, the inequalities $U_j \leq \frac{\gamma r}{\bar{\beta}_j} \sum_{k=1}^n Q_k \quad (j = 1, \dots, n)$ follow, i.e. variables $U_j \quad (j = 1, \dots, n)$ are also bounded. It means that the set \bar{D}' is bounded. The set compactness follows from the fact that inequality (15) is not strict one.

Because set of values $U_k = 0 \quad (k = 1, \dots, 2n + 1)$, $u_{2n+2}(0) = 0$ is accepted in PLP (15), (16) then the set \bar{D}' is non-empty compactum. Because convolution of criteria $\bar{J}'(\mu)$ is a continuous function (at the fixed values of the initial parameters) then there is a solution of the given problem according to Weierstrass theorem. On the other hand any process $\{u, x\}$ admissible in the model A' is also admissible in the model $Z_T A'$ (by the construction of the process), i.e. the following relation is correct

$$\{u, x\} \in D' \Rightarrow U \in \bar{D}' \quad (17)$$

Here $u = \{u_k(t) \quad (k = 1, \dots, n; t = 0, \dots, T - 1); u_{n+k}(t) \quad (k = 1, \dots, n; t = 1, \dots, T - 1); u_{2n+1}(t) \quad (t = 0, \dots, T - 1); u_{2n+2}(0)\}$, $x = \{x_k(t) \quad (k = 1, \dots, n + 3; t = 0, \dots, T)\}$, D' are vectors of control and phase variables and a set of admissible processes of the problem A' , respectively; $U = \{U_k \quad (k = 1, \dots, 2n + 1); u_{2n+2}(0)\}$ is a set of variables of PLP $Z_T A'$ that follows from the set u defined by operator Z_T . The set D' is compact by virtue of relation (17) and because the set \bar{D}' is compact. Since the zeroth control and phase vectors are admissible in the model of A' then the set D' is also a non-empty one. Due to the continuity of the convolution $J'(\mu) \quad (\mu \in (0; 1))$ there is a solution of the problem A' by Weierstrass theorem.

To prove inequality (13) with estimate (14) let us consider the following variants with respect to the sign of $1 - 2\mu$ in (16).

1) If $1 - 2\mu < 0$ then the following inequalities are true

$$-\frac{(1-2\mu)}{r^2} \sum_{k=1}^n \bar{\gamma}_k U_k \leq -\frac{(1-2\mu)}{r^2} \sum_{\bar{\gamma}_k > 0} \bar{\gamma}_k U_k \leq -\frac{(1-2\mu)}{r^2} \max_{k: \bar{\gamma}_k > 0} \bar{\gamma}_k \sum_{\bar{\gamma}_k > 0} U_k$$

$$\text{i.e. } -\frac{(1-2\mu)}{r^2} \sum_{k=1}^n \bar{\gamma}_k U_k \leq -\frac{(1-2\mu)}{r^2} \bar{\gamma}_{max} \sum_{\bar{\gamma}_k > 0} U_k, \quad \text{where } \bar{\gamma}_{max} = \max_{k: \bar{\gamma}_k > 0} \bar{\gamma}_k.$$

Then inequality for convolution of criteria $\bar{J}'(\mu) \quad (\mu \in (0; 1))$ from condition (16) is

$$\bar{J}'(\mu) \leq -\frac{(1-2\mu)}{r^2} \bar{\gamma}_{max} \sum_{\bar{\gamma}_k > 0} U_k + [\mu\gamma + (1-\mu)\rho] \sum_{k=1}^n U_{n+k}. \quad (18)$$

Let us denote $\bar{\beta}_{min} = \min_{k=1, \dots, n} \bar{\beta}_k$. Then it follows from the first and fourth equations (15) (as condition (6) is true) that $\bar{\beta}_{min} \sum_{k=1}^n U_k \leq \sum_{k=1}^n \bar{\beta}_k U_k \leq \gamma r \sum_{k=1}^n U_k \leq \gamma r \sum_{k=1}^n Q_k$, i.e. $\bar{\beta}_{min} \sum_{k=1}^n U_k \leq \gamma r \sum_{k=1}^n Q_k$, whence

$$\sum_{k=1}^n U_k \leq \frac{\gamma r}{\bar{\beta}_{min}} \sum_{k=1}^n Q_k. \tag{19}$$

In particular, since $\sum_{\bar{\gamma}_k > 0} U_k \leq \sum_{k=1}^n U_k$ then inequality $\sum_{\bar{\gamma}_k > 0} U_k \leq \frac{\gamma r}{\bar{\beta}_{min}} \sum_{k=1}^n Q_k$ follows from (19). Then as it follows from the first restriction (15), (18) and the last relation $J'(\mu) \leq \left\{ -\frac{(1-2\mu)\gamma\bar{\gamma}_{max}}{r\bar{\beta}_{min}} + \mu\gamma + (1-\mu)\rho \right\} \sum_{k=1}^n Q_k$ so we obtain the second part of expression (14).

2) If $\mu = 1/2$ then taking into account the first restriction (15), convolution of criteria (16) is $J'(1/2) = \frac{[\gamma + \rho]}{2} \sum_{k=1}^n U_{n+k} \leq \frac{[\gamma + \rho]}{2} \sum_{k=1}^n Q_k$, i.e. $J'(1/2) \leq \frac{[\gamma + \rho]}{2} \sum_{k=1}^n Q_k$. Then the first part of (14) is also proved for $\mu = 1/2$.

3) If $1 - 2\mu > 0$ then the following inequalities are true

$$\begin{aligned} -\frac{(1-2\mu)}{r^2} \sum_{k=1}^n \bar{\gamma}_k U_k &\leq -\frac{(1-2\mu)}{r^2} \sum_{\bar{\gamma}_k < 0} \bar{\gamma}_k U_k \leq \frac{(1-2\mu)}{r^2} \max_{k:\bar{\gamma}_k < 0} (-\bar{\gamma}_k) \sum_{\bar{\gamma}_k < 0} U_k = \\ &-\frac{(1-2\mu)}{r^2} \min_{k:\bar{\gamma}_k < 0} \bar{\gamma}_k \sum_{\bar{\gamma}_k < 0} U_k \text{ i.e. } -\frac{(1-2\mu)}{r^2} \sum_{k=1}^n \bar{\gamma}_k U_k \leq -\frac{(1-2\mu)}{r^2} \min_{k:\bar{\gamma}_k < 0} \bar{\gamma}_k \sum_{\bar{\gamma}_k < 0} U_k, \end{aligned}$$

where $\bar{\gamma}_{min} = \min_{k:\bar{\gamma}_k < 0} \bar{\gamma}_k$. Hence, for convolution of criteria $\bar{J}'(\mu)$ ($\mu \in (0; 1)$) from condition (16) the following inequality is true

$$J'(\mu) \leq -\frac{(1-2\mu)}{r^2} \bar{\gamma}_{min} \sum_{\bar{\gamma}_k < 0} U_k + [\mu\gamma + (1-\mu)\rho] \sum_{k=1}^n U_{n+k}. \tag{20}$$

This inequality is similar to (18). In particular, since $\sum_{\bar{\gamma}_k < 0} U_k \leq \sum_{k=1}^n U_k$ then we obtain $\sum_{\bar{\gamma}_k < 0} U_k \leq \frac{\gamma r}{\bar{\beta}_{min}} \sum_{k=1}^n Q_k$ from (19). Then as it follows from the first restriction (15), (20) and

the last relation $J'(\mu) \leq \left\{ -\frac{(1-2\mu)\gamma\bar{\gamma}_{min}}{r\bar{\beta}_{min}} + \mu\gamma + (1-\mu)\rho \right\} \sum_{k=1}^n Q_k$ so we obtain the first part of expression (14) for $\mu \in (0; 1/2)$.

By combining results of parts 1), 2) and 3) we obtain (14), i.e. theorem 3 is proved.

Corollary 2. *If conditions (10), are satisfied then the solution of problems A and A' exists when $T \rightarrow +\infty$ and the following inequality takes place*

$$J^*(\mu) \leq \Gamma_1(\mu, z) \quad (\mu \in (0; 1)), \tag{21}$$

where

$$\Gamma_1(\mu, z) = \begin{cases} (1 - \mu)(\gamma + \rho) \sum_{k=1}^n Q_k(z) & (\mu \in (0; 1/2]), \\ \{\mu\gamma + (1 - \mu)\rho\} \sum_{k=1}^n Q_k(z) & (\mu \in (1/2; 1)). \end{cases} \quad (22)$$

Note that inequalities (11) and (21) are also correct for the finite planning horizon T according to Theorem 1.

This approach also allows us to evaluate both control and phase variables of problems A and A' (for example, the sales volume and the investment volume for specific types of products), i.e. to evaluate the investment attractiveness of the project.

Let us make some notes about the found estimates: 1. Estimate (9) and (14) obtained for a finite T are rough ones in comparison with (12) and (22) but to use relations (12) and (22) one needs to justify the convergence of the series corresponding to z -images of variables of the initial dynamic models (1) or (1), (2); 2. If estimates (9), (14) or (12), (22) are less than the value a decision-maker (a manufacturer or a tax centre) expects to obtain after the optimum implementation of IP then the proposed project is inefficient otherwise the project can be considered as potentially effective one; 3. If the project is unacceptable for a decision maker on unlimited planning horizon then, according to theorem 1, it is also unacceptable on a finite time interval. Thus, theorems 2 and 3 or consequences 1 and 2 allow us to classify investment projects implemented in accordance with MMPLP A and A' as potentially acceptable or unacceptable at the stage of their preliminary evaluation (without solving the above mentioned multi-step problems).

Theorems 2 and 3 give various analytical estimates of the same value — the optimal value of the project. Therefore to compare these estimates we consider the following example.

Example

This numerical example illustrates the evaluation of the optimal value of the IP considered in theorems 2 and 3, and in consequences 1 and 2. The initial model data used in this example for the problems A and $Z_T A'$ were obtained with the use of software [5]. They are presented in Tab. 1.

Table 1. The initial data for the problems A and $Z_T A'$

n	I_0	K_0	P_1	P_2	T_1	T_2	T	T^1	q_1	q_2	α_2	α_3	α_4	β	r
2	70	30	0.14	0.65	100	100	5	1	10	15	0.02	0.2	0.342	0.05	0.1

Fig. 1 presents the dependences of the optimal convolution values $J^*(\mu)$, $\bar{J}^{I*}(\mu)$, $J^*(\mu, z)$ in problems A , $Z_T A'$, ZA and estimates (14), (22), (9) and (12) on parameter $\mu \in [0; 1]$ (curves 1–7, respectively). As it was shown in [6] when $T \rightarrow +\infty$ the convolution curve $\bar{J}^{I*}(\mu)$ (dotted line 2) tends to convolution curve $J^*(\mu, z)$ (line 3) that is $J^*(\mu, z) = \lim_{T \rightarrow +\infty} \bar{J}^{I*}(\mu)$ and by construction

$$J^*(\mu) \leq J^*(\mu, z) \quad (\mu \in (0; 1)). \quad (23)$$

In its turn, it follows from (3) and (23) that

$$J^{I*}(\mu) \leq J^*(\mu, z) \quad (\mu \in (0; 1)). \quad (24)$$

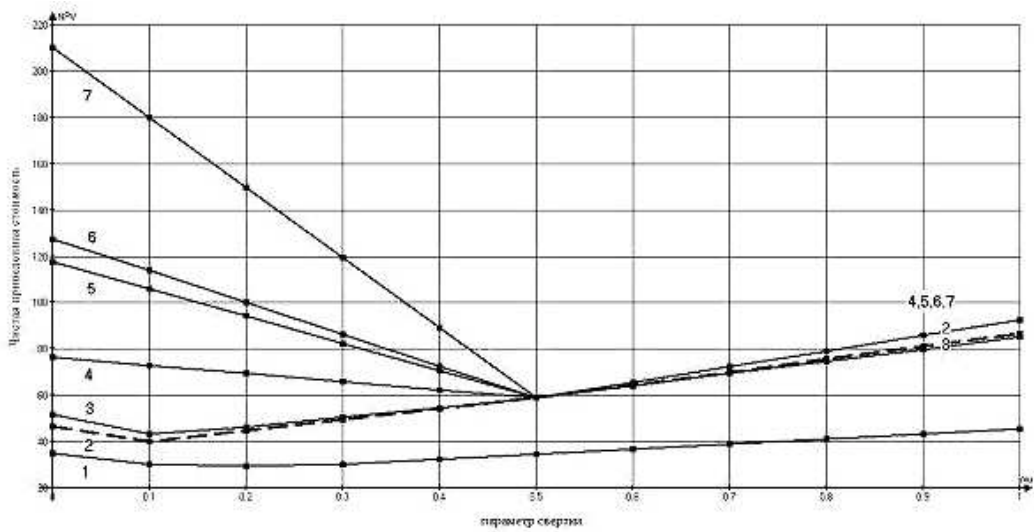


Fig. 1. The dependences of the optimal convolution values in problems A , $Z_T A'$, ZA and estimates (14), (22), (9) and (12) on parameter $\mu \in [0; 1]$

According to (3, curve 1 of the convolution $J^*(\mu)$ is the upper estimate of the optimal value of IP, implemented according to MMPLP (1), (2) and this estimate cannot be exceeded.

In addition, according to conditions (23) and (24) curve 3 is the upper estimate of the optimal value of IP not only in the problem A' but also in the problem A . In this case it is not necessary to solve these multi-step problems. The complexity of these problems is increased with the increase of the planning horizon and a number of products. Taking into account (8), (11), (13) and (21) the upper estimate of the projects value implemented by the models A or A' can be obtained without solving even the static problems $Z_T A'$ and ZA . In the example given above, curve 4 in Fig. 1 corresponds to the best estimate (14). However, under a different set of initial characteristics of the IP any of the estimates (9), (12) and (22) can be the best estimate.

The given example shows that to estimate the optimal value of IP described by the problem A' it is better to apply Theorem 3 other then Theorem 2.

Conclusion

The paper proposes a new approach for obtaining estimates of the optimal value of the projects of the economic system development. The approach is based on the use of the operator that is an analogue of the z -transform for the finite planning horizon. The obtained results allow one to exclude the inefficient projects (when the net present value is less than the value the investor or another participant of the project is expected to obtain) without solving numerically multi-step optimization problems (1) or (1) and (2). The complexity of these problems is increased with the increase of the planning horizon and a number of products. The obtained results allow one to enhance the quality of administrative decisions about selection of effective investment projects. The given above approach based on an operational calculus allows us to simplify the proof of MMPLP solvability and to obtain the upper estimate of the optimal value of IP. The proposed approach can be applied to problems of economic dynamics not only on micro-level [1,7] but also on meso-level [2] and macro-level [3]. Problems of economic dynamics are described in a class of multiobjective linear multi-step problems with discounted multipliers in the objective functions.

The described above dynamic models allow one to obtain optimal values of investment costs

for acquiring BPA and for manufacture of each type of product. The models allow one to perform a parametric analysis of such indicators as demand for manufactured products, price of the products, quantity of the products, productivity and cost of BPA. By raising the discount rate of IP the models also allow one to take into account the risks connected with the project. The risks arise due to inflation, changes in the investor requirements, cases when the demand for the product is uncertain or the maximum productivity of BPA is uncertain, etc.

The proposed operational approach to the analysis of problems of economic dynamics described in the MMPLP class generalizes the method [2] to the case of finite period of time.

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Приложения операционного подхода к оценке проектов развития экономических систем

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На базе оператора, являющегося аналогом z -преобразования для конечного интервала времени, предложен подход для получения аналитических оценок оптимальной стоимости инвестиционных проектов развития экономических систем, которые описываются многокритериальными многошаговыми задачами линейного программирования с дисконтирующими коэффициентами в целевых функциях. Найденные оценки позволяют лицу, принимающему решения, классифицировать рассматриваемый проект как заведомо неприемлемый или потенциально эффективный на стадии предварительного анализа.

Ключевые слова: развитие экономических систем, инвестиционный проект, многокритериальная многошаговая задача линейного программирования, операционный подход.