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On the Existence of Solution of Some Problems for Nonlinear Loaded Parabolic Equations with Cauchy Data

Igor V. Frolenkov*
Maria A. Darzhaa†

Institute of Mathematics and Computer Science,
Siberian Federal University,
Svobodny, 79, Krasnoyarsk, 660041

Russia

Two problems are considered in this paper. First problem is the Cauchy problem for a two-dimensional loaded parabolic equation with coefficients dependent on unknown function and its derivatives. Second problem is the Cauchy problem for one-dimensional equation of the Burgers-type. The sufficient conditions of the existence of solutions of these problems in classes of smooth bounded functions are presented in the paper. The method of weak approximation is used for the purpose of obtaining the proof.

Keywords: inverse problem, direct problem, loaded equation, parabolic equation, equation of the Burgers-type, method of weak approximation.

Introduction

This paper is devoted to an attempt to generalize the method of studying solvability of broad class of auxiliary direct problems for one- and two-dimensional coefficient inverse problems for parabolic equations in unbounded domains with Cauchy data.

Two problems are constructed in this work: special type of the loaded (containing traces of unknown function and its derivatives) two-dimensional parabolic equation and one-dimensional equation of the Burgers-type.

The solution existence of the Cauchy problems for the mentioned above equations was investigated. Coefficient inverse problems with Cauchy data can be reduced to these auxiliary direct problems with the use of overdetermination conditions (some additional information on the solution) assigned at fixed hyperplanes or hypersurfaces. Examples of such methods of studying of inverse problems can be found in [1]. There are also other approaches that reduce an inverse problem to non-linear unloaded equation or to integro-differential equation.

It is necessary to know under what conditions the auxiliary problems are solvable. It is also necessary to know the properties of solutions. The sufficient conditions for the existence of solutions of the problems are obtained in this paper. The method of weak approximation is used to prove the existence of solutions of the given problems. This method is also known as the method of splitting on differential level [2, 3].

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*igor@frolenkov.ru
†darma17@mail.ru

1.   On the special form of two-dimensional loaded semilinear parabolic equation

Let us choose r different points a_k, $k = \overline{1, r}$ of variable x defined on space E_1. We also choose s different points $z = \beta_m$, $m = \overline{1, s}$ of variables z defined on space E_1.

Let us consider in the strip $G_{[0,T]} = \{(t, x, z) | 0 \leqslant t \leqslant T, x \in E_1, z \in E_1\}$ the Cauchy problem for loaded (containing traces of unknown function and its derivatives) non-classical parabolic equation

$$\frac{\partial}{\partial t} u(t, x, z) = a_1(t, x, \overline{w}_0(t)) u_{xx} + a_2(t, z, \overline{w}_0(t)) u_{zz} +$$

$$+ b_1(t, x, z, \overline{w}_0(t)) u_x + b_2(t, x, z, \overline{w}_0(t)) u_z + f(t, x, z, u, \overline{w}_0(t), \overline{w}_1(t, x), \overline{w}_2(t, z)), \quad (1)$$

$$u(0, x, z) = u_0(x, z). \quad (2)$$

The components of vector–function

$$\overline{w}_0(t) = \left(u(t, \alpha_k, \beta_m), \frac{\partial^{j_1 + j_2}}{\partial x^{j_1} \partial z^{j_2}} u(t, \alpha_k, \beta_m) \right), \quad k = \overline{1, r}, \ m = \overline{1, s},$$

$$j_1 = 0, 1, \ldots, p_1, \ j_2 = 0, 1, \ldots, q_1,$$

are traces of function $u(t, x, z)$ and all its derivatives with respect to x up to order p_1 and with respect to z up to order q_1. All traces depend only on variable t.

The vector–function

$$\overline{w}_1(t, x) = \left(u(t, x, \beta_m), \frac{\partial^j}{\partial z^j} u(t, x, \beta_m) \right), \quad m = \overline{1, s}, \ j = 0, 1, \ldots, q_1,$$

consists of the traces of function $u(t, x, z)$ and all its derivatives with respect to z up to order q_1. All traces depend only on variables t and x.

Similarly, the vector–function

$$\overline{w}_2(t, z) = \left(u(t, \alpha_k, z), \frac{\partial^j}{\partial x^j} u(t, \alpha_k, z) \right), \quad k = \overline{1, r}, \ j = 0, 1, \ldots, p_1,$$

consists of the traces of function $u(t, x, z)$ and all its derivatives with respect to x up to order p_1. All traces depend only on variables t and z.

Let us consider a simple example. The following inverse problem for the heat equation is reduced to the direct problem of type (1), (2).

We have the following equation in domain $G_{[0,T]} = \{(t, x, z) \mid 0 < t < T, \ x \in \mathbb{R}, \ z \in \mathbb{R}\}$

$$u_t(t, x, z) = u_{xx}(t, x, z) + u_{zz}(t, x, z) + \lambda(t, x) f(t, x, z), \quad (3)$$

with initial data

$$u(0, x, z) = u_0(x, z), \quad (x, z) \in \mathbb{R}^2. \quad (4)$$

Coefficient $\lambda(t, x)$ should be determined simultaneously with the solution $u(t, x, z)$ of problem (3), (4). The solution satisfies the overdetermination condition

$$u(t, x, \gamma(t)) = \varphi(t, x), \quad 0 \leqslant t \leqslant T, \quad x \in \mathbb{R}. \quad (5)$$

Let the consistency conditions be fulfilled

$$u_0(x, \gamma(0)) = \varphi(0, x), \quad x \in \mathbb{R}.$$

We assume that all input data for the problem are real-valued functions. The functions and all necessary derivatives of these functions are sufficiently smooth and bounded in $G_{[0,T]}$.

Let the following condition be true

$$|f(t, x, \gamma(t))| \geqslant \delta > 0, \quad 0 \leqslant t \leqslant T, \quad x \in \mathbb{R}.$$

The problem (3)–(5) is reduced to the auxiliary direct problem

$$u_t(t, x, z) = u_{xx}(t, x, z) + u_{zz}(t, x, z) + \frac{\psi(t, x) - u_z(t, x, \gamma(t))\gamma'(t) - u_{zz}(t, x, \gamma(t))}{f(t, x, \gamma(t))} f(t, x, z), \quad (6)$$

$$u(0, x, z) = u_0(x, z), \quad (x, z) \in \mathbb{R}^2, \tag{7}$$

where $\psi(t, x) = \varphi'_t(t, x) - \varphi''_{xx}(t, x)$ is the known function.

In this example, in direct problem (6), (7) the functions $a_1(t, x, \overline{w}_0(t))$, $a_2(t, z, \overline{w}_0(t))$, $b_1(t, x, z, \overline{w}_0(t))$, $b_2(t, x, z, \overline{w}_0(t))$ and $f(t, x, z, u, \overline{w}_0(t), \overline{w}_1(t, x), \overline{w}_2(t, z))$ from equation (1) have the following forms:

$$a_1(t, x, \overline{w}_0(t)) = a_2(t, z, \overline{w}_0(t)) = 1,$$

$$b_1(t, x, z, \overline{w}_0(t)) = b_2(t, x, z, \overline{w}_0(t)) = 0,$$

$$f(t, x, z, u, \overline{w}_0(t), \overline{w}_1(t, x), \overline{w}_2(t, z)) = \frac{\psi(t, x) - u_z(t, x, \gamma(t))\gamma'(t) - u_{zz}(t, x, \gamma(t))}{f(t, x, \gamma(t))} f(t, x, z).$$

In what follows we assume that $p \geqslant \max\{2, p_1\}$, $q \geqslant \max\{2, q_1\}$.

Definition 1.1. $Z_{x,z}^{p,q}([0, t^*])$ *denotes the set of functions* $u(t, x, z)$ *that are defined in* $G_{[0,t^*]}$ *and belong to the class*

$$C_{t,x,z}^{1,p,q}(G_{[0,t^*]}) = \left\{ u(t, x, z) \mid \frac{\partial u}{\partial t}, \frac{\partial^{j_1 + j_2} u}{\partial x^{j_1} \partial z^{j_2}} \in C(G_{[0,t^*]}), \ j_1 = \overline{0, p}, \ j_2 = \overline{0, q} \right\},$$

that is, functions and all their derivatives appearing in equation (1) are bounded at $(t, x, z) \in G_{[0,t^*]}$

$$\sum_{j_1=0}^{p} \sum_{j_2=0}^{q} \left| \frac{\partial^{j_1 + j_2}}{\partial x^{j_1} \partial z^{j_2}} u(t, x, z) \right| \leqslant C.$$

Definition 1.2. *A classical solution of problem* (1), (2) *in* $G_{[0,t^*]}$ *is the function* $u(t, x, z) \in Z_{x,z}^{p,q}([0, t^*])$ *which satisfies* (1), (2) *in* $G_{[0,t^*]}$.

Here $0 < t^* \leqslant T$ is a fixed constant. If t^* depends on the constants that bounds the input data and $t^* \leqslant T$ then $u(t, x, z)$ is a solution of problem (1), (2) on a small time interval. If $t^* = T$ for any set of input data that satisfies the condition of solvability then $u(t, x, z)$ is a solution of problem (1), (2) in the whole time interval (or we will use the term "global solvability").

Suppose that the following conditions are true.

Condition 1.1. *The functions* a_1, a_2, b_1, b_2 *are real-valued functions that are defined for all values of their arguments and they are continuous functions. The functions* a_1, a_2 *satisfy conditions* $a_1 \geqslant a_0 > 0$, $a_2 \geqslant a_0 > 0$. *For any* $t_1 \in (0, T]$ *and any function* $u(t, x, z) \in Z_{x,z}^{p+2,q+2}([0, t_1])$ *these functions, as functions of variables* $(t, x, z) \in G_{[0,t_1]}$, *are continuous and they have continuous derivatives that enter into the following inequality*

$$\sum_{j_1=0}^{p+2} \left| \frac{\partial^{j_1}}{\partial x^{j_1}} a_1\big(t, x, \overline{w}_0(t)\big) \right| + \sum_{j_2=0}^{q+2} \left| \frac{\partial^{j_2}}{\partial z^{j_2}} a_2\big(t, z, \overline{w}_0(t)\big) \right| +$$

$$+ \sum_{j_1=0}^{p+2} \sum_{j_2=0}^{q+2} \left(\left| \frac{\partial^{j_1+j_2}}{\partial x^{j_1} \partial z^{j_2}} b_1\big(t, x, z, \overline{w}_0(t)\big) \right| + \left| \frac{\partial^{j_1+j_2}}{\partial x^{j_1} \partial z^{j_2}} b_2\big(t, x, z, \overline{w}_0(t)\big) \right| \right) \leqslant P_{\gamma_1}\left(U(t) \right); \quad (8)$$

Condition 1.2. *The function u_0 is a real-valued function that satisfies the following inequality*

$$\sum_{j_1=0}^{p+2} \sum_{j_2=0}^{q+2} \left| \frac{\partial^{j_1+j_2}}{\partial x^{j_1} \partial z^{j_2}} u_0(x, z) \right| \leqslant C.$$

The function has continuous derivatives that enter into the inequality.

Condition 1.3. *The function f is a real-valued function that is defined for all values of its arguments and it is continuous function. For all $t_1 \in (0, T]$ and any function $u(t, x, z) \in Z_{x,z}^{p+2,q+2}([0, t_1])$ this function, as function of variables $(t, x, z) \in G_{[0,t_1]}$, is continuous and it has continuous derivatives that enter into the following inequality*

$$\sum_{j_1=0}^{p+2} \sum_{j_2=0}^{q+2} \left| \frac{\partial^{j_1+j_2}}{\partial x^{j_1} \partial z^{j_2}} f\big(t, x, z, u, \overline{w}_0(t), \overline{w}_1(t, x), \overline{w}_2(t, z)\big) \right| \leqslant P_{\gamma_2}\left(U(t) \right). \quad (9)$$

In conditions 1.1 и 1.3, γ_1, $\gamma_2 \geqslant 0$ are some fixed integers,

$$P_\zeta(y) = \tilde{C}(1 + y + \cdots + y^\zeta),$$

$\tilde{C} > 1$ is a constant that is independent of the function $u(t, x, z)$ and its derivatives,

$$U(t) = \sum_{j_1=0}^{p+2} \sum_{j_2=0}^{q+2} \sup_{0 < \xi \leqslant t} \sup_{(x,z) \in E_2} \left| \frac{\partial^{j_1+j_2}}{\partial x^{j_1} \partial z^{j_2}} u(\xi, x, z) \right|, \;\; u(t, x, z) \in Z_{x,z}^{p+2,q+2}([0, t_1]).$$

The following theorem is proved in [6].

Theorem 1.1. *Let us assume that conditions 1.1–1.3 are fulfilled.*

1a. If in equation (1) the coefficients a_i, b_i are independent of the space variables:

$$a_1 = a_1(t, \overline{w}_0(t)), \;\; a_2 = a_2(t, \overline{w}_0(t)), \;\; b_1 = b_1(t, \overline{w}_0(t)), \;\; b_2 = b_2(t, \overline{w}_0(t)),$$

and conditions 1.1, 1.3 are fulfilled for $\gamma_1 \geqslant 0$, $0 \leqslant \gamma_2 \leqslant 1$ then the classical solution $u(t, x, z)$ of problem (1), (2) exists in class $Z_{x,z}^{p,q}([0, T])$.

1b. If the coefficients a_i, b_i have the same form as in the case 1a and conditions 1.1, 1.3 are fulfilled for $\gamma_1 \geqslant 0$, $\gamma_2 > 1$ then there is a such constant t^*, $0 < t^* \leqslant T$ dependent on the constant $\tilde{C}$ from (8), (9) that the classical solution $u(t, x, z)$ of problem (1), (2) exists in class $Z_{x,z}^{p,q}([0, t^*])$.

2a. If in equation (1) the coefficients a_i, b_i have the forms:

$$a_1 = a_1(t, x, \overline{w}_0(t)), \;\; a_2 = a_2(t, z, \overline{w}_0(t)),$$

$$b_1 = b_1(t, x, z, \overline{w}_0(t)), \;\; b_2 = b_2(t, x, z, \overline{w}_0(t)),$$

and conditions 1.1, 1.3 are fulfilled for $\gamma_1 = 0$, $0 \leqslant \gamma_2 \leqslant 1$ then the classical solution $u(t, x, z)$ of problem (1), (2) exists in class $Z_{x,z}^{p,q}([0, T])$.

2b. If the coefficients a_i, b_i have the same forms as in the case 2a and conditions 1.1, 1.3 are fulfilled for $\gamma_1 = 0$ but $\gamma_2 > 1$ then there is a such constant t^*, $0 < t^* \leqslant T$, dependent on the constant $\tilde{C}$ from (8), (9) that the classical solution $u(t,x,z)$ of problem (1), (2) exists in class $Z_{x,z}^{p,q}([0,t^*])$.

The fulfillment of the conditions of Theorem 1.1 can be proved for the given above example (direct problem (6), (7)), assuming that input data are sufficiently smooth and bounded functions. For example, the conditions of Theorem 1.1 are fulfilled for the constants $p = q = 4$, $\gamma_1 = 0$ and $\gamma_2 = 1$. Hence, the classical solution $u(t,x,z)$ of problem (6), (7) exists in the class $Z_{x,z}^{4,4}([0,T])$.

2.   On one-dimensional loaded Burgers type equation of the special form

Let us consider the proof of a similar result for the one-dimensional Burgers-type equation. In this equation the coefficient of the first order derivative with respect to space variable depends on the solution and its traces of the specified form.

Let us choose r different points $\alpha_1, \ldots, \alpha_r$ in space E_1.

The following Cauchy problem is considered in the strip $G_{[0,T]} = \{(t,x)|0 \leqslant t \leqslant T, x \in E_1\}$:

$$u_t = a(t)u_{xx} + b(t,x,u(t,x),\omega(t))u_x + f(t,x,u(t,x),\omega(t)), \tag{10}$$

$$u(0,x) = u_0(x), \tag{11}$$

here $\omega(t) = (u(t,\alpha_k), \dfrac{\partial^j}{\partial x^j}u(t,\alpha_k))$, $j = \overline{0,p_1}$, $k = \overline{1,r}$ is a vector–function. The components of this function are the traces of function $u(t,x)$ and all its derivatives with respect to x up to order p_1. The traces depend only on variable t.

Definition 2.1. *$Z_x^p([0,t^*])$ denotes the set of functions $u(t,x)$ defined in $G_{[0,t^*]}$ and they belong to the class*

$$C_{t,x}^{1,p}(G_{[0,t^*]}) = \left\{u(t,x)\,|\,\frac{\partial u}{\partial t}, \frac{\partial^j u}{\partial x^j} \in C(G_{[0,t^*]}),\ j = \overline{0,p}\right\}$$

The functions are bounded at $(t,x) \in G_{[0,t^*]}$ together with the following derivatives

$$\sum_{j=0}^{p}\left|\frac{\partial^j}{\partial x^j}u(t,x)\right| \leqslant C,$$

and $p \geqslant \max\{2,p_1\}$.

Definition 2.2. *A classical solution of problem (10), (11) in $G_{[0,t^*]}$ is the function $u(t,x) \in Z_x^p([0,t^*])$ which satisfies (10) in $G_{[0,t^*]}$. Here $0 < t^* \leqslant T$ is a fixed constant dependent on the input data.*

Suppose that the following conditions are fulfilled.

Condition 2.1. *The functions $b(t,x,u(t,x),\omega(t))$, $f(t,x,u(t,x),\omega(t))$, $u_0(x)$ are real-valued functions that are defined and continuous for all values of their arguments. For all $t_1 \in (0,T]$ and for any $u(t,x) \in Z_x^{p+2}([0,t_1])$ these functions, as functions of variables $(t,x) \in G_{[0,t_1]}$, are continuous and they have continuous derivatives that enter into inequalities (12), (13). The function $a(t) \geqslant a_0 > 0$ is a continuous bounded function on the interval $[0,T]$. The function $u_0(x)$ has continuous derivatives and satisfies the following inequalitie*

$$\sum_{j=0}^{p+2}\left|\frac{d^j}{dx^j}u_0(x)\right|\leqslant\widetilde{C}.$$

Condition 2.2. *Let us introduce the following notations*

$$U_j(0)=\sup_x\left|\frac{d^j}{dx^j}u_0(x)\right|,\ \ j=0,1,\ldots,p+2,$$

$$U_j(t)=\sup_{0<\xi\leqslant t}\left|\frac{\partial^j}{\partial x^j}u(t,x)\right|,\ \ j=0,1,\ldots,p+2,$$

$$U(0)=\sum_{j=0}^{p+2}U_j(0),\quad U(t)=\sum_{j=0}^{p+2}U_j(t).$$

Let us assume that for all $t_1\in(0,T]$, for all $t\in[0,t_1]$ and for any function $u(t,x)\in Z_x^{p+2}([0,t_1])$ the following estimates are hold:

$$\sum_{j=0}^{p+2}\left|\frac{\partial^j}{\partial x^j}b(t,x,u(t,x),\omega(t))\right|\leqslant P_{\gamma_1}(U(t)),\tag{12}$$

$$\sum_{j=0}^{p+2}\left|\frac{\partial^j}{\partial x^j}f(t,x,u(t,x),\omega(t))\right|\leqslant P_{\gamma_2}(U(t)),\tag{13}$$

where $\gamma_1,\ \gamma_2\geqslant0$ *are some fixed integers,* $P_\zeta(y)=\widetilde{C}(1+y+y^2+\ldots+y^\zeta)$ *and* $\widetilde{C}\geqslant1$ *is some constant independent of the function* $u(t,x)$ *and its derivatives.*

Theorem 2.1. *Assume that conditions 2.1 and 2.2 are fulfilled for* $\gamma_1\geqslant0$ *and* $0\leqslant\gamma_2\leqslant1$. *Then a constant* t^*, $0<t^*\leqslant T$ *exists and it depends on the constants* a_0 *and* $\tilde{C}$ *from condition 2.1 and inequalities* (12), (13), *such that the classical solution* $u(t,x)$ *of problem* (10), (11) *exists in class* $Z_x^p([0,t^*])$.

Proof. To prove the existence of a solution of the Cauchy problem (10), (11) we use the method of weak approximation. The original problem is split into three fractional steps on differential level and time shift by $\left(t-\dfrac{\tau}{3}\right)$ is done in the traces of unknown functions and in nonlinear terms:

$$u_t^\tau(t,x)=3a(t)u_{xx}^\tau(t,x),\ \ n\tau<t\leqslant\left(n+\frac{1}{3}\right)\tau;\tag{14}$$

$$u_t^\tau(t,x)=3b\left(t-\frac{\tau}{3},x,u^\tau\left(t-\frac{\tau}{3},x\right),\omega^\tau\left(t-\frac{\tau}{3}\right)\right)u_x^\tau(t,x),\ \ \left(n+\frac{1}{3}\right)\tau<t\leqslant\left(n+\frac{2}{3}\right)\tau;\tag{15}$$

$$u_t^\tau(t,x)=3f\left(t-\frac{\tau}{3},x,u^\tau\left(t-\frac{\tau}{3},x\right),\omega^\tau\left(t-\frac{\tau}{3}\right)\right),\ \ \left(n+\frac{2}{3}\right)\tau<t\leqslant(n+1)\tau;\tag{16}$$

$$u^\tau(0,x)=u_0(x).\tag{17}$$

Let us prove a priori estimates that ensure compactness of the family of solutions $u(t,x)$ of problem (14)–(17) in the class $C_{t,x}^{1,p}(G_{[0,t^*]})$ for some constant $0<t^*\leqslant T$.

Let us introduce the following notations

$$U_j(0)=\sup_x\left|\frac{d^j}{dx^j}u_0(x)\right|,\ \ j=0,1,\ldots,p+2,\tag{18}$$

$$U_j^\tau(t) = \sup_{n\tau < \xi \leqslant t} \left| \frac{\partial^j}{\partial x^j} u(t,x) \right|, \quad t \in (n\tau, (n+1)\tau], \quad j = 0, 1, \ldots, p+2, \tag{19}$$

$$U(0) = \sum_{j=0}^{p+2} U_j(0), \quad U^\tau(t) = \sum_{j=0}^{p+2} U_j^\tau(t). \tag{20}$$

Consider the first fractional step when $n = 0$. On the interval $0 < t \leqslant \dfrac{\tau}{3}$ we have the Cauchy problem (14), (17).

According to the maximum principle we obtain

$$|u^\tau(t,x)| \leqslant \sup_x |u_0(x)|.$$

Upon differentiating problem (14) j times with respect to x, $j = 0, 1, \ldots, p+2$ we obtain

$$\frac{\partial^j}{\partial x^j} u_t^\tau(t,x) = 3a(t) \frac{\partial^j}{\partial x^j} u_{xx}^\tau(t,x),$$

$$\frac{\partial^j}{\partial x^j} u^\tau(0,x) = \frac{d^j}{dx^j} u_0(x).$$

Then, according to the maximum principle, we have the following estimate

$$\left| \frac{\partial^j}{\partial x^j} u^\tau(t,x) \right| \leqslant \sup_x \left| \frac{\partial^j}{\partial x^j} u_0(x) \right|.$$

Taking into account (18)–(20), we obtain

$$U^\tau(t) \leqslant U(0), \quad 0 < t \leqslant \frac{\tau}{3}. \tag{21}$$

Consider the second fractional step when $\dfrac{\tau}{3} < t \leqslant \dfrac{2\tau}{3}$. Then, due to the time shift, the equation (15), is a linear one-dimensional homogeneous partial differential equation.

In this case, the first characteristic equation ([5], п. 2.6) is $t'(\sigma) = 1$. The solution of this equation can be taken in the form $t = \sigma$. The second characteristic equation can be written as

$$\frac{dx}{dt} = -3b\left(t - \frac{\tau}{3}, x, u^\tau\left(t - \frac{\tau}{3}, x\right), \omega^\tau\left(t - \frac{\tau}{3}\right)\right). \tag{22}$$

Considering the assumptions of the theorem and properties of the solution obtained in the first fractional step, assume that $\varphi^\tau(t, \xi, \eta)$ is a characteristic function of equation (22), i.e. $x = \varphi^\tau(t, \xi, \eta)$ is the integral curve of the equation that goes through the point (ξ, η).

Initial data for equation (15) can be written in parametric form as $t = \dfrac{\tau}{3}$, $x = \eta$, $u^\tau = u^\tau\left(\dfrac{\tau}{3}, \eta\right)$ (the function $u^\tau\left(\dfrac{\tau}{3}, \eta\right)$ is taken from the previous fractional step). The solution to this problem exists and can be represented in parametric form

$$u^\tau(t,x) = u^\tau\left(\frac{\tau}{3}, \eta\right), \quad x = \varphi^\tau\left(t, \frac{\tau}{3}, \eta\right),$$

or in the form

$$u^\tau(t,x) = u^\tau\left(\frac{\tau}{3}, \varphi^\tau\left(\frac{\tau}{3}, t, x\right)\right).$$

Hence it follows that

$$U_0^\tau(t) \leqslant U_0^\tau\left(\frac{\tau}{3}\right) \leqslant U(0), \quad \frac{\tau}{3} < t \leqslant \frac{2\tau}{3}. \tag{23}$$

Let us differentiate equation (15) with respect to x and introduce the following notations $z^\tau(t,x) = u_x^\tau(t,x)$,

$$b_0^\tau(t,x) = 3b\left(t - \frac{\tau}{3}, x, u^\tau\left(t - \frac{\tau}{3}, x\right), \omega^\tau\left(t - \frac{\tau}{3}\right)\right),$$

$$b_1^\tau(t,x) = 3\frac{\partial}{\partial x}\left(b\left(t - \frac{\tau}{3}, x, u^\tau\left(t - \frac{\tau}{3}, x\right), \omega^\tau\left(t - \frac{\tau}{3}\right)\right)\right).$$

Then we obtain equation

$$z_t^\tau(t,x) = b_0^\tau(t,x)z_x^\tau + b_1^\tau(t,x)z^\tau.$$

The solution of this equation for $t \in \left(\frac{\tau}{3}, \frac{2\tau}{3}\right]$ can be written in the parametric form ([5], p. 43)

$$z^\tau(t,x) = e^{-F_0^\tau\left(t, \frac{\tau}{3}, \eta\right)}z^\tau\left(\frac{\tau}{3}, \eta\right), \quad x = \varphi^\tau\left(t, \frac{\tau}{3}, \eta\right),$$

where

$$F_0^\tau = F_0^\tau(t, \xi, \eta) = -\int_\xi^t b_1^\tau(t, \varphi^\tau(t, \xi, \eta))\, dt,$$

and $x = \varphi^\tau(t, \xi, \eta)$ is the characteristic function of the equation

$$\frac{dx}{dt} = -b_0^\tau(t,x) = -3b\left(t - \frac{\tau}{3}, x, u^\tau\left(t - \frac{\tau}{3}, x\right), \omega^\tau\left(t - \frac{\tau}{3}\right)\right),$$

i.e. it is the integral curve of the equation passing through the point (ξ, η).

Taking into account that conditions 2.1–2.2 are fulfilled and taking also into account estimate (23) and notations (18)–(20), we obtain

$$U_1^\tau(t) \leqslant U_1^\tau\left(\frac{\tau}{3}\right)\exp\left(P_{\gamma_1}\left(U^\tau\left(t - \frac{\tau}{3}\right)\right)\tau\right) \leqslant U_1^\tau\left(\frac{\tau}{3}\right)e^{P_{\gamma_1}(U(0))\tau}, \tag{24}$$

where $P_\zeta(y) = \widetilde{C}(1 + y + y^2 + \ldots + y^\zeta)$, $\widetilde{C} \geqslant 1$ is polynomial from condition 2.2.

Let us differentiate equation (15) twice with respect to x and introduce the following notation $v^\tau(t,x) = u_{xx}^\tau(t,x)$,

$$c_0^\tau(t,x) = 3b\left(t - \frac{\tau}{3}, x, u^\tau\left(t - \frac{\tau}{3}, x\right), \omega^\tau\left(t - \frac{\tau}{3}\right)\right),$$

$$c_1^\tau(t,x) = 6\frac{\partial}{\partial x}\left(b\left(t - \frac{\tau}{3}, x, u^\tau\left(t - \frac{\tau}{3}, x\right), \omega^\tau\left(t - \frac{\tau}{3}\right)\right)\right),$$

$$c_2^\tau(t,x) = 3\frac{\partial^2}{\partial x^2}\left(b\left(t - \frac{\tau}{3}, x, u^\tau\left(t - \frac{\tau}{3}, x\right), \omega^\tau\left(t - \frac{\tau}{3}\right)\right)\right),$$

Then we obtain equation

$$v_t^\tau(t,x) = c_0^\tau(t,x)v_x^\tau(t,x) + c_1^\tau(t,x)v^\tau(t,x) + c_2^\tau(t,x)u_x^\tau(t,x). \tag{25}$$

The solution of this equation for $t \in \left(\frac{\tau}{3}, \frac{2\tau}{3}\right]$ can be written in the following parametric form ([5], p. 43)

$$v^\tau(t,x) = e^{-G_0^\tau\left(t, \frac{\tau}{3}, \eta\right)}\left(v^\tau\left(\frac{\tau}{3}, \eta\right) + \int_{\frac{\tau}{3}}^t c_2^\tau\left(t, \varphi^\tau\left(t, \frac{\tau}{3}, \eta\right)\right)u_x^\tau\left(t, \varphi^\tau\left(t, \frac{\tau}{3}, \eta\right)\right)e^{G_0^\tau\left(t, \frac{\tau}{3}, \eta\right)}\, dt\right),$$

$$x = \varphi^\tau\left(t, \frac{\tau}{3}, \eta\right),$$

where $G_0^\tau = G_0^\tau(t, \xi, \eta) = -\int_\xi^t c_1^\tau(t, \varphi^\tau(t, \xi, \eta)) \, dt$.

Taking into account that conditions 2.1–2.2 are fulfilled and taking into account estimate (23) and notations (18)–(20), we obtain for $\frac{\tau}{3} < t \leqslant \frac{2\tau}{3}$ the following inequality

$$U_2^\tau(t) \leqslant e^{2P_{\gamma_1}(U(0))\tau} \left(U_2^\tau\left(\frac{\tau}{3}\right) + 3P_{\gamma_1}(U(0))e^{2P_{\gamma_1}(U(0))\tau} \int_{\frac{\tau}{3}}^t U_1^\tau(t) \, dt \right) \leqslant$$

$$\leqslant e^{C_1 P_{\gamma_1}(U(0))\tau} \left(U_2^\tau\left(\frac{\tau}{3}\right) + C_1 P_{\gamma_1}(U(0)) \int_{\frac{\tau}{3}}^t U_1^\tau(t) \, dt \right). \quad (26)$$

Here and further $C_j > 1$ are constants (generally they are different) independent of the parameter τ.

It follows from (23) and (24) that

$$U_2^\tau(t) \leqslant e^{C_2 P_{\gamma_1}(U(0))\tau} \left(U_2^\tau\left(\frac{\tau}{3}\right) + C_2 P_{\gamma_1}(U(0))U_1^\tau\left(\frac{\tau}{3}\right) e^{P_{\gamma_1}(U(0))\tau}\tau \right),$$

$$U_2^\tau(t) \leqslant e^{C_3 P_{\gamma_1}(U(0))\tau} \left(U_2^\tau\left(\frac{\tau}{3}\right) + C_3\tau P_{\gamma_1}(U(0))U_1^\tau\left(\frac{\tau}{3}\right) \right), \quad (27)$$

$$U_2^\tau(t) \leqslant \left(U_1^\tau\left(\frac{\tau}{3}\right) + U_2^\tau\left(\frac{\tau}{3}\right) \right) e^{C_4 P_{\gamma_1}(U(0))\tau} (1 + C_4\tau P_{\gamma_1}(U(0))) \leqslant$$

$$\leqslant \left(U_1^\tau\left(\frac{\tau}{3}\right) + U_2^\tau\left(\frac{\tau}{3}\right) \right) e^{C_4 P_{\gamma_1}(U(0))\tau} e^{C_4 P_{\gamma_1}(U(0))\tau} \leqslant \left(U_1^\tau\left(\frac{\tau}{3}\right) + U_2^\tau\left(\frac{\tau}{3}\right) \right) e^{C_5 P_{\gamma_1}(U(0))\tau}. \quad (28)$$

If we differentiate equation (15) j times with respect to x, $j = 3, 4, \ldots, p+2$ and use the Leibnitz formula for the j-th derivative of the product of two functions then we obtain $w_{j_t}^\tau(t, x) = d_0^\tau(t, x)w_{j_x}^\tau + d_1^\tau(t, x)w_j^\tau + \sum_{k=2}^j d_k^\tau(t, x)w_{j-k+1}^\tau(t, x)$, where

$$d_0^\tau(t, x) = 3b\left(t - \frac{\tau}{3}, x, u^\tau\left(t - \frac{\tau}{3}, x\right), \omega^\tau\left(t - \frac{\tau}{3}\right) \right),$$

$$d_k^\tau(t, x) = 3C_j^k \frac{\partial^k}{\partial x^k} b\left(t - \frac{\tau}{3}, x, u^\tau\left(t - \frac{\tau}{3}, x\right), \omega^\tau\left(t - \frac{\tau}{3}\right) \right).$$

By the arguments used to obtain equation (25) we arrive to the following inequality

$$U_j^\tau(t) \leqslant e^{C_6 P_{\gamma_1}(U(0))\tau} \left(U_j^\tau\left(\frac{\tau}{3}\right) + C_6 P_{\gamma_1}(U(0)) \int_{\frac{\tau}{3}}^t \sum_{k=1}^{j-1} U_k^\tau(t) \, dt \right), \quad j = 3, 4, \ldots, p+2. \quad (29)$$

It follows from (24), (26) and (29) that

$$U_3^\tau(t) \leqslant e^{C_6 P_{\gamma_1}(U(0))\tau} \left(U_3^\tau\left(\frac{\tau}{3}\right) + C_6 P_{\gamma_1}(U(0)) \int_{\frac{\tau}{3}}^t U_1^\tau(t) + U_2^\tau(t) \, dt \right) \leqslant e^{C_6 P_{\gamma_1}(U(0))\tau} \times$$

$$\times \left(U_3^\tau\left(\frac{\tau}{3}\right) + C_6\tau P_{\gamma_1}(U(0)) \left(U_1^\tau\left(\frac{\tau}{3}\right) e^{P_{\gamma_1}(U(0))\tau} + \left(U_1^\tau\left(\frac{\tau}{3}\right) + U_2^\tau\left(\frac{\tau}{3}\right) \right) e^{C_5 P_{\gamma_1}(U(0))\tau} \right) \right) \leqslant$$

$$\leqslant e^{C_7 P_{\gamma_1}(U(0))\tau} \left(U_3^\tau\left(\frac{\tau}{3}\right) + C_7\tau P_{\gamma_1}(U(0)) \left(U_1^\tau\left(\frac{\tau}{3}\right) + U_2^\tau\left(\frac{\tau}{3}\right) \right) \right). \quad (30)$$

Hence we have
$$U_3^\tau(t) \leqslant \left(U_1^\tau\left(\frac{\tau}{3}\right) + U_2^\tau\left(\frac{\tau}{3}\right) + U_3^\tau\left(\frac{\tau}{3}\right) \right) e^{C_8 P_{\gamma_1}(U(0))\tau}. \quad (31)$$

We continue our arguments for $j = 4, \ldots, p+2$ and obtain

$$
U_j^\tau(t) \leqslant e^{C_6 P_{\gamma_1}(U(0))\tau} \left(U_j^\tau\left(\frac{\tau}{3}\right) + C_6 P_{\gamma_1}(U(0)) \int_{\frac{\tau}{3}}^t \sum_{k=1}^{j-1} U_k^\tau(t)\, dt \right) \leqslant e^{C_6 P_{\gamma_1}(U(0))\tau} \times
$$

$$
\times \left(U_j^\tau\left(\frac{\tau}{3}\right) + C_6 \tau P_{\gamma_1}(U(0)) \left(\sum_{k=1}^{j-1} \left(\sum_{l=1}^k U_l^\tau\left(\frac{\tau}{3}\right) \right) e^{A_{lk} P_{\gamma_1}(U(0))\tau} \right) \right) \leqslant
$$

$$
\leqslant e^{B_j P_{\gamma_1}(U(0))\tau} \left(U_j^\tau\left(\frac{\tau}{3}\right) + B_j \tau P_{\gamma_1}(U(0)) \left(\sum_{k=1}^{j-1} U_j^\tau\left(\frac{\tau}{3}\right) \right) \right), \quad (32)
$$

Hence we have

$$
U_j^\tau(t) \leqslant \left(\sum_{k=1}^j U_j^\tau\left(\frac{\tau}{3}\right) \right) e^{D_j P_{\gamma_1}(U(0))\tau}, \tag{33}
$$

here A_{lk}, B_j, D_j are positive constants independent of the parameter τ.

Now we add up inequalities (23), (27) (30) and (32) for $p = 4, \ldots, p+2$. Taking into account notations (18)–(20), we obtain

$$
U^\tau(t) \leqslant e^{C_9 P_{\gamma_1}(U(0))\tau} \left(U^\tau\left(\frac{\tau}{3}\right) + C_9 \tau P_{\gamma_1}(U(0)) U^\tau\left(\frac{\tau}{3}\right) \right) \leqslant U^\tau\left(\frac{\tau}{3}\right) e^{C_{10} P_{\gamma_1}(U(0))\tau}.
$$

On the third fractional step $\frac{2\tau}{3} < t \leqslant \tau$ we consider equation (16) with initial data at the point $\frac{2\tau}{3}$ (the value of function $u^\tau\left(\frac{2\tau}{3}, x\right)$ from the previous fractional step). Upon integrating equation (16) with respect to the time, we get

$$
u^\tau(t, x) = u^\tau\left(\frac{2\tau}{3}, x\right) + 3 \int_{\frac{2\tau}{3}}^t f\left(\eta - \frac{\tau}{3}, x, u^\tau\left(\eta - \frac{\tau}{3}, x\right), \omega^\tau\left(\eta - \frac{\tau}{3}\right) \right) d\eta.
$$

Taking into account this equation, the Theorem condition 2.2 and the previously introduced notations, we arrive to the following inequality

$$
U_0^\tau(t) \leqslant U_0^\tau\left(\frac{2\tau}{3}\right) + C_{11}\tau\left(1 + U_0^\tau\left(\frac{2\tau}{3}\right)\right). \tag{34}
$$

Let us differentiate (16) j times with respect to x, $j = 1, 2, \ldots, p+2$. Taking into account condition 2.2 and the notations (18)–(20), we obtain the following estimates

$$
U_1^\tau(t) \leqslant U_1^\tau\left(\frac{2\tau}{3}\right) + C_{12}\tau\left(1 + U_1^\tau\left(\frac{2\tau}{3}\right)\right), \tag{35}
$$

$$
U_2^\tau(t) \leqslant U_2^\tau\left(\frac{2\tau}{3}\right) + C_{13}\tau\left(1 + U_2^\tau\left(\frac{2\tau}{3}\right)\right). \tag{36}
$$

After combining estimates (34), (35) and (36) on the third fractional step, we obtain

$$
U^\tau(t) \leqslant U^\tau\left(\frac{2\tau}{3}\right) + C_{14}\tau\left(1 + U^\tau\left(\frac{2\tau}{3}\right)\right). \tag{37}
$$

Now considering relation (21), (28) and (37) on the time interval $t \in (0, \tau]$ we obtain the following estimate (constant $C > 0$ does not depend on τ)

$$U^\tau(t) \leqslant U(0)e^{\tau C_{15} P_{\gamma_1}(U(0))} + C_{15}\tau(1 + U(0)e^{\tau C_{15} P_{\gamma_1}(U(0))}) \leqslant$$
$$\leqslant U(0)e^{\tau C_{15} P_{\gamma_1}(U(0))} + 1 - 1 + C_{15}\tau(1 + U(0)e^{\tau C_{15} P_{\gamma_1}(U(0))}) \leqslant$$
$$\leqslant (U(0)e^{\tau C_{15} P_{\gamma_1}(U(0))} + 1)(C_{15}\tau + 1) - 1 \leqslant (U(0) + 1)e^{\tau C_{15} P_{\gamma_1}(U(0))}e^{C_{15}\tau} - 1 \leqslant$$
$$\leqslant (U(0) + 1)e^{\tau C P_{\gamma_1}(U(0)+1)} - 1. \quad (38)$$

Consider the next whole step ($n = 1$). By the given above arguments we obtain the estimate (because constant C does not depend on τ we change $U(0)$ for $U^\tau(\tau)$)

$$U^\tau(t) \leqslant (U^\tau(\tau) + 1)e^{\tau C P_{\gamma_1}(U^\tau(\tau)+1)} - 1, \ \tau < t \leqslant 2\tau. \quad (39)$$

It follows from (38) that $U^\tau(\tau) \leqslant (U(0) + 1)e^{\tau C P_{\gamma_1}(U(0)+1)} - 1$.

Considering this inequality, estimate (39) takes the form

$$U^\tau(t) \leqslant ((U(0) + 1)e^{\tau C P_{\gamma_1}(U(0)+1)} - 1 + 1) \times$$
$$\times \exp\left(\tau C P_{\gamma_1}\left((U(0) + 1)e^{\tau C P_{\gamma_1}(U(0)+1)}\right)\right) - 1 \leqslant$$
$$\leqslant (U(0) + 1)e^{\tau C P_{\gamma_1}(U(0)+1)} \exp\left(\tau C P_{\gamma_1}(U(0) + 1)e^{\tau \gamma_1 C P_{\gamma_1}(U(0)+1)}\right) - 1. \quad (40)$$

Let us take some constant t^* that satisfies the inequality

$$e^{2t^* C \gamma_1 P_{\gamma_1}(U(0)+1)} \leqslant 2. \quad (41)$$

Constant t^* depends on the input data and does not depend on τ. It follow from inequality (41) that

$$e^{(2i-1)\tau C \gamma_1 P_{\gamma_1}(U(0)+1)} \leqslant 2, \ i = \overline{1, k}, \ k\tau = t^*. \quad (42)$$

Taking into consideration (42), we rewrite estimate (40) $U^\tau(t) \leqslant (U(0) + 1)e^{\tau C P_{\gamma_1}(U(0)+1)}$ $e^{2\tau C P_{\gamma_1}(U(0)+1)} - 1 \leqslant (U(0) + 1) \ e^{3\tau C P_{\gamma_1}(U(0)+1)} - 1$. Repeating our arguments, after a finite number of steps we obtain in the interval $((k-1)\tau, k\tau]$

$$U^\tau(t) \leqslant (U(0) + 1)e^{(2k-1)\tau C P_{\gamma_1}(U(0)+1)} - 1 \leqslant (U(0) + 1)e^{2t^* C P_{\gamma_1}(U(0)+1)} - 1 = K.$$

This implies in the strip $G_{[0,t^*]}$ the uniform on τ boundedness of the function u^τ and its derivatives with respect to x up to order $p + 2$ inclusive.

From the above estimates it also follows the uniform in τ boundedness of the derivatives $\dfrac{\partial}{\partial t}\dfrac{\partial^j u^\tau}{\partial x^j}, \dfrac{\partial}{\partial x}\dfrac{\partial^j u^\tau}{\partial x^j}, \ j = 0, 1, \ldots, p$. It presents sufficient condition in order for sets of functions $u^\tau, u^\tau_x, u^\tau_{xx}, \ldots, \dfrac{\partial^p u^\tau}{\partial x^p}$ to be equicontinuous in $G^N_{[0,t^*]} = \{(t,x) | t \in [0, t^*], |x| \leqslant N\}$ for any fixed constant N.

By Arzela's theorem, there is some subsequence u^{τ_k} of sequence u^τ that converges in $G^N_{[0,t^*]}$ with its derivatives to order p to certain function $u(t,x)$. By the convergence theorem of the method of weak approximation, the function $u(t,x) = \lim_{k \to \infty} u^{\tau_k}(t,x)$. By virtue of the arbitrariness of N it belongs to the class

$$C^{1,p}_{t,x}(G_{[0,t^*]}) = \{u(t,x) | \frac{\partial u}{\partial t}, \frac{\partial^j}{\partial x^j}u(t,x) \in C(G_{[0,t^*]}), j = 0, 1, \ldots, p\}, \quad (43)$$

and it is a solution of (10), (11). The inequality $\sum\limits_{j=0}^{p}\left|\dfrac{\partial^j}{\partial x^j}u(t,x)\right| \leqslant C$, is true, i.e. $u(t,x) \in Z^p_x([0,t^*])$. $\qquad\square$

3.   Example

As an example, we consider the inverse problem for the Burgers-type equation which has been studied earlier by Belov and Korshun.

In strip $\Pi_{[0,T]} = \{(t,x)|0 \leqslant t \leqslant T, -\infty < x < \infty\}$ we consider the following Burgers-type equation

$$u_t(t,x) = \mu(t)u_{xx} + A(t)uu_x + B(t)u + C(t) + g(t)f(t,x), \qquad (44)$$

where $A(t)$, $B(t)$, $C(t)$ and $f(t,x)$ are given functions and initial condition is

$$u(0,x) = u_0(x), \quad -\infty < x < \infty. \qquad (45)$$

The functions $u(t,x) and\ g(t)$ are unknown. Let us assume that the overdetermination conditions are

$$u(t,x_0) = \phi(t), \quad x_0 = \text{const}, \qquad (46)$$

and consistency condition is $\phi(0) = u_0(x_0)$.

We also suggest that input data satisfy the following conditions

$$\sum_{k=0}^{6}\left|\frac{d^k u_0(x)}{dx^k}\right| + \sum_{k=0}^{6}\left|\frac{\partial^k f(t,x)}{\partial x^k}\right| + |A(t)| + |B(t)| + |C(t)| + |\psi(t)| \leqslant K, |f(t,x_0)| \geqslant \frac{1}{K}, \ K = \text{const} > 0,$$

where $\psi(t) = \phi'(t) - B(t)\phi(t) - C(t)$.

With the use of the overdetermination conditions (46) problem (44), (45) is reduced to the auxiliary direct problem of the form

$$u_t(t,x) = \mu(t)u_{xx} + A(t)uu_x + B(t)u + C(t) +$$
$$+ \frac{\psi(t) - \mu(t)u_{xx}(t,x_0) + A(t)\phi(t)u_x(t,x_0)}{f(t,x_0)}f(t,x), \quad (47)$$

$$u(0,x) = u_0(x). \qquad (48)$$

In this example the functions $b(t,x,u(t,x),\omega(t))$ and $f(t,x,u(t,x),\omega(t))$ from equation (10) have the form $b(t,x,u(t,x),\omega(t)) = A(t)u(t,x)$,

$$f(t,x,u(t,x),\omega(t)) = B(t)u + C(t) + \frac{\psi(t) - \mu(t)u_{xx}(t,x_0) + A(t)\phi(t)u_x(t,x_0)}{f(t,x_0)}f(t,x).$$

Conditions 2.1 and 2.2 are fulfilled. Parameter $\gamma_1 = 1$ in condition 2.2 and we have

$$\sum_{k=0}^{4}\left|\frac{\partial^k}{\partial x^k}\left(B(t)u + C(t) + \frac{\psi(t) - \mu(t)u_{xx}(t,x_0) + A(t)\phi(t)u_x(t,x_0)}{f(t,x_0)}f(t,x)\right)\right| =$$

$$= |C(t)| + \sum_{k=0}^{4}\left|B(t)\frac{\partial^k}{\partial x^k}u + \frac{\psi(t) - \mu(t)u_{xx}(t,x_0) + A(t)\phi(t)u_x(t,x_0)}{f(t,x_0)}\frac{\partial^k}{\partial x^k}f(t,x)\right| \leqslant$$

$$\leqslant C + CU(t) + \frac{C + CU(t) + CU(t)}{1/K}C \leqslant C(1 + U(t)).$$

Therefore, parameter $\gamma_2 = 1$ in condition 2.2. The conditions of Theorem 2 are fulfilled. Thus, there is such constant t^*: $0 < t^* \leqslant T$ (it depends on the constants that bound the input data) that the classical solution $u(t, x)$ of direct problem (47), (48) exists in class $Z_x^4([0, t^*])$.

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О существовании решений некоторых задач для нелинейных нагруженных параболических уравнений с данными Коши

Игорь В. Фроленков
Мария А. Даржаа

В данной статье приведены достаточные условия существования решения в классе гладких ограниченных функций задачи Коши для двумерного нагруженного параболического уравнения специального вида (коэффициенты при старших, младших членах и правой части зависят от следов неизвестной функции и ее производных), а также получен аналогичный результат для одномерного нагруженного уравнения типа Бюргерса (уравнение дополнительно содержит нелинейность относительно решения при младшей производной по пространственной переменной) с данными Коши. Для доказательства используется метод слабой аппроксимации.

Ключевые слова: обратная задача, прямая задача, нагруженное уравнение, параболическое уравнение, уравнение типа Бюргерса, метод слабой аппроксимации.