УДК 517.9

Heterogeneous System MMPP/GI(2)/∞ with Random Customers Capacities

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Received 28.11.2018, received in revised form 14.01.2019, accepted 20.02.2019

A heterogeneous queuing system with an infinite number of servers is considered in this paper. Customers arrive in the system according to a Markov Modulated Poisson Process. The type of incoming customer is defined as i-type with probability $p_i\,(i = 1, 2)$. *Each customer carries a random quantity of work (capacity of the customer). In this study service time does not depend on the customers capacities. It is shown that the joint probability distribution of the customers number and total capacities in the system is multidimensional Gaussian distribution under the asymptotic condition of an infinitely growing service time. Simulation results allow us to determine an applicability area of the asymptotic result.*

Keywords: infinite-server queueing system, random capacity of customers, Markov Modulated Poisson Process.
DOI: 10.17516/1997-1397-2019-12-2-231-239.

Queueing systems represent a powerful mathematical tool for investigating the performance of a wide variety of real-life systems ranging from telecommunication networks to financial markets, from computer architectures to supply chain management and airplane traffic control. Analytical tractability of the corresponding models strongly depends on the nature of the underlying processes: mathematical model of arrival process, discipline of service, and on the system structure. A new trend in the study of resource queueing systems is the analysis of systems with non-Poisson arrivals and non-exponential service time. Although physical resources are always finite, quite often it is easier to study queueing systems in which it is assumed that the corresponding parameters can reach infinite values. The fundamentals of the theory of queueing systems for random-capacity customers can be found in [13, 14]. The single-server queueing systems for random-capacity customers with Poisson input flow were considered [15] where the servicing time is distributed exponentially and arbitrarily under the assumption that the customer capacity and

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the servicing time are independent, and the AQM mechanism is realized in them. Similar results were established for the infinite-server system with exponentially and arbitrary distributed servicing time [2, 9–11]. We note that the queueing system with random-capacity customers, processor sharing, and limited capacity of memory using specific algorithm was discussed in detail [1]. In this work we consider an infinite-server queueing system fed by non-Poisson arrivals with random customers capacities. Queues with random customers capacities are useful for analysis and design issues in high-performance computer and communication systems, in which service time and customer volume are independent quantities. For instance, performance analysis of LTE (Long Term Evolution) networks was carried out in terms of flow level dynamics and the amount of required radio resources does not depend on the duration of the flow [7]. Such queues are also important in modeling devices, where it is necessary to calculate a sufficient volume of buffer for data storing [7, 12]. A new trend in the study of queueing systems is the analysis of the systems with non-Poisson arrivals and non-exponential service time. The main contribution of this paper is to extend such analysis, focusing on the properties of the multidimensional process of the number of customers and the total capacity in the system when an infinite-server queue is fed by MMPP arrivals with random capacities and non-exponential service time distribution.

1. Matematical model

Let us consider the queueing system with unlimited number of servers of two different types [10, 11]. Each customer carries a random quantity of work (capacity of the customer) [2] (Fig.1).

Customers arrive in the system according to a Markov Modulated Poisson Process (MMPP). The input process is defined by its generator matrix $\mathbf{Q} = ||q_{ij}||$ of size $K \times K$ and by conditional rates $\lambda_1, \ldots, \lambda_K$ typically composed into the diagonal matrix $\mathbf{\Lambda} = \mathrm{diag}\{\lambda_1, \ldots, \lambda_K\}$. Let us denote the underlying Markov chain of the MMPP by $k(t) \in 1, 2, \ldots, K$. At the time of occurrence of the event in the MMPP-flow only one customer flows in the system. The type of incoming customer is defined as i-type with probability p_i $(i = 1, 2)$. It goes to the appropriate device type, where its service is performed during a random time with distribution function $B_i(x)$ $(i = 1, 2)$, according to the type of the customer. Let us assume that each customer of i-type has some random capacity $v_i > 0$ $(i = 1, 2)$ with distribution function $G_i(y)$ $(i = 1, 2)$.

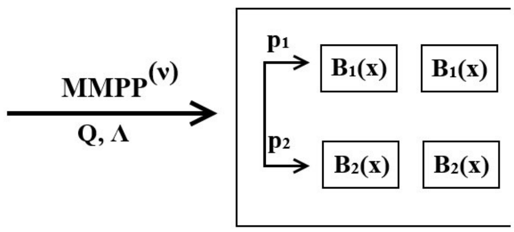

Fig. 1. Heterogeneous queue MMPP/GI/ ∞ with random customers capacities

Denote the number of each type‘s customers in the system at time t by $\{i_1(t), i_2(t)\}$ and their total capacity by $\{V_1(t), V_2(t)\}$. Four-dimensional stochastic process $\{i_1(t), i_2(t), V_1(t), V_2(t)\}$ is the goal of the study. This process is not Markovian, therefore, we use the dynamic screening method [6] for its investigation.

Let the system be empty at moment t_0, and let us fix some arbitrary moment T in the future. Consider three time axes that are numbered from 0 to 2 (Fig.2). Let axis 0 shows the epochs of customers' arrivals, while axes 1 and 2 correspond to two-dimensional screened process. Let $S_1(t)$ be a probability that the arriving customer generates a point on the first axis of the screened process and $S_2(t)$ be a probability that it generates a point on the second axis. The customer does not generate any points with probability $1 - S_1(t) - S_2(t)$. Probability $S_i(t)$ is equal to the probability that a customer of i-type ($i = 1,\ 2$) arriving at time t is serviced in the system at the moment T, $S_i(t) = 1 - B_i(T - t)$, $i = 1, 2$, for $t_0 \leqslant t \leqslant T$.

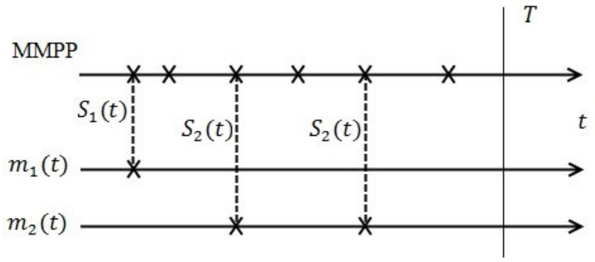

Fig. 2. Screening of the customers arrivals

Let us denote the number of arrivals screened before the moment t and their total capacity by $m_1(t), m_2(t)$ and $w_1(t), w_2(t)$,respectively. As it was shown [5], the probability distribution of the number of customers in the system at the moment T coincides with the probability distribution of the number of screened arrivals on the axis

$$P\{i_1(T) = m_1, i_2(T) = m_2\} = P\{m_1(T) = m_1, m_2(T) = m_2\}$$

for all $m = 0, 1, 2, \ldots$

It is easy to prove the same property for the extended process $\{i_1(t), i_2(t), V_1(t), V_2(t)\}$:

$$P\{i_1(T) = m_1, i_2(T) = m_2, V_1(T) < w_1, V_2(T) < w_2\} =$$
$$= P\{m_1(T) = m_1, m_2(T) = m_2, w_1(T) < w_1, w_2(T) < w_2\}$$

for $m = 0, 1, 2, \ldots$ and $z \geqslant 0$.

2. Kolmogorov differential equations

Let us consider the five-dimensional Markovian process $\{k(t), m_1(t), m_2(t), w_1(t), w_2(t)\}$. The probability distribution of this process is

$$P(k, m_1, m_2, w_1, w_2, t) = P\{k(t) = k, m_1(t) = m_1, m_2(t) = m_2, w_1(t) < w_1, w_2(t) < w_2\}.$$

Taking into account the formula of total probability, we can write the following system of Kolmogorov differential equations

$$\frac{\partial P(k, m_1, m_2, w_1, w_2, t)}{\partial t} = \lambda_k[(p_1(1 - S_1(t)) + p_2(1 - S_2(t)) - 1) P(k, m_1, m_2, w_1, w_2, t) +$$
$$+ p_1 S_1(t) \int_0^{w_1} P(k, m_1 - 1, m_2, w_1 - y, w_2, t) dG_1(y) +$$
$$+ p_2 S_2(t) \int_0^{w_2} P(k, m_1, m_2 - 1, w_1, w_2 - y, t) dG_2(y)] + \sum_v P(\nu, m_1, m_2, w_1, w_2, t) q_{\nu k}$$

for $k = 1, \ldots, K,\ m_i = 0, 1, 2, \ldots,\ w_i > 0,\ i = 1, 2.$

We introduce the partial characteristic function

$$h(k, u_1, u_2, z_1, z_2, t) = \sum_{m_1=0}^{\infty} \sum_{m_2=0}^{\infty} e^{ju_1 m_1} e^{ju_2 m_2} \int_0^{\infty} e^{jz_1 w_1} e^{jz_2 w_2} P(k, m_1, m_2, dw_1, dw_2, t),$$

where $j=\sqrt{-1}$ is the imaginary unit. Then we can write the following equations

$$\frac{\partial h(k,u_1,u_2,z_1,z_2,t)}{\partial t} = h(k,u_1,u_2,z_1,z_2,t)\lambda_k\big[p_1S_1(t)(e^{ju_1}G_1^*(z_1)-1)+$$
$$+p_2S_2(t)(e^{ju_2}G_2^*(z_2)-1)\big]+\sum_\nu h(\nu,u_1,u_2,z_1,z_2,t)q_{\nu k}$$

for $k=1,\dots,K$, where $G_i^*(z_i)=\int\limits_0^\infty e^{jz_iy}dG(y)\, i=1,2$.

Let us rewrite this system in the matrix form

$$\frac{\partial \mathbf{h}(u_1,u_2,z_1,z_2,t)}{\partial t} = \mathbf{h}(u_1,u_2,z_1,z_2,t)\left[\sum_{i=1}^{2}p_i(e^{ju_i}G_i^*(z_i)-1)S_i(t)\mathbf{\Lambda}+\mathbf{Q}\right] \tag{1}$$

with the initial condition

$$\mathbf{h}(u_1,u_2,z_1,z_2,t_0)=\mathbf{r}, \tag{2}$$

where $\mathbf{h}(u_1,u_2,z_1,z_2,t)=[h(1,u_1,u_2,z_1,z_2,t),\dots,h(K,u_1,u_2,z_1,z_2,t)]and\mathbf{r}=[r(1),\dots,r(K)]$ represents the stationary distribution of the underlying Markov chain, i.e., vector $\mathbf{r}$ satisfies the following linear system

$$\begin{cases}\mathbf{rQ}=\mathbf{0},\\ \mathbf{re}=1,\end{cases} \tag{3}$$

where $\mathbf{e}$ is a column vector with all entries equal to 1.

In general, the exact solution of equation (1) is not available but it may be found subject to asymptotic conditions. We consider in the paper the case of infinitely growing service time.

3. Asymptotic analysis

We formulate and prove the following statements.

Theorem 3.1. *The first-order asymptotic characteristic function of the probability distribution of the process $\{k(t),m_1(t),m_2(t),w_1(t),w_2(t)\}$ has the form*

$$\mathbf{h}^{(1)}(u_1,u_2,z_1,z_2,t)=\exp\left\{j\kappa_1\sum_{i=1}^{2}p_i(u_i+z_ia_{1i})b_i\right\},$$

where $\kappa_1=\mathbf{r\Lambda e}$, $b_i=\int\limits_0^\infty(1-B_i(x))dx$ and $a_{1i}=\int\limits_0^\infty ydG_i(y)$ is the mean customer capacity.

Proof. Let us perform substitutions

$$\frac{1}{b_i}=\varepsilon,\quad t\varepsilon=\tau,\quad S_i(t)=\widetilde{S}_i(\tau),\quad u_i=\varepsilon x_i,\quad z_i=\varepsilon y_i,$$
$$\mathbf{h}(u_1,u_2,z_1,z_2,t)=\mathbf{f}_1(x_1,x_2,y_1,y_2,\tau,\varepsilon).$$

Then problem (1) takes the form

$$\varepsilon\frac{\partial \mathbf{f}_1(x_1,x_2,y_1,y_2,\tau,\varepsilon)}{\partial\tau}=\mathbf{f}_1(x_1,x_2,y_1,y_2,\tau,\varepsilon)\left[\mathbf{Q}+\sum_{i=1}^{2}p_i(e^{j\varepsilon x_i}G_i^*(\varepsilon y_i)-1)\,\widetilde{S}_i(\tau)\Lambda\right], \tag{4}$$

with the initial condition

$$\mathbf{f}_1(x_1,x_2,y_1,y_2,\tau_0,\varepsilon)=\mathbf{r}. \tag{5}$$

Let us find the asymptotic solution (where $\varepsilon \to 0$,) of problem (4)–(5). Let $\varepsilon \to 0$ in (5). Then we obtain

$$\mathbf{f}_1(x_1, x_2, y_1, y_2, \tau)\mathbf{Q} = \mathbf{0}.$$

Comparing this equation with the first one in (3), we can conclude that $\mathbf{f}_1(x_1, x_2, y_1, y_2, \tau)$ can be expressed as

$$\mathbf{f}_1(x_1, x_2, y_1, y_2, \tau) = \mathbf{r}\Phi_1(x_1, x_2, y_1, y_2, \tau), \tag{6}$$

where $\Phi_1(x_1, x_2, y_1, y_2, \tau)$ is some scalar function that satisfies the condition

$$\Phi_1(x_1, x_2, y_1, y_2, \tau_0) = 1.$$

Let us multiply (4) by vector $\mathbf{e}$, substitute (6), divide the result by ε and assume that $\varepsilon \to 0$. Then, taking into account (4), we obtain the following differential equation for $\Phi_1(x_1, x_2, y_1, y_2, \tau)$

$$\frac{\partial \Phi_1(x_1, x_2, y_1, y_2, \tau)}{\partial \tau} = j\kappa_1 \Phi_1(x_1, x_2, y_1, y_2, \tau) \sum_{i=1}^{2} p_i \, \widetilde{S}_i(\tau)(x_i + y_i a_{1i}).$$

Taking into account the initial condition, we obtain the solution of this equation

$$\Phi_1(x_1, x_2, y_1, y_2, \tau) = \exp\left\{ j\kappa_1 \sum_{i=1}^{2} p_i(x_i + y_i a_{1i}) \int_{\tau_0}^{\tau} \widetilde{S}_i(z)dz \right\}.$$

Using substitutions, we can write first-order asymptotic characteristic function of the probability distribution of the process $\{k(t), m_1(t), m_2(t), w_1(t), w_2(t)\}$

$$\mathbf{h}^{(1)}(u_1, u_2, z_1, z_2, t) = \exp\left\{ j\kappa_1 \sum_{i=1}^{2} p_i(u_i + z_i a_{1i}) b_i \right\}.$$

The theorem is proved.

The main result of the paper is the following theorem.

Theorem 3.2. *The second-order asymptotic characteristic function of the probability distribution of the process* $\{k(t), m_1(t), m_2(t), w_1(t), w_2(t)\}$ *is*

$$\mathbf{h}^{(2)}(u_1, u_2, z_1, z_2, t) = \exp\left\{ j\kappa_1 \sum_{i=1}^{2} p_i(u_i + z_i a_{1i}) b_i + \frac{j^2}{2}\kappa_1 \sum_{i=1}^{2} p_i(u_i^2 + z_i^2 a_{2i}) b_i + \right.$$
$$\left. + j^2\kappa_1 \sum_{i=1}^{2} p_i u_i z_i a_{1i}^2 b_i + j^2\kappa_2 \left[\sum_{i=1}^{2} p_i^2 (u_i + z_i a_{1i}) b_{2i} + \sum_{i=1}^{2} \sum_{g=1, g\neq i}^{2} p_i p_g (u_i + z_i a_{1i})(u_g + z_g a_{1g}) b_i b_g \right] \right\},$$

where $\kappa_2 = \mathbf{d}(\mathbf{\Lambda} - \kappa_1 \mathbf{I})\mathbf{e}$, $a_{2i} = \int\limits_0^\infty y^2 dG_i(y)$, $b_{2i} = \int\limits_0^\infty (1 - B_i(x))^2 dx$ *and the row vector* $\mathbf{d}$ *satisfies the linear matrix system*

$$\begin{cases} \mathbf{dQ} = \mathbf{r}(\kappa_1 \mathbf{I} - \mathbf{\Lambda}), \\ \mathbf{de} = 0. \end{cases}$$

Proof. Let us represent the original characteristic function $\mathbf{h}(u_1, u_2, z_1, z_2, t)$ in the form

$$\mathbf{h}(u_1, u_2, z_1, z_2, t) = \mathbf{h}_2(u_1, u_2, z_1, z_2, t) \exp\left\{ j\kappa_1 \sum_{i=1}^{2} p_i(u_i + z_i a_{1i}) \int_{\tau_0}^{\tau} \widetilde{S}_i(z)dz \right\},$$

where $\mathbf{h}_2(u_1, u_2, z_1, z_2, t)$ is some vector function to be defined.

Substituting this expression into (1)–(2), we obtain

$$\begin{gathered}\frac{\partial \mathbf{h}_2(u_1, u_2, z_1, z_2, t)}{\partial t} = \mathbf{h}_2(u_1, u_2, z_1, z_2, t)\Big[\mathbf{Q} + \sum_{i=1}^{2} p_i \left(\exp^{ju_i} G_i^*(z_i) - 1\right) \widetilde{S}_i(t)\mathbf{\Lambda} - \\ -j\kappa_1 \sum_{i=1}^{2} p_i(u_i + z_i a_{1i})\, \widetilde{S}_i(t)\mathbf{I}\Big], \\ \mathbf{h}_2(u_1, u_2, z_1, z_2, t_0) = \mathbf{r}.\end{gathered} \tag{7}$$

Let us use the following substitutions

$$\frac{1}{q_i b} = \varepsilon^2, \quad b_i = q_i b, \quad t\varepsilon^2 = \tau, \quad S_i(t) = \widetilde{S}_i(\tau), \quad u_i = \varepsilon x_i, \quad z_i = \varepsilon y_i,$$
$$\mathbf{h}_2(u_1, u_2, z_1, z_2, t) = \mathbf{f}_2(x_1, x_2, y_1, y_2, \tau, \varepsilon),$$

where $b_i = \int\limits_0^\infty (1 - B_i(x))dx$, $i = 1, 2$. Using these notations, problem (7) can be rewritten in the form

$$\begin{gathered}\varepsilon^2 \frac{\mathbf{f}_2(x_1, x_2, y_1, y_2, \tau, \varepsilon)}{\partial \tau} = \mathbf{f}_2(x_1, x_2, y_1, y_2, \tau, \varepsilon)\Big[\mathbf{Q} + \sum_{i=1}^{2} p_i \left(e^{ju_i} G^*(z_i) - 1\right) \widetilde{S}_i(t)\Lambda - \\ -j\kappa_1 \sum_{i=1}^{2} p_i(u_i + z_i a_{1i})\, \widetilde{S}_i(t)\mathbf{I}\Big].\end{gathered} \tag{8}$$

Let us find the asymptotic solution (where $\varepsilon \to 0$) of the problem. Let $\varepsilon \to 0$. Then we obtain

$$\mathbf{f}_2(x_1, x_2, y_1, y_2, \tau)\mathbf{Q} = \mathbf{0}.$$

Comparing this equation with the first one in (3), we can conclude that $\mathbf{f}_2(x_1, x_2, y_1, y_2, \tau)$ can be expressed as

$$\mathbf{f}_2(x_1, x_2, y_1, y_2, \tau) = \mathbf{r}\Phi_2(x_1, x_2, y_1, y_2, \tau),$$

where $\Phi_2(x_1, x_2, y_1, y_2, \tau)$ is some scalar function that satisfies the condition

$$\Phi_2(x_1, x_2, y_1, y_2, \tau_0) = 1.$$

So function $\mathbf{f}_2(x_1, x_2, y_1, y_2, \tau, \varepsilon)$ can be represented in the expansion form

$$\mathbf{f}_2(x_1, x_2, y_1, y_2, \tau, \varepsilon) = \Phi_2(x_1, x_2, y_1, y_2, \tau)\Big[\mathbf{r} + \sum_{i=1}^{2} p_i \left(j\varepsilon x_i + j\varepsilon y_i a_{1i}\right) \widetilde{S}_i(\tau)\mathbf{d}\Big] + O(\varepsilon^2). \tag{9}$$

Then substituting this expansion into equation (8), we obtain

$$\begin{gathered}O(\varepsilon^2) = \Phi_2(x_1, x_2, y_1, y_2, \tau)\Big[\mathbf{rQ} + \sum_{i=1}^{2} p_i \left(j\varepsilon x_i + j\varepsilon y_i a_{1i}\right) \mathbf{r}\left(\mathbf{\Lambda} - \kappa_1 \mathbf{I}\right) \widetilde{S}_i(\tau) + \\ + \sum_{i=1}^{2} p_i \left(j\varepsilon x_i + j\varepsilon y_i a_{1i}\right) \widetilde{S}_i(\tau)\mathbf{dQ}\Big],\end{gathered}$$

where $\mathbf{d}$ is some row vector that satisfies the following system

$$\begin{cases} \mathbf{dQ} + \mathbf{r}\left(\Lambda - \kappa_1 \mathbf{I}\right) = 0, \\ \mathbf{de} = 0. \end{cases}$$

We multiply equation (8) by vector $\mathbf{e}$ and assume that $\varepsilon \to 0$. The solution of the latter equation with the available initial condition $\Phi_2(x_1, x_2, y_1, y_2, \tau) = 1$ gives

$$\Phi_2(x_1, x_2, y_1, y_2, \tau) = \exp\left\{\frac{j^2}{2}\kappa_1 \sum_{i=1}^{2} p_i({x_i}^2 + a_{2i}{y_i}^2 + 2x_i y_i {a_{1i}}^2)\int_{\tau_0}^{\tau} \tilde{S}_i(z)dz + \right.$$
$$\left. +j^2\kappa_2\left[\sum_{i=1}^{2} {p_i}^2(x_i + y_i a_{1i})^2 \int_{\tau_0}^{\tau} \tilde{S}_i^2(z)dz + \sum_{i=1}^{2}\sum_{\substack{g=1\\ g\neq i}}^{2} p_i p_g (x_i + y_i a_{1i})(x_g + y_g a_{1g}) \int_{\tau_0}^{\tau} \tilde{S}_i(z) S_g(z)dz\right]\right\}.$$

Performing inverse substitutions, we obtain the following expression for the asymptotic characteristic function of the process $\{k(t), m_1(t), m_2(t), w_1(t), w_2(t)\}$:

$$\mathbf{h}^{(2)}(u_1, u_2, z_1, z_2, t) = \exp\left\{j\kappa_1 \sum_{i=1}^{2} p_i(u_i + z_i a_{1i})b_i + \frac{j^2}{2}\kappa_1 \sum_{i=1}^{2} p_i(u_i^2 + z_i^2 a_{2i})b_2 i + \right.$$
$$\left. +j^2\kappa_1 \sum_{i=1}^{2} p_i u_i z_i a_{1i}^2 b_i + j\kappa_2\left[\sum_{i=1}^{2} p_i^2(u_i + z_i a_{1i})b_i + \sum_{i=1}^{2}\sum_{\substack{g=1\\ g\neq i}}^{2} p_i p_g (u_i + z_i a_{1i})(u_g + z_g a_{1g}) b_i b_g\right]\right\}.$$

The theorem is proved. □

It is clear from the form of characteristic function that the four-dimensional process is asymptotically Gaussian.

4. Numerical example

Result of Theorem 3.2 is obtained subject to the asymptotic condition $b_i \to \infty$ ($\varepsilon \to 0$). Therefore, it may be used just as an approximation when b_i is large enough. To test its practical applicability, we considered several numerical examples with various system parameters (including the distributions of the service time and of the customer capacity). Since all simulation sets led to similar results, for sake of brevity, we discuss in detail just one of them. In particular, we assume that the input MMPP is characterized by the following matrices

$$\mathbf{Q} = \begin{bmatrix} -11 & 5 & 6 \\ 0.5 & -1 & 0.5 \\ 2.5 & 2.5 & -5 \end{bmatrix}, \quad \Lambda = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1.5 & 0 \\ 0 & 0 & 2 \end{bmatrix}.$$

The type of incoming customer is defined as i-type with probabilities $p_1 = 0.4$, $p_2 = 0.6$. Volumes of customers has exponential distribution with the rate equal to 2 for type 1 and with the rate equal to 1 for type 2. Service time has gamma distribution with shape and inverse scale parameters $a_1 = 1.5$ and $a_2 = 0.5$, $b_1 = a_1/N$, $b_2 = a_2/N$, $N = 1, 10, 15, 20, 50, 100$.

Our goal is to find a lower bound of parameter N to obtain applicable approximation. To find it, we carried out series of simulations for increasing values of N and compared the asymptotic distributions with empiric ones by using the Kolmogorov distance

$$\Delta = \sup_x |F(x) - A(x)|$$

as an accuracy measure. Here $F(x)$ is the cumulative distribution function build on the basis of simulation results, and $A(x)$ is the Gaussian approximation based on Theorem 2. Values of the Kolmogorov distance for the total capacity of different types of customers for various values of parameter N are presented in Tab. 1.

Table 1. Kolmogorov distances Δ_1, Δ_2 between simulation results and asymptotic values for the total capacity of customers of type 1 and 2

N	1	10	15	20	50	100
Δ_1	0.243	0.091	0.032	**0.021**	**0.017**	**0.012**
Δ_2	0.363	0.034	**0.025**	**0.019**	**0.013**	**0.010**

One can notice that asymptotic results become more accurate when parameter N increases. If we suppose that an approximation is applicable when its Kolmogorov distance is less than 0.03, then we can conclude that asymptotic results are applicable when N is about 15 or more (marked by boldface in the tables).

Conclusion

The queue with MMPP arrivals, infinite number of servers and non-exponential service time is considered in the paper. Moreover, random capacities of customers independent of their service time are taken into account. The analysis is performed subject to the asymptotic condition of an infinitely growing service time. It was shown that multidimensional probability distribution of the number of customers and total capacity in the system is four-dimensional Gaussian distribution subject to this asymptotic condition. Numerical results show that asymptotic results are sufficiently accurate when scale parameter of service time N is greater than 15.

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Ресурсные неоднородные СМО MMPP/GI(2)/∞

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Рассматривается неоднородная бесконечнолинейная система массового обслуживания. На вход системы поступает MMPP-поток требований случайного объема, время обслуживания требований не зависит от их объёма. Требования, поступившие в систему, с вероятностью p_i ($i = 1, 2$) определяются как требования типа i. Показано, что совместное распределение вероятностей числа занятых приборов и суммарного объема занятого ресурса в системе является многомерным гауссовским при асимптотическом условии эквивалентного роста времени обслуживания на приборах разного типа.Результаты моделирования позволяют определить область применимости асимптотического метода.

Ключевые слова: бесконечнолинейные СМО, случайный объём требований, марковский модулированный пуассоновский процесс.