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On a Problem of Cross-Diffusion with Nonlocal Boundary Conditions

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Condition of global existence of solution of a non-linear system of cross-diffusion with non-linear boundary conditions is studied in the paper. Critical exponents of Fujita type and critical exponents of global existence of solution are established.

Keywords: nonlinear parabolic system, cross-diffusion, critical Fujita curve, blow-up.

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1. Introduction and preliminaries

Let us consider the following nonlinear system of cross-diffusion equations with nonlinear boundary conditions

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial x} \left(v^{m_1-1} \frac{\partial u}{\partial x} \right), \quad \frac{\partial v}{\partial t} = \frac{\partial}{\partial x} \left(u^{m_2-1} \frac{\partial v}{\partial x} \right), \quad x \in R_+, \quad t > 0, \quad (1)$$

$$-v^{m_1-1} \frac{\partial u}{\partial x} (0, t) = u^{q_1} (0, t), \quad -u^{m_2-1} \frac{\partial v}{\partial x} (0, t) = v^{q_2} (0, t), \quad t > 0, \quad (2)$$

$$u(x, 0) = u_0(x), \quad v(x, 0) = v_0(x), \quad x \in R_+, \quad (3)$$

where $m_i > 1$, $q_i > 0$ ($i = 1, 2$), u_0 and $v_0(x)$ are non-negative continuous functions with compact support in R_+ .

The process of cross-diffusion means that the spatial displacement of one object, described by one of the variables, occurs due to diffusion of another object described by another variable.

Cross-diffusion models are used in various fields of natural science. For example, in physical systems (plasma physics) [1–3], in chemical systems (dynamics of electrolytic solutions), in biological systems (cross-diffusion transport, dynamics of population), in ecology (dynamics of the forest structure), in seismology — the Burridge-Knopoff model which describes the interaction of tectonic plates [4–7]. In recent years, mathematical models with cross-diffusion have been actively used to study biological populations and the motion of tectonic plates [4, 5].

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It is known that a system of degenerate equations may not have a classical solution in the region where $u, v \equiv 0$. In this case, the generalized solution of system (1) is studied in a class where

$$u(x, t), v(x, t) \geq 0, \quad v^{m_1-1} \frac{\partial u}{\partial x}, \quad u^{m_2-1} \frac{\partial v}{\partial x} \in C(R \times (0, +\infty)),$$

and system (1) is satisfied from the point of distribution [1, 3].

In recent years, the condition for global existence of solutions and the condition for the emergence of a blow-up regime have been intensively studied (see [5–16]). The following system of equations was considered [8, 9]

$$u_t = u_{xx}, \quad v_t = v_{xx}, \quad x > 0, \quad 0 < T < \infty, \quad (4)$$

$$-u_x(0, t) = u^\alpha v^p, \quad -v_x(0, t) = u^q v^\beta, \quad 0 < t < T, \quad (5)$$

$$u(x, 0) = u_0(x), \quad v(x, 0) = v_0(x), \quad x > 0. \quad (6)$$

The conditions for global solvability in terms of time were studied and the estimation of the solution near explosion time was stated. It was proved that if $pq \leq (1 - \alpha)(1 - \beta)$ then each solution of problem (4)–(6) is global.

The following problem was studied [10]

$$u_t = (u^n)_{xx}, \quad v_t = (v^k)_{xx}, \quad x \in R_+, \quad t > 0, \quad (7)$$

$$-(u^n)_x(0, t) = v^p(0, t), \quad -(v^k)_x(0, t) = u^q(0, t), \quad t > 0, \quad (8)$$

$$u(x, 0) = u_0(x), \quad v(x, 0) = v_0(x), \quad x \in R_+. \quad (9)$$

It was shown that solution of problem (7)–(9) is global if $pq \leq (n + 1)(k + 1)/4$. Conditions were obtained for the numerical parameters of system (7)–(9) wherein the solution of the problem blows-up in a finite time.

System (7) was considered also with the following boundary conditions [11]

$$-(u^n)_x(0, t) = u^\alpha v^p(0, t), \quad -(v^k)_x(0, t) = u^q v^\beta(0, t), \quad t > 0.$$

It was shown that $\min\{y_1 - r_1, y_2 - r_2\} = 0$, where

$$r_1 = \frac{2p + k + 1 - 2\beta}{4pq - (k + 1 - 2\alpha)(n + 1 - 2\beta)}, \quad r_2 = \frac{2p + n + 1 - 2\beta}{4pq - (k + 1 - 2\alpha)(n + 1 - 2\beta)},$$

$$y_1 = \frac{1 - r_1(n - 1)}{2}, \quad y_2 = \frac{1 - r_2(k - 1)}{2},$$

are critical exponents of Fujita type.

Definition 1 ([1]). *The solution $\hat{u}(x, t)$, $\hat{v}(x, t)$ is called lower solution (upper solution) in $Q_T = \{(x, t) : x \in R_+, 0 < t < +\infty\}$ if $\hat{u}(x, t) = 0$ and $\hat{v}(x, t) = 0$ in $Q_T \setminus D_t$, $D_t = \{|x| < \rho(t)\} \times (0, +\infty)$, $\hat{u}(x, t), \hat{v}(x, t) \in C_{t,x}^{1,2}(D) \cap C(Q_T)$, $\hat{v}^{m_1-1} \frac{\partial \hat{u}}{\partial x}, \hat{u}^{m_2-1} \frac{\partial \hat{v}}{\partial x} \in C(Q_T)$ and it satisfies*

$$\frac{\partial \hat{u}}{\partial t} \leq (\geq) \frac{\partial}{\partial x} \left(\hat{v}^{m_1-1} \frac{\partial \hat{u}}{\partial x} \right), \quad \frac{\partial \hat{v}}{\partial t} \leq (\geq) \frac{\partial}{\partial x} \left(\hat{u}^{m_2-1} \frac{\partial \hat{v}}{\partial x} \right),$$

$$-\hat{v}^{m_1-1} \frac{\partial \hat{u}}{\partial x}(0, t) \leq (\geq) \hat{u}^{q_1}(0, t), \quad -\hat{u}^{m_2-1} \frac{\partial \hat{v}}{\partial x}(0, t) \leq (\geq) \hat{v}^{q_2}(0, t),$$

$$\hat{u}(x, 0) \leq (\geq) u_0(x), \quad \hat{v}(x, 0) \leq (\geq) v_0(x), \quad x \in R.$$

The purpose of this paper is to find conditions for existence of solutions of problem (1)–(3) in time on the basis of self-similar analysis. Various self-similar solutions of problem (1)–(3) were constructed, the assessment and asymptotic solutions were obtained, critical exponents of Fujita type and critical exponents of global existence of the solution were established.

2. Main results

Theorem 2.1. *If $q_1 \leq 1$, $q_2 \leq 1$ then each solution of problem (1)–(3) is global.*

Proof. Sufficient conditions for the solvability in time of problem (1)–(3) are determined by constructing its bounded upper solutions. The bounded upper solutions of the problem are taken in the following self-similar form

$$\begin{cases} \widehat{u}(x, t) = e^{L_1 t} (K + e^{-M_1 \xi_1}), & \xi_1 = x e^{-J_1 t}, \quad x \geq 0, \quad t \geq 0, \\ \widehat{v}(x, t) = e^{L_2 t} (K + e^{-M_2 \xi_2}), & \xi_2 = x e^{-J_2 t}, \quad x \geq 0, \quad t \geq 0, \end{cases} \quad (10)$$

where $K > \max\{\|u_0\|_\infty, \|v_0\|_\infty\}$, $M_1 = (K+1)^{\frac{q_1}{m_1-1}}$, $M_2 = (K+1)^{\frac{q_2}{m_2-1}}$, $L_1 = \frac{M_1(K+1)^{m_1-2}(M_1 K + M_2(m_1-1))}{K+1}$, $L_2 = \frac{M_2(K+1)^{m_2-2}(M_2 K + M_1(m_2-1))}{K+1}$, $J_1 = L_2(m_1-1)$, $J_2 = L_1(m_2-1)$.

Let us show that functions (10) are upper solutions of problem (1)–(3). According to the comparison principle of solutions [1, pp. 21–22], they must satisfy the following system of inequalities

$$\begin{cases} \frac{\partial \widehat{u}}{\partial t} \geq \frac{\partial}{\partial x} \left(\widehat{v}^{m_1-1} \frac{\partial \widehat{u}}{\partial x} \right), & x > 0, \quad t > 0, \\ \frac{\partial \widehat{v}}{\partial t} \geq \frac{\partial}{\partial x} \left(\widehat{u}^{m_2-1} \frac{\partial \widehat{v}}{\partial x} \right), & x > 0, \quad t > 0, \end{cases} \quad (11)$$

$$\begin{cases} -\widehat{v}^{m_1-1} \frac{\partial \widehat{u}}{\partial x} \Big|_{x=0} \geq \widehat{u}^{q_1}(0, t), & t > 0, \\ -\widehat{u}^{m_2-1} \frac{\partial \widehat{v}}{\partial x} \Big|_{x=0} \geq \widehat{v}^{q_2}(0, t), & t > 0. \end{cases} \quad (12)$$

Taking into account that

$$\begin{aligned} \widehat{u}_t &= L_1 e^{L_1 t} (K + e^{-M_1 \xi_1}) + M_1 J_1 x e^{(L_1 - J_1)t} \geq L_1 e^{L_1 t} (K + 1), \\ \widehat{v}^{m_1-1} \widehat{u}_x &= M_1 e^{(L_2(m_1-1) + L_1 - J_1)t} (K + e^{-M_2 \xi_2})^{m_1-1} e^{-M_1 \xi_1}, \\ \left(\widehat{v}^{m_1-1} \widehat{u}_x \right)_x &= M_1 e^{(L_2(m_1-1) + L_1 - J_1)t} (K + e^{-M_2 \xi_2})^{m_1-2} \times \\ &\quad \times \left(M_1 e^{-(M_1 \xi_1 + J_1 t)} (K + e^{-M_2 \xi_2}) + (m_1 - 1) M_2 e^{-J_2 t} \right) \leq \\ &\leq M_1 e^{(L_2(m_1-1) + L_1 - J_1)t} (K + 1)^{m_1-2} (M_1 (K + 1) + (m_1 - 1) M_2), \\ \widehat{v}_t &= L_2 e^{L_2 t} (K + e^{-M_2 \xi_2}) + M_2 J_2 x e^{(L_2 - J_2)t} \geq L_2 e^{L_2 t} (K + 1), \\ \widehat{u}^{m_2-1} \widehat{v}_x &= M_2 e^{(L_1(m_2-1) + L_2 - J_2)t} (K + e^{-M_1 \xi_1})^{m_2-1} e^{-M_2 \xi_2}, \end{aligned}$$

$$\begin{aligned}
 \left(\widehat{u}^{m_2-1} \widehat{v}_x \right)_x &= M_2 e^{(L_1(m_2-1)+L_2-J_2)t} (K + e^{-M_1 \xi_1})^{m_2-2} \times \\
 &\times \left(M_2 e^{-(M_2 \xi_2+J_2)t} (K + e^{-M_1 \xi_1}) + (m_2-1) M_1 e^{-J_1 t} \right) \leq \\
 &\leq M_2 e^{(L_1(m_2-1)+L_2-J_2)t} (K+1)^{m_2-2} (M_2 (K+1) + (m_2-1) M_1)
 \end{aligned}$$

and considering definitions of M_i , J_i , L_i ($i = 1, 2$) and K , it is easy to prove that at $q_1 \leq 1$, $q_2 \leq 1$ systems of inequalities (11) and (12) are always true. $\square$

Theorem 2.2. *Let $q_1 > 1$, $q_2 > 1$. Then each solution of problem (1)–(3) is unbounded at sufficiently large initial data.*

Proof. To prove Theorem 2.2 the solution of problem (1)–(3) is taken in the following self-similar form

$$\begin{cases} \underline{u}(x, t) = (T - t)^{-\alpha_1} \varphi(\xi), \\ \underline{v}(x, t) = (T - t)^{-\alpha_2} \phi(\xi), \quad \xi = x(T - t)^{-\beta}, \end{cases} \quad (13)$$

where $T > 0$, $\beta = \frac{q_1 - m_2}{2q_1 - m_2 - 1} = \frac{q_2 - m_1}{2q_2 - m_1 - 1}$, $\alpha_1 = \frac{1}{2q_1 - m_2 - 1}$, $\alpha_2 = \frac{1}{2q_2 - m_1 - 1}$. Functions $(\varphi(\xi), \phi(\xi))$ are solutions of the following problems

$$\begin{cases} \frac{d}{d\xi} \left(\phi^{m_1-1} \frac{d\varphi}{d\xi} \right) - \beta \xi \frac{d\varphi}{d\xi} - \alpha_1 \varphi = 0, \\ \frac{d}{d\xi} \left(\varphi^{m_2-1} \frac{d\phi}{d\xi} \right) - \beta \xi \frac{d\phi}{d\xi} - \alpha_2 \phi = 0, \end{cases} \quad (14)$$

$$\begin{cases} -\phi^{m_1-1} \frac{d\varphi}{d\xi} (0) = \varphi^{q_1} (0), \\ -\varphi^{m_2-1} \frac{d\phi}{d\xi} (0) = \phi^{q_2} (0), \end{cases} \quad (15)$$

which are obtained after substituting (13) into (1)–(3) and some simplifications. Conditions can be attained wherein solution (13) is an unbounded lower solution of problem (1)–(3). The following functions are compared

$$\begin{cases} \tilde{\varphi}(\xi) = A_1(a - \xi)^{\frac{1}{m_2-1}}, \\ \tilde{\phi}(\xi) = A_2(a - \xi)^{\frac{1}{m_1-1}}, \end{cases} \quad (16)$$

where $A_i > 0$ ($i = 1, 2$). Then in order to use the comparison theorem the following inequalities should be satisfied

$$\begin{cases} \frac{d}{d\xi} \left(\tilde{\phi}^{m_1-1} \frac{d\tilde{\varphi}}{d\xi} \right) - \beta \xi \frac{d\tilde{\varphi}}{d\xi} - \alpha_1 \tilde{\varphi} \geq 0, \\ \frac{d}{d\xi} \left(\tilde{\varphi}^{m_2-1} \frac{d\tilde{\phi}}{d\xi} \right) - \beta \xi \frac{d\tilde{\phi}}{d\xi} - \alpha_2 \tilde{\phi} \geq 0, \\ -\tilde{\phi}^{m_1-1} \frac{d\tilde{\varphi}}{d\xi} (0) \leq \tilde{\varphi}^{q_1} (0), \\ -\tilde{\varphi}^{m_2-1} \frac{d\tilde{\phi}}{d\xi} (0) \leq \tilde{\phi}^{q_2} (0). \end{cases}$$

This is equivalent to stating that the following inequalities

$$\begin{cases} \frac{A_2^{m_1-1}}{(m_1-1)^2} - \alpha_1 a + \left(\alpha_1 + \frac{\beta_1}{m_2-1} \right) \xi \geq 0, \\ \frac{A_1^{m_2-1}}{(m_2-1)^2} - \alpha_2 a + \left(\alpha_2 + \frac{\beta_2}{m_1-1} \right) \xi \geq 0, \end{cases}$$

$$\begin{cases} A_1 A_2^{m_1-1} a^{\frac{1}{m_2-1}} \leq A_1^{q_1} a^{\frac{q_1}{m_2-1}}, \\ A_2 A_1^{m_2-1} a^{\frac{1}{m_1-1}} \leq A_2^{q_2} a^{\frac{q_2}{m_1-1}} \end{cases}$$

are true. It is clear that the first systems of inequalities are true if the following sufficient conditions

$$a \leq \min \left\{ \frac{A_1^{m_1-1}}{\alpha_2(m_1-1)^2}, \frac{A_2^{m_2-1}}{\alpha_1(m_2-1)^2} \right\}, \quad q_1 > 1, \quad q_2 > 1$$

are satisfied.

From the second systems of inequalities the following restrictions on a are obtained

$$a \geq \max \left\{ \frac{A_1^{\frac{(m_1-1)(m_2-1)}{q_2-1}}}{(m_1-1)^{\frac{m_1-1}{q_2-1}} A_2^{m_1-1}}, \frac{A_2^{\frac{(m_1-1)(m_2-1)}{q_1-1}}}{(m_2-1)^{\frac{m_2-1}{q_1-1}} A_1^{m_2-1}} \right\}.$$

Then, choosing values of A_1, A_2, a , we can always achieve that the last system of inequalities are satisfied at $q_1 > 1, q_2 > 1$. According to the comparison principle of solutions, the lower self-similar solutions (13), (16) proved conditions for the initial data:

$$\begin{cases} u_0(x) \geq T^{-\alpha_1} A_1 (a - x^2 T^{-2\beta})_+^{\frac{1}{m_2-1}}, \\ v_0(x) \geq T^{-\alpha_2} A_2 (a - x^2 T^{-2\beta})_+^{\frac{1}{m_1-1}}. \end{cases}$$

Thus, by comparison we conclude that

$$\begin{cases} u(x, t) \geq (T-t)^{-\alpha_1} \tilde{\varphi}(0) \rightarrow \infty, \quad t \rightarrow T, \\ v(x, t) \geq (T-t)^{-\alpha_2} \tilde{\phi}(0) \rightarrow \infty, \quad t \rightarrow T, \end{cases}$$

at $q_1 > 1$ and $q_2 > 1$. It implies that u and v are unbounded. The theorem is proved.

Theorem 2.3. Suppose that $q_1 > m_2 + 1, q_2 > m_1 + 1$ and initial data are sufficiently small. Then each solution of problem (1)–(3) is global.

Proof. Conditions for the solvability in time of problem (1)–(3) are determined by constructing bounded upper solutions. They are taken in the following self-similar form:

$$\begin{cases} u_+(x, t) = (T+t)^{-\alpha_1} f(\xi) \\ v_+(x, t) = (T+t)^{-\alpha_2} g(\xi), \quad \xi = x(T+t)^{-\beta}, \end{cases} \quad (17)$$

where $T > 0$. Taking into account the comparison theorem of solutions, functions $(f(\xi), g(\xi))$ should satisfy the system of inequalities

$$\begin{cases} \frac{d}{d\xi} \left(g^{m_1-1} \frac{df}{d\xi} \right) + \beta \xi \frac{df}{d\xi} + \alpha_1 f \leq 0, \\ \frac{d}{d\xi} \left(f^{m_2-1} \frac{dg}{d\xi} \right) + \beta \xi \frac{dg}{d\xi} + \alpha_2 g \leq 0, \end{cases} \quad (18)$$

$$\begin{cases} -g^{m_1-1} \frac{df}{d\xi}(0) \geq f^{q_1}(0), \\ -f^{m_2-1} \frac{dg}{d\xi}(0) \geq g^{q_2}(0). \end{cases} \quad (19)$$

Let us consider the following functions

$$\begin{cases} \bar{f}(\xi) = A_1 \left(a - (\xi + h_1)^2 \right)^{\frac{1}{m_2-1}}, \\ \bar{g}(\xi) = A_2 \left(a - (\xi + h_2)^2 \right)^{\frac{1}{m_1-1}}, \end{cases} \quad (20)$$

where $h_1, h_2 \in (0, \sqrt{a})$, $a > 0$, $A_1 = \left(\frac{\beta(m_1-1)}{2} \right)^{\frac{1}{m_2-1}}$, $A_2 = \left(\frac{\beta(m_2-1)}{2} \right)^{\frac{1}{m_1-1}}$. Systems of inequalities (18), (19) are solvable with respect to unknowns a , h_1 and h_2 at $q_1 > m_2 + 1$, $q_2 > m_1 + 1$. Then, substituting functions (20) in (18) and (19), we obtain

$$\begin{cases} (\alpha_1 - \beta) \left(a - (\xi + h_1)^2 \right) \leq 0, \\ (\alpha_2 - \beta) \left(a - (\xi + h_2)^2 \right) \leq 0. \end{cases}$$

Then necessary conditions $q_1 > m_2 + 1$, $q_2 > m_1 + 1$ and conditions for numerical parameters a , h_1 , h_2

$$a \leq \min \left\{ h_1^2 + \left(A_2^{m_1-1} A_1^{1-q_1} \frac{2h_1}{m_2-1} \right)^{\frac{1}{q_1+m_2-2}}, h_2^2 + \left(A_1^{m_2-1} A_2^{1-q_2} \frac{2h_2}{m_1-1} \right)^{\frac{1}{q_2+m_1-2}} \right\} \quad (21)$$

are obtained. Thus, if $q_1 > m_2 + 1$, $q_2 > m_1 + 1$, and initial functions $u_0(x)$, $v_0(x)$ satisfy conditions

$$\begin{cases} u_0(x) \leq T^{-\alpha_1} A_1 \left(a - (xT^{-\beta} + h_1)^2 \right)^{\frac{1}{m_2-1}}, \\ v_0(x) \leq T^{-\alpha_2} A_2 \left(a - (xT^{-\beta} + h_2)^2 \right)^{\frac{1}{m_1-1}}, \end{cases}$$

where a , h_1 , h_2 are chosen from condition (21), then the solution of problem (1)–(3) is global.

Remark 1. Theorem 2.1 shows that critical exponents of global existence of solution are $q_{10} = 1$, $q_{20} = 1$.

Remark 2. Theorem 2.3 shows that critical exponents of Fujita type are $q_{1c} = m_2 + 1$, $q_{2c} = m_1 + 1$.

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Об одной задаче кросс-диффузии с нелокальными граничными условиями

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В работе изучается условие глобального существования и несуществования решения нелинейной системы кросс-диффузии с нелинейными граничными условиями. Установлены критические экспоненты типа Фуджита и критические экспоненты для глобального существования решения.

Ключевые слова: нелинейная параболическая система, кросс-диффузия, критические экспоненты типа Фуджита, режим с обострением.